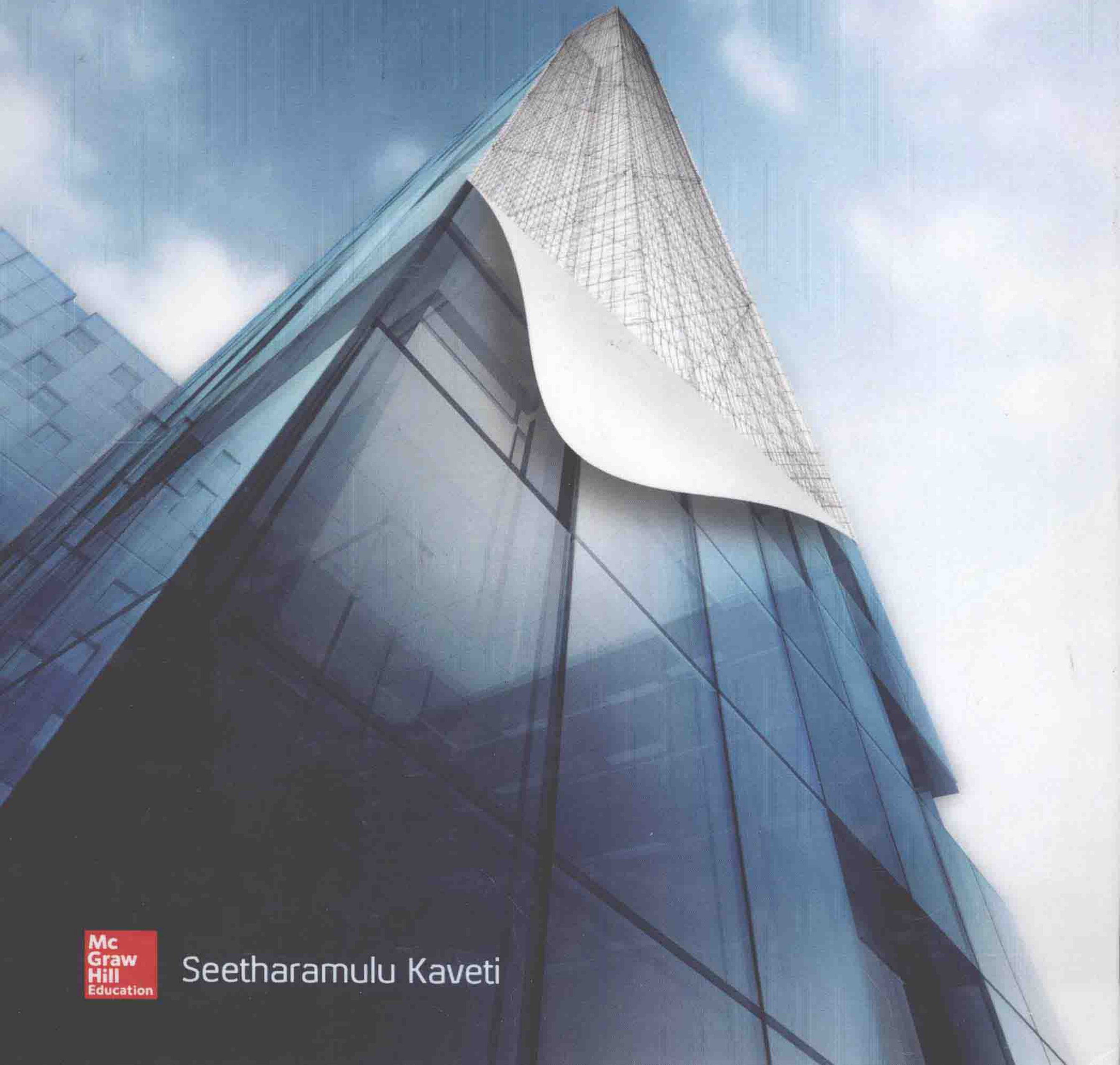
Dynamic Analysis of SKELETAL STRUCTURES

Force and Displacement Methods and Iterative Techniques



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Force and Displacement Methods and Iterative Techniques

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To

My wife Kanchanamala

Preface

The proposed book, even though is essentially Analysis of Civil Engineering Skeletal Structures, the Design and Construction aspects are indicated for a proper appreciation, relevance and motivation for the analytical coverage. The Skeletal Structures constitute an idealization of Structures. The conceptual background of which simplifies the analysis without sacrificing accuracy required for the Design purposes. The skeletal modeling of the structures assumes the cross-sectional dimensions are 'small' compared to the length, and it covers practically all types of structures encountered in the design and construction practices of structures. The numerical examples are selectively chosen as illustrations to complement the theory. In the computer era, too many numerical examples obscure the basic concepts.

It was way back in 1864, J.C. Maxwell suggested 'Force Method', and in 1862, A. Clebsch presented Displacement Method. In the later years, because of over enthusiasm with various simplifications for design solutions, the emphasis shifted to different procedures of analysis suitable for the commonly used structures. In the case of structures with constraints more than the minimum required for stability (indeterminate structures), a justifiable attempt was to suitably locate points of inflection (zero curvature), points of zero moments, which facilitated the evaluation of internal forces from simple statics. The slope-deflection method is a simplified version of the displacement method where the skeletal members are assumed inextensible; the axial deformations have negligible effect on the flexural forces-bending moments and shears. The difficulty of evaluating the large number of unknown displacements was simplified by the iterative procedure like "The Moment Distribution" based on the assumption of inextensibility.The numerical algorithm of the moment distribution procedure is important for the structural understanding it gives from the structural behavior introduced by way of clamping of the joints, releasing and balancing of the joints in a sequence, and carrying over the effect of balancing.

The book is structured to highlight the unification of structural analysis based on, (1) Force Method, and (2) Displacement Method. The few examples chosen are complementary to the understanding of concepts introduced. The shear walls in the tall buildings are treated as a particular type of skeletal member. The member flexibility and the member stiffness are illustrated so as to prepare the reader for *finite element analysis*. There are a number of good books available on the finite element analysis and to limit the scope of the book, the finite element idealization has not been included. The modern trend of the earthquake design is to base the earthquake analysis on *capacity* considerations. The uniqueness theorem used for enabling plastic analysis is yet another great concept which is adequately explained by the Force Method. The plastic analysis has been included with an additional objective of preparing the readers for a proper appreciation of capacity of structures for the earthquake design of tall structures. The Force method is ideally suited for the comprehension of all these concepts. An important aspect of stiffening of structures for their increased capacities is included. The stiffening of the framed structures cannot be divorced from an important aspect of ductility. It is the ductility that ensures the structure reaching its full capacity, the collapse load. A good design limits the development of the plastic hinges in the girders. The shear effect in mobilizing the full plastic moment of resistance is not crucial. An Introduction to applied dynamics has also been included for preparing the students to earthquake design of tall buildings. A section has been devoted to the maintenance of large structural systems. The Tall Buildings constitute such a system.

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Introduction

The contents are broadly categorized under Force Method, Displacement Method, and Iterative Techniques; with a brief introduction to Structural Dynamics and its application to a dedicated structure, Tall Buildings. The force method is also known as compatibility method because compatibility conditions are used while forming the equations for the unknown forces. The number of (redundant) unknown forces is equal to the number of excess constraints over and above the minimum constraints required for achieving a stable structure. Whereas the displacement method, also known as equilibrium method, is used for solving the unknown displacements equal to the degrees of freedom of a chosen system. The system of equations obtained from equilibrium conditions of the free bodies, which are generally the joints or appropriately chosen parts of the total structure. Therefore, the displacement method is also known as the 'Equilibrium Method'. The force method is more conceptual as it helps to determine whether the structure to be analyzed is stable or unstable. It is only when the structure is stable; we proceed to find the unknown redundant forces. The number of the unknowns defines the degree of indeterminacy of any given structure. The degree of indeterminacy is *unique*. But the type of releases, their number equal to the degree of indeterminacy, depends on the choice by the engineer analyzing the structure. A proper choice of the releases results in the compatibility equations that are well conditioned (dominance of the leading diagonals of the equations). The method, therefore, demands a conceptual preparation from the engineers while choosing the system of releases. It is this aspect of analysis that prepares a thinking designer leading to good conceptualization.

In the case of displacement method, generally, the equilibrium equations do not result in any kind of ill conditioning of the stable structures. The method, however presupposes the structures are stable. The indicators whether the structure is stable or not, are provided by the unpredictable results; deflections, internal forces, and unusual values of frequencies. The method does not put any kind of demand on the engineers while making the choice of the nodes and the corresponding unknown displacements and their number. The degrees of freedom equal to the number of unknown displacements are essentially caused by the applied static loads or the externally applied forces. In the case of time varying loads, the analysis is referred as the dynamic analysis for determining the corresponding time dependent displacements. An important concept of geometric loading is introduced as one of the loading cases. It is sometimes expedient to use a mixed method which combines the two basic methods, i.e., force and displacement methods. These methods are developed. The examples chosen complement the presentation of the basic theories. The examples are selectively chosen which provide leads to the design of dedicated structures like tall buildings or bridges or large span structures, etc.

Appropriate techniques for evaluating the unknowns are developed if the solution of the equations is to be obtained from the manual calculations. These are essentially based on numerical techniques; iteration (Kani's Method) and incremental iteration (Hardy Cross Moment-Distribution Method). The conceptualization of structural analysis is incomplete without discussing the energy principles. The contents include: 1) Virtual work principles and 2) Complementary virtual work principles.

The entire book is based on linear analysis, i.e., Small Deformations. The other aspect of linearity pertains to the material linearity (elasticity). Except for the plastic analysis of structures where the plasticity is assumed to be localized at the plastic hinges, the skeletal members are assumed elastic. The basic concepts developed are important for the design of structures which are not only safe but result in structurally efficient and by and large economic structures. The topics pertaining to the structures which undergo large deformations, like cable structures (a typical case of skeletal configuration) and tension structures (a typical example is the textile tension membranes and the cutting patterns) being geometrically nonlinear, do not fall under the scope of the book.

The book comprises two parts. The part-A pertains to Force Method and it consists of five chapters.

Part A

Chapter 1–General Concepts and Energy Principles

After introducing general concepts pertaining to broad classification of structures and basic assumptions, the energy principles are introduced which are later used as virtual work principles. The principles are used in the development of analysis, particularly Chapter 4, based on the energy principles for presenting a unified force method.

Chapter 2–Force Method

The concepts pertaining to kinematical constraint conditions are fundamental to arriving at stable structures, trusses, beams, plane frames, grids, and space frames. A good understanding of stable and unstable structures is achieved while determining, Determinacy, and Indeterminacy. The contents of the chapter are oriented to lead the reader to plastic analysis in Chapter 3 and matrix approach in Chapter 4. A proper appreciation and understanding of the Chapter 2 are the essential requisites in the making of a Structural Engineer.

Chapter 3–Plastic Analysis

The scope of the plastic analysis is limited to skeletal structures and it is based on small deformations. The number of basic mechanisms is related to the degree of determinacy of skeletal structures. The lower bound approach is explained comprehensively with the help of technique based on virtual stiffening. The concept of the virtual stiffening is used for enhancing the collapse load by appropriately stiffening at the zones of maximum moments. This concept of stiffening, however, reduces the overall ductility which is an important parameter for the pushover design of earthquake resistant structures. The Uniqueness theorem and the Upper bound and Lower bound are the important conceptual contents.

Chapter 4–Matrix Approach for Force Method

It is an alternative to the scalar approach developed in the Chapters 2 and 3.

For the computer based analysis of structures, it is a requirement to present the steps of analysis in the matrix form. This has been achieved by presenting the matrix formulation analogous to scalar approach presented in Chapter 2. An alternative formulation has been presented as a unified force method based on energy principles. The typical examples of the indeterminate structures considered to illustrate the method are hinged trusses, pontoons treated as beams resting on interior supports as flexible and end supports hinged, rigid frames like plane frames, grids, space frames, rigid cum hinged frames likes trussed beams or trussed frames.

Chapter 5–Approximate Methods of Analysis

It is an illustration of various approximations introduced for the analysis of a dedicated structure. Here in, the dedicated structure chosen is a tall skeletal building frame; the concepts are introduced by way of appropriately assuming the inflexion points (zero bending moment). The location of inflexion points are enabled by sketching deflected shapes (elastic lines).The elastic line of tall skeletal structures is used not only for the bending moment diagrams but also for obtaining the influence lines. The concept is used for the loading patterns for the design purposes. To enable critical moments, shears and thrusts at chosen points, this important aspect remains totally obscured when a designer is totally dependent on the commercially available computer software. The principle of virtual work is employed to compute floor deflections.

Part B

It comprises Displacement Method, Iterative Techniques, and Introduction of Applied Dynamics and Earthquake Design of Tall Structures.

Chapter 6–Displacement Method and Chapter 7–Displacement Method-Illustrations

In as much as choice of release system is important for the force method, the modeling of a given structure before the analysis in respect of nodes and joints need to be done with care while using the Displacement Method. The degrees of freedom equal to the unknown displacements depends on the number of joints and locations of the nodes at locations other than the joints. The illustrations considered are mostly straight prismatic skeletal members, and these include trusses, beams, plane frames, grids, and space frames. The degrees of freedom are governed by approximation of inextensibility, that is negligible effect of axial deformations on bending moments and shears. For manual calculations, particularly the assumption of inextensibility reduces the computational effort quite significantly. Wherever inextensibility is used, the load matrix and the corresponding degrees of freedom need to be understood properly. All these aspects have been discussed while formulation of the equations for the unknown displacements. The slope deflection method is illustrated as a particular case of displacement method applied to small frames with the assumed inextensibility. In the case of beams, however, axial deformations get delinked from flexural deformations. While discussing trusses with hinged joints, the reader is cautioned against likely mechanisms. Sketching of such mechanisms of unstable trusses is also included. Sometimes a hybrid approach, combined force and displacement methods are computationally very efficient. The types of examples suitable for the mixed methods are illustrated.

Chapter 8–Iterative Techniques

The iterative techniques used for analysis are: Moment Distribution Method and Kani's Method .These methods are essentially numerical techniques for solving the equations of unknown displacements. The Kani's Method is an iterative technique and similarly, moment distribution method is an incremental iterative technique. These being the numerical techniques, typical numerical examples are presented for illustrating the techniques. It is particularly pointed out that the Hardy Cross Method from the considerations of incremental clamping and balancing of the moments at the ends of the members meeting the joint being released, makes it highly conceptual.

Chapter 9–Introduction to Applied Dynamics and Earthquake Design of Tall Frames

Tall buildings constitute an example of a dedicated structure. The various appropriate approximations are explained. The degrees of freedom being large in number, numericals are limited. The formulations are explained with the help of flow charts frequently used in the computer programming. The Shear walls are extensively used in the tall buildings and in the formulation of analysis for tall building frames using shear walls are explained as wide-column skeletal member. The floor slabs are assumed rigid in their own plain and therefore undergo in-plane rigid body displacements. Such a formulation is considered for writing down the dynamic equations of equilibrium and the mode shapes corresponding to horizontal degrees of freedom which are the dominant displacements for earthquake design. The numerical modeling for the analysis of tall buildings by, (i) Clough Wilson and King, and (ii) Weaver and Nelson are explained. Basics of Structural Dynamics are included with an objective of introducing earthquake design of tall buildings. Typical skeletal frames have been used as illustrations for computing mode shapes and arriving at the earthquake loads. The scope is limited to elastic analysis using Houser's Spectra. The push over analysis based on capacity of the structure and on the inelasticity/collapse is not included. However, a reader with the background of dynamics and plastic collapse, and the computations pertaining to angle discontinuities at the location of the plastic hinges should be able to comprehend capacities of the structures.

Appendix–Mathematical Preliminaries

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To all my students who directly and indirectly contributed to the development of the book. The students of the IIT Roorkee contributed on the basics of Mechanics, IIT Bombay on the elasticity and structural systems, and IIT Delhi on the core of the book pertaining to the Skeletal Structures, Plastic Analysis, Tall Buildings and Dynamics. My wife, companion, Kanchanamala (1958 onwards), has been my inspiration and motivation. It is her desire that I am fulfilling by presenting the book.

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Brief Profile of Mr. Subhash C. Mehrotra

Mr. Subhash Mehrotra Graduated in Civil Engineering in 1972 and subsequently did M. Tech. in Structural Engineering from IIT Delhi. His professional experience of 38 years includes structural designs for commercial, residential, multistoried buildings, hospitals, malls, industrial structures, and public infrastructures, including 600 tall buildings in the range of 10–40 storeys and 30 Airport terminal buildings in India. He has published many technical papers relating to civil engineering and consultancy profession and his designs have won several prestigious awards.

He is the Principal Consultant of M/s Mehro Consultants, which is one of the leading Structural Engineering Consulting Firms of India. He was the President of Consulting Engineers Association of India during the period December 2003–December 2005. He is presently Vice President of Indian Association of Structural Engineers (IAStructE) for the period of 2011–2013.

He is first Indian and first South Asian who got elected as Executive Committee member of FIDIC, for the period September 2005 to September 2009. He is currently Chairperson of Membership Committee of FIDIC for the period 2009–2012.

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