

Sensors

Science and Engineering

Marvin Heather



Sensors: Science and Engineering

Edited by Marvin Heather

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Sensors: Science and Engineering

Preface

Detecting events or changes in machines, equipment or environment is becoming highly critical. Such equipment needs reliable, robust and accurate sensors in order to avert risks that could endanger human lives, environment etc. This book discusses the theories and concepts of sensors. Most of the topics introduced herein cover new technologies and applications of sensors. The various studies that are constantly contributing towards advancing technologies and evolution of this field are examined in detail. The extensive content of this book provides the readers with a thorough understanding of working of sensors and its application.

The information contained in this book is the result of intensive hard work done by researchers in this field. All due efforts have been made to make this book serve as a complete guiding source for students and researchers. The topics in this book have been comprehensively explained to help readers understand the growing trends in the field.

I would like to thank the entire group of writers who made sincere efforts in this book and my family who supported me in my efforts of working on this book. I take this opportunity to thank all those who have been a guiding force throughout my life.

Editor

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Fault-Tolerant Algorithms for Connectivity Restoration in Wireless Sensor Networks

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Abstract: As wireless sensor network (WSN) is often deployed in a hostile environment, nodes in the networks are prone to large-scale failures, resulting in the network not working normally. In this case, an effective restoration scheme is needed to restore the faulty network timely. Most of existing restoration schemes consider more about the number of deployed nodes or fault tolerance alone, but fail to take into account the fact that network coverage and topology quality are also important to a network. To address this issue, we present two algorithms named Full 2-Connectivity Restoration Algorithm (F2CRA) and Partial 3-Connectivity Restoration Algorithm (P3CRA), which restore a faulty WSN in different aspects. F2CRA constructs the fan-shaped topology structure to reduce the number of deployed nodes, while P3CRA constructs the dual-ring topology structure to improve the fault tolerance of the network. F2CRA is suitable when the restoration cost is given the priority, and P3CRA is suitable when the network quality is considered first. Compared with other algorithms, these two algorithms ensure that the network has stronger fault-tolerant function, larger coverage area and better balanced load after the restoration.

Keywords: wireless sensor networks; connectivity restoration; fault tolerance

1. Introduction

Wireless sensor networks (WSNs) are known for their wide use in industry, military, and environmental monitoring applications [1]. They are usually deployed in harsh environments, where nodes are subjected to failures and the networks are easy to be partitioned into disjoint segments. Therefore, fault tolerance becomes a critical issue for WSNs and numerous restoration algorithms are proposed [2–6] to address this issue. In order to achieve fault tolerance when restoring a faulty WSN, one approach is to deploy additional relay nodes to provide k ($k > 1$) vertex-disjoint paths (hereinafter referred to as k -connectivity) between every pair of network nodes (segments and relay nodes). In this way, the restored network can survive the failure of fewer than k nodes, which is more practical for WSNs. In this paper, we adopt this approach to repair the faulty network which is divided into many segments.

However, deploying additional relay nodes for network restoration brings us two conflicting requirements: On the one hand, it needs to spend some money to purchase the equipment. In order to save the cost, it is required to place as few nodes as possible to repair the faulty network. On the other hand, as a wireless sensor network is easy to fail, the network after the restoration is required to be with fault-tolerant function so that it can resist the attack and damage in the future. The network, which is constructed by using as few nodes as possible, may not be fault-tolerant, but the network with fault-tolerant function needs to deploy more relay nodes and costs more money. Hence, these two requirements are contradictory. In addition, as for a network, network coverage and topology quality are also important to a network. Therefore, when designing the restoration scheme, we should

consider not only the cost and network fault tolerance, but also the other aspects. Only in this way can the network after the restoration be more practical.

1.1. Our Contributions

In this paper, we comprehensively consider the restoration cost, fault tolerance, network coverage and topology quality. We seek to use fewer nodes to establish a network with fault-tolerant function under the premise of multiple segments that are unable to communicate with each other. Meanwhile, except for the restoration cost and fault tolerance, we also consider the network coverage, the quality of topology and others in this paper, so as to ensure that the network can not only has better fault tolerance, but also has stronger robustness and higher coverage after the restoration. Certainly, these performances are not considered fully in the existing literature. The algorithms we propose in this paper are summarized as follows:

- (1) Full 2-Connectivity Restoration Algorithm (F2CRA) provides two vertex-disjoint paths between every pair of network nodes. This algorithm is suitable when the cost is considered first.
- (2) Partial 3-Connectivity Restoration Algorithm (P3CRA) provides three vertex-disjoint paths between every pair of segments and at least two vertex-disjoint paths between every pair of relay nodes. This algorithm is suitable when the fault tolerance, network coverage and topology quality are considered first.

1.2. Paper Organization

The remainder of this paper is organized as follows. Section 2 reviews some related works. Section 3 proposes the system model and preliminaries. Our algorithms are introduced in Section 4. Sections 5 and 6 conduct the theory and simulation analysis for our algorithms, respectively. Finally, we conclude this paper in Section 7.

2. Related Work

WSNs are prone to failures due to the hostile environments where they are deployed. How to recover a faulty WSN is an important issue that has attracted numerous researches. We summarize some existing restoration algorithms in Table 1. In the connected relay node placement problem, the aim is to ensure the network is connected ($k = 1$) [7–13], while in the survivable relay node placement problem, the aim is to ensure k -connectivity ($k > 1$) [2–6,14–18]. k -connectivity can be either full or partial [19]. Full k -connectivity implies that k node-disjoint paths exist between every pair of nodes, while partial fault-tolerance requires k -connectivity between original nodes (segments) only.

Table 1. Relay placement algorithms.

Algorithms	k	Deployment Locations	Fault-Tolerance	Network Types
Lloyd [9]	$k = 1$	Unconstrained	No	Homogeneous
Li [10]	$k = 1$	Unconstrained	No	Heterogeneous
Bhattacharya [13]	$k = 1$	Constrained	No	Homogeneous
Yang [11]	$k = 1, 2$	Constrained	Full	Hierarchical
Hao [2]	$k > 1$	Unconstrained	Partial	Hierarchical
Zhang [3]	$k = 2$	Unconstrained	Full	Hierarchical
Han [4]	$k > 1$	Unconstrained	Full, Partial	Heterogeneous
Senel [5]	$k = 2$	Unconstrained	Full	Homogeneous
Our algorithms	$k = 2, 3$	Unconstrained	Full, Partial	Homogeneous

In connectivity problems, most algorithms restore a faulty network by finding the minimum spanning tree or Steiner tree. Lin and Xue [7] show that the STP-MSP problem is NP-hard. They also show that the approximation obtained from the minimum spanning tree has a worst-case performance ratio at most 5, while Chen *et al.* [8] point out that this approximation has a performance

ratio exactly 4. Chen *et al.* also present a new polynomial-time approximation with a performance ratio at most 3. Yang *et al.* [11] study two-tiered constrained relay node placement problems and propose polynomial time approximation algorithms with $O(1)$ -approximation ratios. Lloyd *et al.* [9] study two versions of relay node placement problems, but the same objective of these two versions is to deploy the minimum number of relay nodes. Li *et al.* [10] also has the same objective as [9], but they study the placement problem in a heterogeneous WSN. Although easy to implement, these algorithms are usually not efficient when a failure occurs in the network.

In survivable problems, most algorithms aim to construct a fault-tolerant network topology in a WSN. Hao and Tang *et al.* [2,14] study a fault-tolerant relay node placement problem in a two-tiered network, while Zhang *et al.* [3] study the problem in both single and two tiered networks. Smith *et al.* [2] is further extended to cover k -connectivity in heterogeneous wireless sensor networks in [4] where sensor nodes possess different transmission radii. The same as [4], Misra *et al.* [15] study the placement problem in heterogeneous wireless sensor networks, but [15] studies a constrained version in which relay nodes can only be placed at a set of candidate locations. As many algorithms do in connectivity problem, many restoration algorithms in survivable problem also try to place fewest number of relay nodes in a WSN like [6,16,17].

In a word, most of the aforementioned algorithms try to place minimum relay nodes in a WSN. However, none of them take network quality into account which is also crucial in terms of application-level performance. Therefore, Senel and Lee *et al.* [5,18] opt to reestablish connectivity using the least number of relays while ensuring a certain quality in the formed topology. However, their algorithms produce many overlapped areas and cannot be practical in multiple node failures caused by aftermath. To address these issues, we jointly consider establishing fault-tolerant connectivity and providing large coverage area which has not been studied.

3. System Model and Preliminaries

3.1. System Model

WSN is often deployed in the hostile environment, and sometimes it may suffer from the large-scale damage, resulting in the entire network being divided into multiple segments which cannot communicate with each other. In this paper, the problem we consider is how to repair the faulty WSN composed of multiple segments. As mentioned above, our scheme is to deploy relay nodes between each segment, but this scheme brings us two contradictory requirements: One is to minimize the number of nodes, and the other is to construct a fault-tolerant network. If the segments are regarded as a node, the set of these nodes is defined as S and the set of the deployed relay nodes is defined as P , then our problem can be transformed into the following.

Given a set of nodes (segments) S on a plane with a random distribution, the nodes in the set S cannot communicate with each other. After the set of relay nodes P being added on the plane for the restoration, all the nodes $S \cup P$ can communicate with each other. It requires that (1) the number of relay nodes is minimized; and (2) the network is fault-tolerance after the restoration.

3.2. Preliminaries

Definition 1. *Minimum Convex Hull:* Given a set of nodes S on a plane with a random distribution, the minimal convex polygon which contains all the points is called minimum convex hull (Figure 1a). In this paper, we need to construct two convex hulls. To describe them conveniently, we call this minimum convex hull as outer convex hull here. The outer convex hull is composed of the isolated segments in the network.

Definition 2. *Inner Convex Hull:* In this paper, when the minimum convex hull is found, in order to ensure the network has fault tolerance after the restoration, the other convex hull is built inside the minimum convex hull. This convex hull is called inner convex hull (Figure 1b).

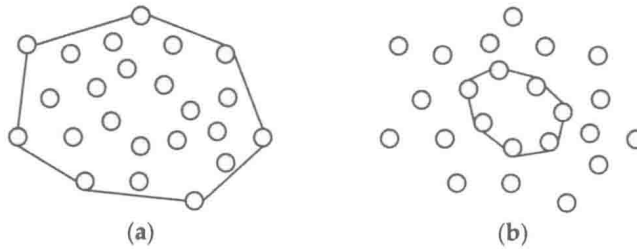


Figure 1. (a) Minimum convex hull; and (b) Inner convex hull.

Definition 3. *Corner Point of Convex Hull:* The points to make up convex hull are called corner points. In outer convex hull, corner points are the isolated segments, while in inner convex hull, corner points are relay nodes.

The notations used in this paper are as follows (Table 2).

Table 2. Notations.

Notation	Description
OCH	Outer Convex Hull
ICH	Inner convex hull
CP	Corner point
O	Center of OCH
R	Radius of relay node
P	Set of relay nodes
S	Set of n segments, $S = \{s_1, s_2, \dots, s_n\}$. The corresponding coordinate set of these n segments is $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$.

4. Algorithms

This section illustrates two proposed algorithms in detail.

4.1. Full 2-Connectivity Restoration Algorithm

Many schemes place too many relay nodes for improving network fault tolerance performance, but other performances like network coverage and topology quality have not been optimized. To address this issue, we propose a new restoration algorithm named Full 2-Connectivity Restoration Algorithm (F2CRA). This algorithm aims to deploy the minimum number of relay nodes to form a full 2-vertex connected network. Meanwhile, the restored network has a larger coverage area and a more balanced load than other schemes. The flow chart of F2CRA is shown in Figure 2.

The steps of F2CRA are as follows (Algorithms 1):

Given the scattered segments set S on the plane, in Step 1 the minimum convex hull composed of these segments is found by the method of Graham scan algorithm. The time complexity of Graham scan algorithm is $O(n \log n)$. By calculating the length from CPs to the center of OCH, in Step 2, we obtain the number of nodes and the accurate deployment position between each CP and O. Step 2 enables the nodes on ICH to form 3-connectivity, and Steps 3 and 4 enable the remaining segments on the plane to form 2-connectivity. Steps 2–4 make the network topology have the fan-shaped structure after the restoration. Compared with other algorithms, the network topology with such structure has better fault tolerance, larger coverage and more balanced load. The time complexity from Step 2 to Step 4 is $O(n)$; therefore, the time complexity of F2CRA algorithm is $O(n \log n)$.

Algorithms 1 F2CRA

INPUT: $R, S = \{s_1, s_2, \dots, s_n\}$ and $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$. P is null.

OUTPUT: A set of relay nodes P .

Step 1. Find OCH.

(1) Adopt the Graham scan algorithm to find OCH in S .

Suppose that CPs set of OCH is $\{s_1, s_2, \dots, s_m\} \subset S$ and their corresponding coordinate is

$\{(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m)\}$. The remaining segments set inside OCH is $\{s_{m+1}, s_{m+2}, \dots, s_n\}$.

(2) Calculate the coordinate (x_0, y_0) of O .

$$x_0 = \frac{1}{m} \sum_{i=1}^m x_i$$

$$y_0 = \frac{1}{m} \sum_{i=1}^m y_i$$

(3) Calculate each side length of OCH.

for $i = 1$ to m **do**

$j = i + 1$

if $i = m$ **then**

$j = 1$

end if

$$side_i = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

end for

Then the set of the side lengths of OCH is $\{side_1, side_2, \dots, side_m\}$.

Step 2. Find ICH.

(1) Line each CP with O , respectively, and calculate the length of each line:

for $i = 1$ to m **do**

$$l_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}$$

Then the set of these lines is $\{l_1, l_2, \dots, l_m\}$

(2) Calculate the angle between two adjacent lines:

for $i = 1$ to m **do**

$j = i + 1$

if $i = m$ **then**

$j = 1$

end if

$$\theta_i = \arccos \left(\frac{l_i^2 + l_{i+1}^2 - side_i^2}{2 \times l_i \times l_{i+1}} \right)$$

Calculate the number of relay nodes to be deployed in each line:

$$x'_i = x_0 \pm \frac{R \times |x_0 - x_i|}{2 \times l_i \times \sin \frac{\theta_i}{2}}$$

$$y'_i = y_0 \pm \frac{R \times |y_0 - y_i|}{2 \times l_i \times \sin \frac{\theta_i}{2}}$$

$$n'_i = \left\lceil \frac{\sqrt{(x_i - x'_i)^2 + (y_i - y'_i)^2}}{R} \right\rceil$$

Start with a CP of OCH s_i , and deploy relay nodes towards O with one relay node every distance R , then get the corresponding deployment position of n'_i nodes. Add these nodes into the set of P and set the last node p_i , the coordinate (x'_i, y'_i) .

end for

Then the CPs set of ICH is $\{p_1, p_2, \dots, p_m\}$.

(3) Deploy the nodes along with the edge of ICH.

for $i = 1$ to m **do**

if $i = m$ **then**

$j = 1$

end if

Deploy relay nodes between p_i and p_{i+1} , and add these nodes into the set of P .

end for

Step 3. Establish 2-connectivity for the segments on OCH.

if $m \% 2 = 0$ **then**

Select $\frac{m}{2}$ non-adjacent sides of shortest total length from the side set $\{side_1, side_2, \dots, side_m\}$,

deploy relay nodes along with these sides, and add these nodes into the set of P .

else

There will be a vertex $u \in S$ not forming 2-connectivity. At this time, find a node v on ICH that is

else

closest to u but not collinear with u , deploy the nodes uniformly between v and u , and add these nodes into the set of P .

end if

Step 4. Establish 2-connectivity for the isolated segments on the plane

for $k = m + 1$ to n **do**

Find the nearest two nodes u' and v' for s_k , $u', v' \in P$. Deploy relay nodes uniformly in s_k and u', s_k and v' , and add these nodes into set P .

end for

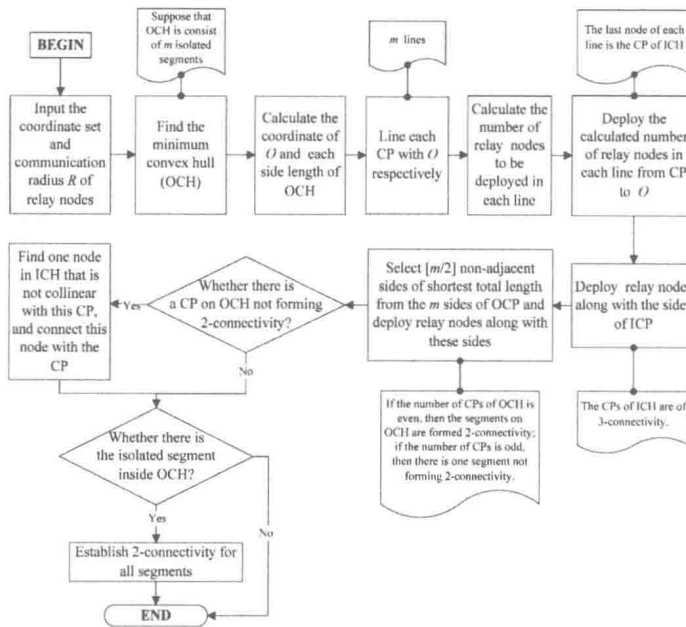


Figure 2. Flow chart of Full 2-Connectivity Restoration Algorithm.

4.2. Partial 3-Connectivity Restoration Algorithm

F2CRA uses fewer nodes to establish a network topology with fault tolerance. Therefore, F2CRA is suitable for the case when the number of available relay nodes is small. When the number of available nodes is sufficient, we can extend F2CRA, so that the network topology can have the stronger fault tolerance after the restoration. Here, we propose an improved algorithm Partial 3-Connectivity Restoration Algorithm (P3CRA). P3CRA is similar to F2CRA, but the network restored by P3CRA will have partial 3-connectivity structure. Partial 3-connectivity means that after the restoration, all the segments have 3-connectivity at least, and the deployed relay nodes have 2-connectivity at least. The network restored by P3CRA has larger coverage and better fault tolerance than that by F2CRA. However, P3CRA needs to deploy more nodes; therefore, P3CRA is suitable when the network quality is taken into consideration first, and F2CRA is suitable when the cost is in consideration. P3CRA flow chart is shown in Figure 3.

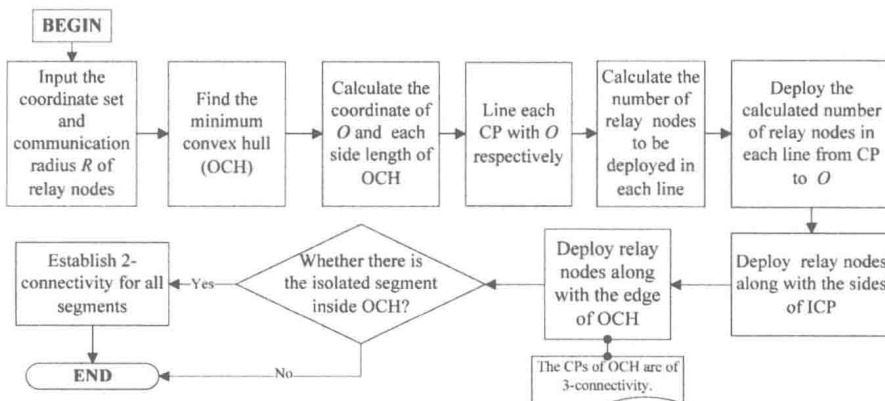


Figure 3. Flow chart of Partial 3-Connectivity Restoration Algorithm.

The steps of P3CRA are as follows (Algorithms 2):

Algorithms 2 P3CRA

INPUT: $R, S = \{s_1, s_2, \dots, s_n\}$ and $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$. P is null.

OUTPUT: A set of relay nodes P .

Step 1 and Step 2 are the same with F2CRA's.

Step 3. Establish 3-connectivity for the segments on OCH.

for $i = 1$ to m **do**

 Deploy relay nodes between s_i and s_{i+1} . (if $i = m$, then $i + 1 = 1$)

end for

Step 4. Establish 3-connectivity for the isolated segments on the plane.

for $k = m + 1$ to n **do**

 Find the nearest three nodes u', v' and w' for s_k . (u', v', w' are not on the same line)

 Deploy nodes uniformly in s_k and u', s_k and v', s_k and w' .

end for

The first two steps are consistent in F2CRA algorithm and P3CRA algorithm, but in Step 3, P3CRA algorithm directly deploys nodes along the edge of OCH. At that time, all CPs (segments) on OCH form 3-connectivity. In Step 4, all segments on the plane eventually form 3-connectivity. Like F2CRA algorithm, the time complexity of P3CRA algorithm is also $O(n \log n)$.

To summarize, the network topology repaired by F2CRA algorithm has 2-connectivity. As it needs fewer nodes, this algorithm is suitable when the cost is considered first. Compared with F2CRA algorithm, P3CRA algorithm needs to deploy more nodes. Due to the stronger fault tolerance, larger coverage and more balanced load, P3CRA algorithm is applicable when the performance of network is considered first.

5. Algorithm Analysis

It is known that the coordinates of CPs and the center coordinate of OCH are $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$ and (x_0, y_0) , respectively, and the value of communication radius of relay nodes is R . Assume that nodes are deployed every distance R . When the CP coordinate of ICH is $(x_0 \pm \frac{R \times |x_0 - x_i|}{2 \times l_i \times \sin \frac{\theta_i}{2}}, y_0 \pm \frac{R \times |y_0 - y_i|}{2 \times l_i \times \sin \frac{\theta_i}{2}})$, the restoration algorithm will use the minimum number of relay nodes.

Proof. As shown in Figure 4, we assume the coordinates of point A, B , and O are $(x_1, y_1), (x_2, y_2)$, and (x_0, y_0) , respectively. Here point A and point B represent the different CPs of OCH, and point O represents the center of OCH. Our algorithm deploys relay nodes from the CPs (A and B) to the central point (O). As the values of AB, AO, BO and R are fixed, to minimize the nodes, it requires the total length of AD and BE to be the shortest, that is, the total length of OD and OE is the longest. Consequently, the problem is transformed into: Seeking the coordinate values of point D and E when the total length of OD and OE is the longest.

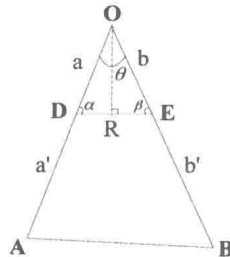


Figure 4. Diagram of triangle.

Set $\angle AOB = \theta$, the lengths of DE , OD and are R , a and b , respectively. $\angle ODE = \alpha$, $\angle OED = \beta$, a , b , α , β are unknown. When a is equal to b , $a + b$ reaches the maximum value, which means the total length of OD and OE is the longest. The detailed argument is relegated to the Appendix.

When $a = b$, by trigonometric function, we have:

$$a = b = \frac{\frac{R}{2}}{\sin \frac{\theta}{2}} = \frac{R}{2\sin \frac{\theta}{2}} \quad (1)$$

Assume the coordinate of point D is (x'_1, y'_1) , then:

$$\sqrt{(x'_1 - x_0)^2 + (y'_1 - y_0)^2} = \frac{R}{2\sin \frac{\theta}{2}} \quad (2)$$

As D is on line AO , then we have:

$$\frac{y_0 - y_1}{x_0 - x_1} = \frac{y_0 - y'_1}{x_0 - x'_1} \quad (3)$$

By Equations (2) and (3), we have:

$$x'_1 = x_0 \pm \frac{R \times |x_0 - x_1|}{2 \times l_1 \times \sin \frac{\theta}{2}} \quad (4)$$

$$y'_1 = y_0 \pm \frac{R \times |y_0 - y_1|}{2 \times l_1 \times \sin \frac{\theta}{2}} \quad (5)$$

If $x_1 < x_0$, then:

$$x'_1 = x_0 - \frac{R \times |x_0 - x_1|}{2 \times l_1 \times \sin \frac{\theta}{2}} \quad (6)$$

Otherwise:

$$x'_1 = x_0 + \frac{R \times |x_0 - x_1|}{2 \times l_1 \times \sin \frac{\theta}{2}} \quad (7)$$

y'_1 is similar to x'_1 , and the method to get the coordinate of E is similar to that of D . That is, when the coordinates of D and E are, respectively, $(x_0 \pm \frac{R \times |x_0 - x_1|}{2 \times l_1 \times \sin \frac{\theta}{2}}, y_0 \pm \frac{R \times |y_0 - y_1|}{2 \times l_1 \times \sin \frac{\theta}{2}})$ and $(x_0 \pm$

$\frac{R \times |x_0 - x_2|}{2 \times l_2 \times \sin \frac{\theta}{2}}, y_0 \pm \frac{R \times |y_0 - y_2|}{2 \times l_2 \times \sin \frac{\theta}{2}})$, the total length of OD and OE is the longest, the total length of AD and BE is the shortest, and the number of nodes is the least.

To summarize, the number of the nodes can be the least when the CP coordinate of ICH is

$$(x_0 \pm \frac{R \times |x_0 - x_i|}{2 \times l_i \times \sin \frac{\theta_i}{2}}, y_0 \pm \frac{R \times |y_0 - y_i|}{2 \times l_i \times \sin \frac{\theta_i}{2}})$$