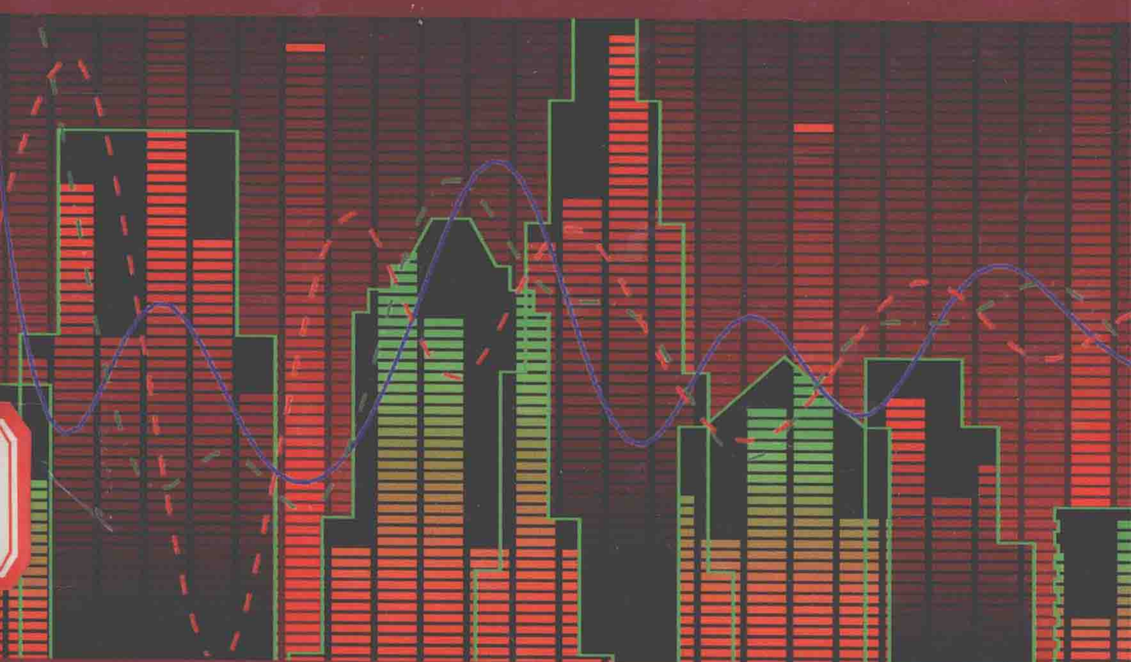
MECHANICAL ENGINEERING AND SOLID MECHANICS SERIES

Structural Dynamic Analysis with Generalized Damping Models

Analysis

Sondipon Adhikari



ISTE

WILEY

Structural Dynamic Analysis with Generalized Damping Models

Analysis

Sondipon Adhikari

Series Editor
Noël Challamel

iSTE

WILEY

First published 2014 in Great Britain and the United States by ISTE Ltd and John Wiley & Sons, Inc.

ISTE Ltd
27-37 St George's Road
London SW19 4EU
UK

www.iste.co.uk

John Wiley & Sons, Inc.
111 River Street
Hoboken, NJ 07030
USA

www.wiley.com

Library of Congress Control Number: 2013948547

British Library Cataloguing-in-Publication Data
A CIP record for this book is available from the British Library
ISBN: 978-1-84821-521-4

MIX
Paper from responsible sources
FSC® C013604

Printed and bound in Great Britain by CPI Group (UK) Ltd., Croydon, Surrey CR0 4YY

Preface

Among the various ingredients of structural dynamics, damping is one of the least understood topics. The main reason is that unlike the stiffness and inertia forces, damping forces cannot always be obtained from "first principles". The past two decades have seen significant developments in the modeling and analysis of damping in the context of engineering dynamic systems. Developments in composite materials including nanocomposites and their applications in advanced structures, such as new generation of aircrafts and large wind turbines, have led to the need for understanding damping in a better manner. Additionally, the rise of vibration energy harvesting technology using piezoelectric and electromagnetic principles further enhanced the importance of looking at damping more rigorously. The aim of this book is to systematically present the latest developments in the modeling and analysis of damping in the context of general linear dynamic systems with multiple degrees-of-freedom. The focus has been on the mathematical and computational aspects. This book will be relevant to aerospace, mechanical and civil engineering disciplines and various sub-disciplines within them. The intended readers of this book include senior undergraduate students and graduate students doing projects or doctoral research in the field of damped vibration. Researchers, professors and practicing engineers working in the field of advanced vibration will find this book useful. This book will also be useful for researchers working in the fields of aeroelasticity and hydroelasticity, where complex eigenvalue problems routinely arise due to fluid–structure interactions.

There are some excellent books which already exist in the field of damped vibration. The book by Nashif *et al.* [NAS 85] covers various material damping models and their applications in the design and analysis of dynamic systems. A valuable reference on dynamic analysis of damped structures is [SUN 95]. The book by Beards [BEA 96] takes a pedagogical approach toward structural vibration of damped systems. The handbook by Jones [JON 01] focuses on viscoelastic damping and analysis of structures with such damping models. These books represented the state of the art at the time of their publications. Since these publications, significant research works have gone into the dynamics of damped systems. The aim of this book is to cover some of these latest developments. The attention is mainly limited to theoretical and computational aspects, although some references to experimental works are given.

One of the key features of this book is the consideration of general non-viscous damping and how such general models can be seamlessly integrated into the framework of conventional structural dynamic analysis. New results are illustrated by numerical examples and, wherever possible, connections are made to well-known concepts of viscously damped systems. The related title, *Structural Dynamic Analysis with Generalized Damping Models: Identification* [ADH 14b], is complementary to this book, and, indeed, they could have been presented together. However, for practical reasons, it has proved more convenient to present the material separately. There are ten chapters and one appendix in the two books combined covering analysis and identification of dynamic systems with viscous and non-viscous damping.

This current book, *Structural Dynamic Analysis with Generalized Damping Models: Analysis*, deals with the analysis of linear systems with general damping models. Chapter 1 gives an introduction to the various damping models. Dynamics of viscously damped systems are discussed in Chapter 2. Chapter 3 considers dynamics of non-viscously damped single-degree-of-freedom systems in detail. Chapter 4 discusses non-viscously damped multiple degree-of-freedom systems. Linear systems with general non-viscous damping are studied in Chapter 5. Chapter 6 proposes reduced computational methods for damped systems. A method to deal with general asymmetric systems is described in the appendix.

The related book *Structural Dynamic Analysis with Generalized Damping Models: Identification* [ADH 14b] deals with the identification and quantification of damping. Chapter 1 describes parametric sensitivity of damped systems. Chapter 2 describes the problem of identification of viscous damping. The identification of non-viscous damping is detailed in Chapter 3. Chapter 4 gives some tools for the quantification of damping.

This book is the result of the last 15 years of research and teaching in the area of damped vibration problems. Initial chapters started taking shape when I offered a course on advanced vibration at the University of Bristol. The later chapters originated from the research works with numerous colleagues, students, collaborators and mentors. I am deeply indebted to all of them for numerous stimulating scientific discussions, exchanges of ideas and, on many occasions, direct contributions toward the intellectual content of the book. I am grateful to my teachers Professor C. S. Manohar (Indian Institute of Science, Bangalore), Professor R. S. Langley (University of Cambridge) and, in particular, Professor J. Woodhouse (University of Cambridge), who was heavily involved with the works reported in Chapters 2–4 of

[ADH 14b]. I am very thankful to my colleague Professor M. I. Friswell with whom I have a long-standing collaboration. Some joint works are directly related to the content of this book (Chapter 1 of [ADH 14b] in particular). I would also like to thank Professor D. J. Inman (University of Michigan) for various scientific discussions during his visits to Swansea. I am thankful to Professor A. Sarkar (Carleton University) and his doctoral student M. Khalil for joint research works. I am deeply grateful to Dr A. S. Phani (University of British Columbia) for various discussions related to damping identification and contributions toward Chapters 2 and 5 of this book and Chapter 2 of [ADH 14b]. Particular thanks go to Dr N. Wagner (Intes GmbH, Stuttgart) for joint works on non-viscously damped systems and contributions in Chapter 4 of this book. I am also grateful to Professor F. Papai for involving me in research works on damping identification. My former PhD students B. Pascual (contributed in Chapter 6), J. L. du Bois and F. A. Diaz De la O deserve particular thanks for various contributions throughout their time with me and putting up with my busy schedules. I am grateful to Dr Y. Lei (University of Defense Technology, Changsha) for carrying out joint research with me on non-viscously damped continuous systems. I am grateful to Professor A. W. Lees (Swansea University), Professor N. Lieven, Professor F. Scarpa (University of Bristol), Professor D. J. Wagg (University of Sheffield), Professor S. Narayanan (Indian Institute of Technology (IIT) Madras), Professor G. Litak (Lublin University), E. Jacquelin (Université Lyon), Dr A. Palmeri (Loughborough University), Professor S. Bhattacharya (University of Surrey), Dr S. F. Ali (IIT Madras), Dr R. Chowdhury (IIT Roorkee), Dr P. Duffour (University College London), and Dr P. Higino, Dr G. Caprio and Dr A. Prado (Embraer Aircraft) for their intellectual contributions and discussions at different times. Besides the names mentioned here, I am also thankful to many colleagues, fellow researchers and students working in this field of research around the world, whose names cannot be listed here due to page limitations. The lack of explicit mentions by no means implies that their contributions are any lesser. The opinions presented in the book are entirely mine, and none of my colleagues, students, collaborators and mentors have any responsibility for any shortcomings.

I have been fortunate to receive grants from various companies, charities and government organizations including an Advanced Research Fellowship from UK Engineering and Physical Sciences Research Council (EPSRC), the Wolfson Research Merit Award from the Royal Society and the Philip Leverhulme Prize from the Leverhulme Trust. Without these findings, it would have been impossible to have conducted the works leading to this book. Finally, I want to thank my colleagues at the College of Engineering at Swansea University. Their support proved to be a key factor in materializing the idea of writing this book.

Last, but by no means least, I wish to thank my wife Sonia and my parents for their constant support, encouragement and putting up with my ever-increasing long periods of "non-engagement" with them.

Sondipon ADHIKARI
October 2013

Nomenclature

C'_{jj}	diagonal element of the modal damping matrix
$\alpha_k^{(j)}$	terms in the expansion of approximate complex modes
α_1, α_2	proportional damping constants
α_j	coefficients in Caughey series, $j = 0, 1, 2, \cdots$
$\mathbf{0}_j$	a vector of j zeros
$\mathbf{A}$	state-space system matrix
$\mathbf{a}_j$	a coefficient vector for the expansion of jth complex mode
$\boldsymbol{\alpha}$	a vector containing the constants in Caughey series
$\bar{h}(\mathrm{i}\omega)$	frequency response function of an SDOF system
$\mathbf{B}$	state-space system matrix
$\mathbf{b}_j$	a vector for the expansion of jth complex mode
$\bar{\mathbf{f}}(s)$	forcing vector in the Laplace domain
$\bar{\mathbf{f}}'(s)$	modal forcing function in the Laplace domain
$\bar{\mathbf{p}}(s)$	effective forcing vector in the Laplace domain
$\bar{\mathbf{q}}(s)$	response vector in the Laplace domain
$\bar{\mathbf{u}}(s)$	Laplace transform of the state-vector of the first-order system
$\bar{\mathbf{y}}(s)$	modal coordinates in the Laplace domain
$\bar{\mathbf{y}}_k$	Laplace transform of the internal variable $\mathbf{y}_k(t)$
$\mathbb{R}^+$	positive real line
$\mathbf{C}$	viscous damping matrix
$\mathbf{C}'$	modal damping matrix
$\mathbf{C}_0$	viscous damping matrix (with a non-viscous model)
$\mathbf{C}_k$	coefficient matrices in the exponential model for $k = 0, ..., n$, where n is the number of kernels

$\mathcal{G}(t)$	non-viscous damping function matrix in the time domain
$\Delta\mathbf{K}$	error in the stiffness matrix
$\Delta\mathbf{M}$	error in the mass matrix
β	non-viscous damping factor
β_c	critical value of β for oscillatory motion, $\beta_c = \frac{1}{3\sqrt{3}}$
$\beta_i(\bullet)$	proportional damping functions (of a matrix)
$\beta_k(s)$	coefficients in the state-space modal expansion
β_{mU}	the value of β above which the frequency response function always has a maximum
$\mathbf{F}$	linear matrix pencil with time step in state-space, $\mathbf{F} = \mathbf{B} - \frac{h}{2}\mathbf{A}$
$\mathbf{F}_1, \mathbf{F}_2$	linear matrix pencils with time step in the configuration space
$\mathbf{F}_j$	regular linear matrix pencil for the jth mode
$\mathbf{f}'(t)$	forcing function in the modal coordinates
$\mathbf{f}(t)$	forcing function
$\mathbf{G}(s)$	non-viscous damping function matrix in the Laplace domain
$\mathbf{G}_0$	the matrix $\mathbf{G}(s)$ at $s \to 0$
$\mathbf{G}_\infty$	the matrix $\mathbf{G}(s)$ at $s \to \infty$
$\mathbf{H}(s)$	frequency response function matrix
$\hat{\mathbf{u}}_j$	real part of $\hat{\mathbf{z}}_j$
$\hat{\mathbf{v}}_j$	imaginary part of $\hat{\mathbf{z}}_j$
$\hat{\mathbf{z}}_j$	jth measured complex mode
$\mathbf{I}$	identity matrix
$\mathbf{K}$	stiffness matrix
$\mathbf{M}$	mass matrix
$\mathbf{O}_{ij}$	a null matrix of dimension $i \times j$
$\mathbf{\Omega}$	diagonal matrix containing the natural frequencies
$\mathbf{p}$	parameter vector (in [ADH 14b], Chapter 1)
$\mathbf{P}_j$	a diagonal matrix for the expansion of jth complex mode
$\boldsymbol{\phi}_j$	eigenvectors in the state-space
$\boldsymbol{\psi}_j$	left eigenvectors in the state-space
$\mathbf{q}(t)$	displacement response in the time domain
$\mathbf{q}_0$	vector of initial displacements
$\mathbf{Q}_j$	an off-diagonal matrix for the expansion of jth complex mode
$\mathbf{r}(t)$	forcing function in the state-space
$\mathbf{R}_k$	rectangular transformation matrices (in Chapter 4)

$\mathbf{R}_k$	residue matrix associated with pole s_k
$\mathbf{S}$	a diagonal matrix containing eigenvalues s_j
$\mathbf{T}$	a temporary matrix, $\mathbf{T} = \sqrt{\mathbf{M}^{-1}\mathbf{K}}$ ([ADH 14b], Chapter 2)
$\mathbf{T}_k$	Moore-Penrose generalized inverse of $\mathbf{R}_k$
$\mathbf{T}_k$	a transformation matrix for the optimal normalization of the kth complex mode
$\mathbf{\Theta}$	normalization matrix
$\mathbf{u}(t)$	the state-vector of the first-order system
$\mathbf{u}_0$	vector of initial conditions in the state-space
$\mathbf{u}_j$	displacement at the time step j
$\mathbf{v}(t)$	velocity vector $\mathbf{v}(t) = \dot{\mathbf{q}}(t)$
$\mathbf{v}_j$	a vector of the j-modal derivative in Nelson's methods (in [ADH 14b], Chapter 1)
$\mathbf{v}_j$	velocity at the time step j
ε_j	error vector associated with jth complex mode
$\varphi_k(s)$	eigenvectors of the dynamic stiffness matrix
$\mathbf{W}$	coefficient matrix associated with the constants in Caughey series
$\mathbf{X}$	matrix containing the undamped normal modes $\mathbf{x}_j$
$\mathbf{x}_j$	undamped eigenvectors, $j = 1, 2, \cdots, N$
$\mathbf{y}(t)$	modal coordinate vector (in Chapter 2)
$\mathbf{y}_k(t)$	vector of internal variables, $k = 1, 2, \cdots, n$
$\mathbf{y}_{k,j}$	internal variable $\mathbf{y}_k$ at the time step j
$\mathbf{Z}$	matrix containing the complex eigenvectors $\mathbf{z}_j$
$\mathbf{z}_j$	complex eigenvectors in the configuration space
ζ	diagonal matrix containing the modal damping factors
ζ_v	a vector containing the modal damping factors
χ	merit function of a complex mode for optimal normalization
χ_R, χ_I	merit functions for real and imaginary parts of a complex mode
Δ	perturbation in the real eigenvalues
δ	perturbation in complex conjugate eigenvalues
$\dot{q}_0$	initial velocity (SDOF systems)
ϵ	small error
η	ratio between the real and imaginary parts of a complex mode
$\mathcal{F}$	dissipation function
γ	non-dimensional characteristic time constant

γ_j	complex mode normalization constant
γ_R, γ_I	weights for the normalization of the real and imaginary parts of a complex mode
$\hat{\theta}(\omega)$	frequency-dependent estimated characteristic time constant
$\hat{\theta}_j$	estimated characteristic time constant for jth mode
$\hat{t}$	an arbitrary independent time variable
κ_j	real part of the complex optimal normalization constant for the jth mode
λ	complex eigenvalue corresponding to the oscillating mode (in Chapter 3)
λ_j	complex frequencies MDOF systems
$\mathcal{M}_r$	moment of the damping function
$\mathcal{D}$	dissipation energy
$\mathcal{G}(t)$	non-viscous damping kernel function in an SDOF system
$\mathcal{T}$	kinetic energy
$\mathcal{U}$	potential energy
μ	relaxation parameter
μ_k	relaxation parameters associated with coefficient matrix $\mathbf{C}_k$ in the exponential non-viscous damping model
ν	real eigenvalue corresponding to the overdamped mode
$\nu_k(s)$	eigenvalues of the dynamic stiffness matrix
ω	driving frequency
ω_d	damped natural frequency of SDOF systems
ω_j	undamped natural frequencies of MDOF systems, $j = 1, 2, \cdots, N$
ω_n	undamped natural frequency of SDOF systems
$\omega_{\max}$	frequency corresponding to the maximum amplitude of the response function
ω_{d_j}	damped natural frequency of MDOF systems
ρ	mass density
i	unit imaginary number, $\mathrm{i} = \sqrt{-1}$
τ	dummy time variable
θ_j	characteristic time constant for jth non-viscous model
$\tilde{\mathbf{f}}(t)$	forcing function in the modal domain
$\tilde{\omega}$	normalized frequency ω/ω_n
ς_j	imaginary part of the complex optimal normalization constant for the jth mode

ϑ	phase angle of the response of SDOF systems
ϑ_j	phase angle of the modal response
ψ	a trail complex eigenvector (in Chapter 2)
$\widehat{\mathbf{A}}$	asymmetric state-space system matrix
$\widehat{\mathbf{C}}$	fitted damping matrix
$\widehat{f}(\omega_j)$	fitted generalized proportional damping function (in [ADH 14b], Chapter 2)
$\widetilde{\mathbf{A}}$	state-space system matrix for rank-deficient systems
$\widetilde{\mathbf{B}}$	state-space system matrix for rank-deficient systems
$\widetilde{\mathbf{i}}_r$	integration of the forcing function in the state-space for rank-deficient systems
$\widetilde{\mathbf{i}}_r$	integration of the forcing function in the state-space
$\widetilde{\mathbf{\Phi}}$	matrix containing the state-space eigenvectors for rank-deficient systems
$\widetilde{\phi}_j$	eigenvectors in the state-space for rank-deficient systems
$\widetilde{\mathbf{r}}(t)$	forcing function in the state-space for rank-deficient systems
$\widetilde{\mathbf{u}}(t)$	the state vector for rank-deficient systems
$\widetilde{\mathbf{y}}_k(t)$	vector of internal variables for rank-deficient systems, $k = 1, 2, \cdots, n$
$\widetilde{\mathbf{y}}_{k,j}$	internal variable $\mathbf{y}_k$ at the time step j for rank-deficient systems
$\widetilde{\boldsymbol{y}}_{k_j}$	jth eigenvector corresponding to the kth the internal variable for rank-deficient systems
ξ	a function of ζ defined in equation [3.132]
ζ	viscous damping factor
ζ_c	critical value of ζ for oscillatory motion, $\zeta_c = \frac{4}{3\sqrt{3}}$
ζ_j	modal damping factors
ζ_L	lower critical damping factor
ζ_n	equivalent viscous damping factor
ζ_U	upper critical damping factor
ζ_{mL}	the value of ζ below which the frequency response function always has a maximum
a_k, b_k	non-viscous damping parameters in the exponential model
B	response amplitude of SDOF systems
B_j	modal response amplitude
c	viscous damping constant of an SDOF system
c_k	coefficients of exponential damping in an SDOF system
c_{cr}	critical damping factor

d_j	a constant of the j-modal derivative in Nelson's methods
E	Young's modulus
$f(t)$	forcing function (SDOF systems)
$f_d(t)$	non-viscous damping force
$G(\mathrm{i}\omega)$	non-dimensional frequency response function
$G(s)$	non-viscous damping kernel function in the Laplace domain (SDOF systems)
$g_{(i)}$	scalar damping functions, $i = 1, 2, \cdots$
h	constant time step
$h(t)$	impulse response function of SDOF systems
$h(t)$	impulse response function
I_k	non-proportionally indices, $k1 = 1, 2, 3, 4$
k	spring stiffness of an SDOF system
L	length of the rod
l_e	length of an element
m	dimension of the state-space for non-viscously damped MDOF systems
m	mass of an SDOF system
N	number of degrees of freedom
n	number of exponential kernels
n_d	number of divisions in the time axis
p	any element in the parameter vector **p** (in [ADH 14b], Chapter 1)
$q(t)$	displacement in the time domain
q_0	initial displacement (SDOF systems)
Q_{nc_k}	non-conservative forces
$R(\mathbf{x})$	Rayleigh quotient for a trail vector **x**
R_1, R_2, R_3	three new Rayleigh quotients
r_j	normalized eigenvalues of non-viscously damped SDOF systems (in Chapter 3)
r_k	rank of $\mathbf{C}_k$ matrices
s	Laplace domain parameter
s_j	eigenvalues of dynamic systems
t	time
T_n	natural time period of an undamped SDOF system
T_{min}	minimum time period for the system
$varrho_j$	complex optimal normalization constant for the jth mode

x	normalized frequency-squared, $x = \omega^2/\omega_n^2$ (in Chapter 3)
y_j	modal coordinates (in Chapter 2)
$\bar{f}(s)$	forcing function in the Laplace domain
$\bar{q}(s)$	displacement in the Laplace domain
$\hat{\mathbf{U}}$	matrix containing $\hat{\mathbf{u}}_j$
$\hat{\mathbf{V}}$	matrix containing $\hat{\mathbf{v}}_j$
$\mathbf{\Phi}$	matrix containing the eigenvectors $\boldsymbol{\phi}_j$
$\dot{\mathbf{q}}_0$	vector of initial velocities
$\mathcal{F}_i(\bullet, \bullet)$	non-viscous proportional damping functions (of a matrix)
$\boldsymbol{Y}_k$	a matrix of internal eigenvectors
$\boldsymbol{y}_{kj}$	jth eigenvector corresponding to the kth the internal variable
PSD	power spectral density
$\mathbf{0}$	a vector of zeros
$\mathcal{L}$	Lagrangian (in Chapter 3)
$\delta(t)$	Dirac-delta function
δ_{jk}	Kroneker-delta function
$\Gamma(\bullet)$	gamma function
γ	Lagrange multiplier (in Chapter 3)
$(\bullet)^*$	complex conjugate of $(\bullet)$
$(\bullet)^T$	matrix transpose
$(\bullet)^{-1}$	matrix inverse
$(\bullet)^{-T}$	matrix inverse transpose
$(\bullet)^H$	Hermitian transpose of $(\bullet)$
$(\bullet)_e$	elastic modes
$(\bullet)_{nv}$	non-viscous modes
$(\dot{\bullet})$	derivative with respect to time
$\mathbb{C}$	space of complex numbers
$\mathbb{R}$	space of real numbers
$\perp$	orthogonal to
$\mathcal{L}(\bullet)$	Laplace transform operator
$\mathcal{L}^{-1}(\bullet)$	inverse Laplace transform operator
$\det(\bullet)$	determinant of $(\bullet)$
$\mathrm{diag}\,[\bullet]$	a diagonal matrix
$\forall$	for all
$\Im(\bullet)$	imaginary part of $(\bullet)$

$\in$	belongs to
$\notin$	does not belong to
$\otimes$	Kronecker product
$\overline{(\bullet)}$	Laplace transform of $(\bullet)$
$\Re(\bullet)$	real part of $(\bullet)$
vec	vector operation of a matrix
$O(\bullet)$	in the order of
ADF	anelastic displacement field model
adj$(\bullet)$	adjoint matrix of $(\bullet)$
GHM	Golla, Hughes and McTavish model
MDOF	multiple-degree-of-freedom
SDOF	single-degree-of-freedom

Table of Contents