

**MATHEMATICAL MODELS IN WATER  
POLLUTION CONTROL**

*Edited by*

**A. James**

# **Mathematical Models in Water Pollution Control**

*Edited by*

**A. James**

*Head of Division of Public Health Engineering  
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*A Wiley-Interscience Publication*

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## Preface

The technology of water pollution control has developed over the last hundred years in an evolutionary manner. Empirical ideas have been tested and the successful ones incorporated into treatment practice. An early example of such developments was the evolution of percolating filters from land treatment. The main difficulties encountered with land treatment were clogging of the pores, so the initial experiments examined the drainage characteristics of other material such as crushed rock. Beds of stone known as contact beds were successfully used on a fill-and-draw basis and then, with the introduction of continuous inflow and removal, developed into percolating filters. Similarly, with the development of the activated sludge process, theory often lagged behind practice.

Theoretical developments have been slow because of the great degree of variation in natural waters and treatment plants. The action of random biological populations on wastes of variable composition produces data from which it is difficult to discern any general principles. Even in processes like sedimentation which have a physico-chemical rather than a biological basis the complicating effects of flocculation have obstructed the development of a coherent hydraulic description. It is in such a situation that the techniques of mathematical modelling can offer considerable assistance. For example simulation can be used to mathematically reconstruct a chain of causal connections where the individual relationships contain nonlinear elements, or are interdependent, or contain some other type of complexity which prevents a classical solution. Numerical methods and computing may then be used to find answers to the sets of equations which would be insoluble by analytical methods.

In this way the behaviour of complex natural systems like estuaries or a treatment process like percolating filters can be studied theoretically and alternative management policies can be explored. Other techniques like optimization may then be used to find the best combination of management strategies for combinations of treatment units or for water resource networks. Mathematical modelling is, therefore, an extremely valuable aid in speeding up the theoretical understanding of water pollution and its control.

This book is based upon papers presented at a conference, 'The Use of Mathematical Modelling in Water Pollution Control', held at the University of Newcastle upon Tyne in September, 1973. Since then interest in modelling techniques for pollution control has increased and it was felt that the publication of a revised version of the proceedings would usefully summarize the present state of knowledge.

It is noteworthy that authors of the chapters have a wide spectrum of backgrounds—engineers, mathematicians, chemists, and biologists. The rapid growth of modelling has resulted from an interdisciplinary approach. It is the aim of this book to encourage such cooperation because only in this way can the possibilities and limitations of modelling be properly appreciated.

*A. James*

The authors of the chapters have been selected on the basis of their distinguished work in their respective fields. The book is an excellent example of such interdisciplinary cooperation. The authors of the chapters have been selected on the basis of their distinguished work in their respective fields. The book is an excellent example of such interdisciplinary cooperation.

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# *Application of some Systems Engineering Concepts and Tools to Water Pollution Control Systems*

JOHN F. ANDREWS

## 1.1 INTRODUCTION

The word 'systems' is one of the most popular words of our time and has pervaded all fields of science and engineering as well as popular thinking and the mass media. Professions and job titles have appeared in recent years under names such as systems science, systems theory, systems analysis, systems engineering, and others. However, very few people, even among scientists and engineers, can give a concise, accurate definition of these terms. This confusion, which is both natural and not very serious, has been one of many factors contributing to a reluctance on the part of some engineers to adopt and use some of the basic concepts and tools of systems engineering.

The word 'systems' would lead one to think that there should be as many kinds of systems engineers as there are systems and this is indeed the case. Since water pollution control systems have been around for many years, the question might fairly be asked "Why include the word systems?" The author can answer this question only by stating that he has attempted to adopt some of the key concepts and analytical techniques found in the systems engineering literature and apply these to the analysis, design, and operation of wastewater treatment plants. Included among these concepts and analytical techniques are:

- (1) Looking at a system as an integrated whole yet with recognition of the interactions between the elements in a system and between the system and its environment. A good example is the activated sludge process in which the interactions between the aeration basin and the secondary settler (elements) must be clearly defined in order for the process (system) to function adequately. The activated sludge process is also influenced by the temperature of the environment in which it is operated.
- (2) Recognition of the universality of characteristics among systems. Even though two systems may appear vastly different, there are some basic common characteristics which can be used to obtain a better understanding of the systems. For example, biological processes used in wastewater

treatment have many common characteristics since the basic principles of microbiology apply to all of these processes.

- (3) An increased awareness of the importance of dynamic behaviour, information handling needs, and an orderly examination of alternate ways of accomplishing objectives including establishment of those factors of major importance in comparing the value of different alternatives. An example is the use of control systems, which involve information handling, to improve the dynamic behaviour of wastewater treatment plants. This would be an alternate to increasing the size of the plant as is usually the case in conventional plant design.
- (4) A team or interdisciplinary approach to the analysis, design, or operation of a system. From the viewpoint of the author, the most important member of this team is that person who is intimately familiar and experienced with the system to be analysed, designed, or operated. However, the talents of a wide variety of disciplines are needed in the analysis, design, or operation of water pollution control systems and included among these are most branches of engineering and the sciences.
- (5) The engineer involved in a systems study has at his disposal a wide variety of relatively new analytical tools. Included among these are:
  - (a) Mathematical modelling
  - (b) Computer simulation
  - (c) Transient response analysis
  - (d) Control theory
  - (e) Optimization techniques

The author is a relatively new user of systems engineering concepts and techniques and makes no claim to be an expert in any of the techniques illustrated in this chapter. Moreover, his experience with these concepts and techniques has been primarily restricted to the analysis, design, and operation of wastewater treatment plants and most of the examples presented herein are therefore drawn primarily from this field. For a more detailed discussion of systems theory and systems engineering as applied to a variety of fields, the reader is referred to the books of von Bertalanffy (1968) and Chestnut (1965). Motard (1966) has presented an excellent discussion of how systems engineering concepts and techniques are being incorporated into engineering curricula in U.S. Universities.

## 1.2 SYSTEM DEFINITION AND STRUCTURE

There are many different types of water pollution control systems ranging from large complex regional systems for river basins down to septic tanks for individual dwellings. For analysis, design, or operation of these systems it is first necessary to delineate their boundaries. The more complex systems are then structured by identification of individual elements or subsystems. These subdivisions are made in a fashion such that the important inputs and outputs

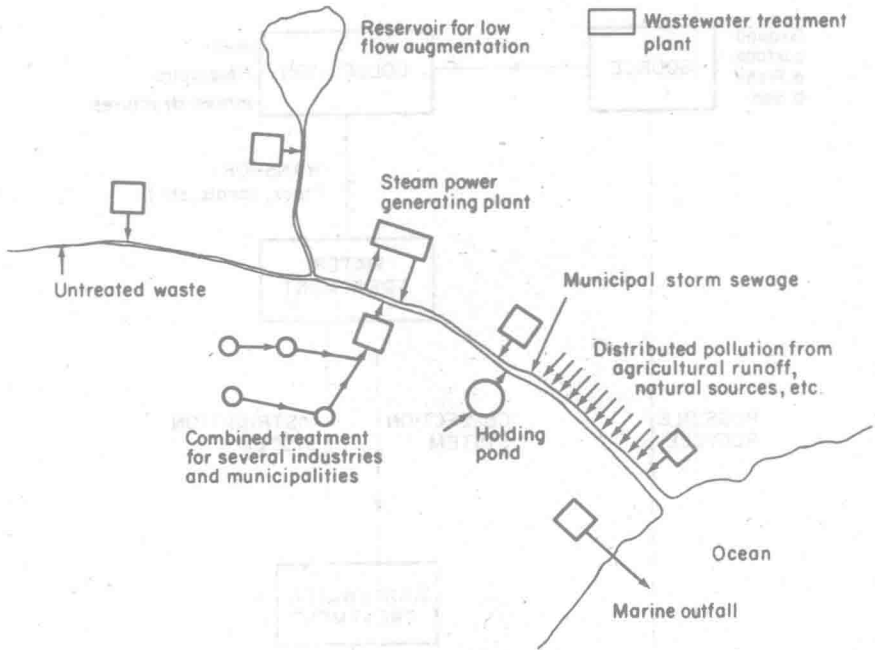


Figure 1.1 River basin pollution control system

as well as the environment in which the subsystem must operate can be reasonably well defined. Each subsystem should also have well-defined objectives which contribute to the overall objective of the system.

The breakdown of a system into its subsystems is illustrated in Figures 1.1, 1.2, and 1.3. One of the subsystems for the river basin shown in Figure 1.1 is the water and wastewater system for a municipality as presented in Figure 1.2. The wastewater treatment plant shown in Figure 1.3, the system with which the author is most familiar, is a subsystem for the municipality or could be a subsystem for the water and wastewater system of an industry. The treatment plant, in its turn, can be subdivided into individual processes such as primary sedimentation, anaerobic digestion, and so on. However, in discussing these individual processes, one must always be aware that they are part of a larger system, the treatment plant, and be conscious of the interactions between the individual processes as well as the contribution of the individual processes to the overall objectives of the plant.

The process of subdividing a system into its components and reconstituting the system from these components may be compared with viewing an American football game through the zoom lens of a camera. This is a sport in which both team effort and a high level of individual performance are essential for success. Focusing on an individual player (component) enables one to thoroughly analyse the performance of that player; however, the objective of the football team (system) is to score touchdowns or prevent the other team from scoring



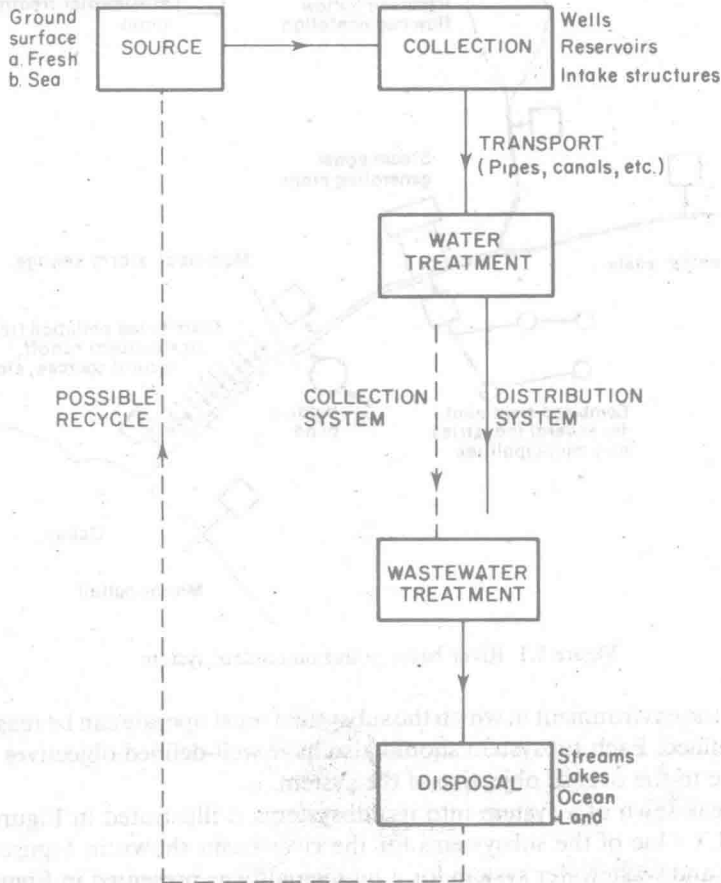


Figure 1.2 Water and wastewater system for a municipality

touchdowns and one occasionally suffers the frustration of the camera being focused on one player while another scores the touchdown. Stated in another and more old fashioned way, 'you can't see the forest for the trees.'

When the camera is focused on the entire playing field, the team objective and some of the interactions between the players become much clearer; however, one loses much of the detail of the game, such as the individual performance of some of the players and close observation of the interactions between individual players. A play may fail or succeed due to the performance of a single player and in order to improve the team (system) performance, a coach needs to know the strengths and weaknesses of each player (component). Another old saying is that 'a chain is no stronger than its weakest link'.

The systems engineering approach has sometimes failed to give reasonable predictions concerning the behaviour of a system because of inadequate attention to description of the individual components. This type of superficial treatment of a system has caused some people to oppose the systems engineering