Wiley Handbooks in Financial Engineering and Econometrics

ADVANCES IN HEAVY TAILED RISK MODELING

A HANDBOOK OF OPERATIONAL RISK

Gareth W. Peters
Pavel V. Shevchenko

WILEY

Advances in Heavy Tailed Risk Modeling

A Handbook of Operational Risk

GARETH W. PETERS
PAVEL V. SHEVCHENKO

WILEY

Published by John Wiley & Sons, Inc., Hoboken, New Jersey
Published simultaneously in Canada

Library of Congress Cataloging-in-Publication Data:

Peters, Gareth W., 1978-
Advances in heavy tailed risk modeling : a handbook of operational risk / Gareth W. Peters, Department of Statistical Science, University College of London, London, United Kingdom, Pavel V. Shevchenko., Division of Computational Informatics, The Commonwealth Scientific and Industrial Research Organization, Sydney, Australia.
pages cm
Includes bibliographical references and index.
ISBN 978-1-118-90953-9 (hardback)
1. Risk management. 2. Operational risk. I. Shevchenko, Pavel V. II. Title.
HD61.P477 2014
658.15′5–dc23

2014015418

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Gareth W. Peters

This is dedicated to three very inspirational women in my life: Chen Mei–Peters, my mother Laraine Peters and Youxiang Wu; your support, encouragement and patience has made this possible. Mum, you instilled in me the qualities of scientific inquiry, the importance of questioning ideas and scientific rigour. This is especially for my dear Chen who bore witness to all the weekends in the library, the late nights reading papers and the ups and downs of toiling with mathematical proofs across many continents over the past few years.

Pavel V. Shevchenko

To my dear wife Elena

Embarking upon writing this book has proven to be an adventure through the landscape of ideas. Bringing forth feelings of adventure analogous to those that must have stimulated explorers such as Columbus to voyage to new lands.

In the depth of winter, I finally learned that within me there lay an invincible summer.
Albert Camus.

Preface

This book covers key mathematical and statistical aspects of the quantitative modeling of heavy tailed loss processes in operational risk (OpRisk) and insurance settings. OpRisk has been through significant changes in the past few years with increased regulatory pressure for more comprehensive frameworks. Nowadays, every mid-sized and larger financial institution across the planet would have an OpRisk department. Despite the growing awareness and understanding of the importance of OpRisk modeling throughout the banking and insurance industry there is yet to be a convergence to a standardization of the modeling frameworks for this new area of risk management. In fact to date the majority of general texts on this topic of OpRisk have tended to cover basic topics of modeling that are typically standard in the majority of risk management disciplines. We believe that this is where the combination of the two books *Fundamental Aspects of Operational Risk and Insurance Analytics: A Handbook of Operational Risk* (Cruz, Peters and Shevchenko, 2015) and the companion book *Advances in Heavy Tailed Risk Modeling: A Handbook of Operational Risk* will play an important role in better understanding specific details of risk modeling directly aimed to specifically capture fundamental and core features specific to OpRisk loss processes.

These two texts form a sequence of books which provide a detailed and comprehensive guide to the state of the art OpRisk modeling approaches. In particular, this second book on heavy tailed modeling provides one of the few detailed texts which is aimed to be accessible to both practitioners and graduate students with quantitative background to understand the significance of heavy tailed modeling in risk and insurance, particularly in the setting of OpRisk. It covers a range of modeling frameworks from general concepts of heavy tailed loss processes, to extreme value theory, how dependence plays a role in joint heavy tailed models, risk measures and capital estimation behaviors in the presence of heavy tailed loss processes and finishes with simulation and estimation methods that can be implemented in practice. This second book on heavy tailed modeling is targetted at a PhD or advanced graduate level quantitative course in OpRisk and insurance and is suitable for quantitative analysts working in OpRisk and insurance wishing to understand more fundamental properties of heavy tailed modeling that is directly relevant to practice. This is where the *Advances in Heavy-Tailed Risk Modeling: A Handbook of Operational Risk* can add value to the industry. In particular, by providing a clear and detailed coverage of modeling for heavy tailed OpRisk losses from both a rigorous mathematical as well as a statistical perspective.

More specifically, this book covers advanced topics on risk modeling in high consequence low frequency loss processes. This includes splice loss models and motivation for heavy tailed risk models. The key aspects of extreme value theory and their development in loss distributional approach modeling are considered. Classification and understanding of different classes of heavy tailed risk process models is discussed; this leads to topics on heavy tailed closed-form loss distribution approach models and flexible heavy tailed risk models such as α-stable, tempered stable, g-and-h, GB2 and Tukey quantile transform based models. The remainder of the chapters covers advanced topics on risk measures and asymptotics for heavy tailed compound process models. Then the final chapter covers advanced topics including forming links between actuarial compound process recursions and Monte Carlo numerical solutions for capital risk measure estimations.

The book is primarily developed for advanced risk management practitioners and quantitative analysts. In addition, it is suitable as a core reference for an advanced mathematical or statistical risk management masters course or a PhD research course on risk management and asymptotics.

As mentioned, this book is a companion book of *Fundamental Aspects of Operational Risk and Insurance Analytics: A Handbook of Operational Risk* (Cruz, Peters and Shevchenko, 2015). The latter covers fundamentals of the building blocks of OpRisk management and measurement related to Basel II/III regulation, modeling dependence, estimation of risk models and the four-data elements (internal data, external data, scenario analysis and business environment and internal control factors) that need to be used in the OpRisk framework.

Overall, these two books provide a consistent and comprehensive coverage of all aspects of OpRisk management and related insurance analytics as they relate to loss distribution approach modeling and OpRisk – organizational structure, methodologies, policies and infrastructure – for both financial and non-financial institutions. The risk measurement and modeling techniques discussed in the book are based on the latest research. They are presented, however, with considerations based on practical experience of the authors with the daily application of risk measurement tools. We have incorporated the latest evolution of the regulatory framework. The books offer a unique presentation of the latest OpRisk management techniques and provide a unique source of knowledge in risk management ranging from current regulatory issues, data collection and management, technological infrastructure, hedging techniques and organizational structure.

We would like to thank our families for their patience with our absence whilst we were writing this book.

Gareth W. Peters and Pavel V. Shevchenko
London, Sydney, March 2015

Acknowledgments

Dr. Gareth W. Peters acknowledges the support of the Institute of Statistical Mathematics, Tokyo, Japan and Prof. Tomoko Matsui for extended collaborative research visits and discussions during the development of this book.

Acronyms

ABC	approximate Bayesian computation
ALP	accumulated loss policy
a.s.	almost surely
AMA	advanced measurement approach
APT	arbitrage pricing theory
BCRLB	Bayesian Cramer–Rao lower bound
BCBS	Basel Committee on Banking Supervision
BIS	Bank for International Settlements
CV	co-variation
CD	co-difference
CRLB	Cramer–Rao lower bound
CLP	combined loss policy
CVaR	conditional value at risk
DFT	discrete Fourier transform
EVT	extreme value theory
EVI	extreme value index
ES	expected shortfall
FFT	fast Fourier transform
GLM	generalized linear models
GAM	generalized additive models
GLMM	generalized linear mixed models
GAMM	generalized additive mixed models
GAMLSS	generalized additive models for location scale and shape
HMCR	higher moment coherent risk measure
HILP	haircut individual loss policy
ILPU	individual loss policy uncapped
ILPC	individual loss policy capped
i.i.d.	independent and identically distributed
LDA	loss distribution approach
MCMC	Markov chain Monte Carlo
MC	Monte Carlo
MLE	maximum likelihood estimator
MPT	modern portfolio theory
OpRisk	operational risk
PMCMC	particle Markov chain Monte Carlo
r.v.	random variable
SMC	sequential Monte Carlo

SRM	spectral risk measure
SLA	Single Loss Approximation
s.t.	such that
TCE	tail conditional expectation
TTCE	tempered tail conditional expectation
VaR	value at risk
Vco	variational coefficient
w.r.t.	with respect to

Symbols

$\forall$	for all
$\exists$	there exists
$\cup$	union of two sets
$\cap$	intersection of two sets
$*$	convolution operator
$F(\cdot)$	probability distribution function
$C(u_1, u_2, \ldots, u_d)$	d-dimensional Copula probability distribution function.
$\overline{F}$	tail function $\overline{F}(x) = 1 - F(x)$
$f(\cdot)$	probability density function
$h(\cdot)$	hazard rate given by $h(\cdot) = \frac{f(\cdot)}{\overline{F}(\cdot)}$
$\Phi_X[\theta]$	characteristic function for random variable X
$M_X(t)$	moment-generating function of random variable X
$F^{(n)*}(\cdot)$	n-fold convolution of a distribution function with itself
$F^{-1}(\cdot)$	inverse distribution function (quantile function)·
$Q(\alpha)$	quantile function
$U(y) = Q(1 - 1/y)$	tail quantile function
$F^{\leftarrow}(\cdot)$	generalized inverse
$g(\cdot) \sim f(\cdot)$	function g is asymptotic equivalent to f at infinity (unless specified otherwise)
$X \sim F(\cdot)$	random variable X is distributed according to F
$X_{(k,n)}$	kth largest sample from n samples, that is, the kth order statistic
$\text{VaR}_\alpha[\cdot]$	value at risk for level α
$\text{ES}_\alpha[\cdot]$	expected shortfall for level α
$\text{SRM}_\alpha[\cdot]$	spectral risk measure for level α
$\mathbb{J}$	space of integers
$\mathbb{R}$	real line
$\mathbb{C}$	complex plane
$\mathscr{R}\text{e}\{\cdot\}$	real component of a complex number or function
$\mathscr{I}\text{m}\{\cdot\}$	imaginary component of a complex number
$\mathbb{E}[\cdot]$	expectation operator
$\mathbb{P}\text{r}[\cdot]$	probability
$f(\cdot) \in RV(\rho)$	function f is in the class of Regularly Varying functions with index of regular variation ρ
$L(\cdot)$	slowly varying function, that is, function $L(\cdot) \in RV(0)$
$\Phi^{(d)}$	dth derivative of function Φ
$\mathbb{I}[\cdot]$	indicator function of an event

$\delta(\cdot)$	Dirac delta function
$o(\cdot)$	little 'Oh' Landau asymptotic notation
$O(\cdot)$	big 'Oh' Landau asymptotic notation
$<<$	Vinogradov's asymptotic notation for big 'Oh' notation
$f \circ g$	f is a composite function of g, that is, $f(g(\cdot))$
$\mathscr{W}[x]$	Lambert W function
$\mathscr{E}[n, x]$	Misra function

List of Distributions

Distribution Name	Distribution Symbol
α-Stable	$S_\alpha(\cdot)$
Asymmetric Laplace	$AssymetricLaplace(\cdot)$
Beta	$Beta(\cdot)$
Chi Squared	$ChiSquared(\cdot)$
Dagum or Burr Type XII	$BurrXII(\cdot)$
Double $h-h$ distributions	$T_{h,h}(\cdot)$
Exponential	$Exp(\cdot)$
Gamma	$Gamma(\cdot)$
g distributions	$T_g(\cdot)$
g-and-h distributions	$T_{g,h}(\cdot)$
g-and-k distributions	$T_{g,k}(\cdot)$
Generalized gamma	$GG(\cdot)$
Generalized beta	$GB2(\cdot)$
Generalized Hyperbolic	$GH(\cdot)$
Generalized Inverse Guassian	$GIG(\cdot)$
Generalized Log-Logistic	$Gen(\cdot)$
Geometric stable	$GeoStable(\cdot)$
h distributions	$T_h(\cdot)$
Halphen type A	$Halphen(\cdot)$
Inverse Gamma	$InverseGamma(\cdot)$
Inverse Gaussian	$InverseGaussian(\cdot)$
Laplace	$Laplace(\cdot)$
Log-Cauchy	$LogCauchy(\cdot)$
LogNormal	$LogNormal(\cdot)$
Log-t	Log-$t(\cdot)$
Lomax	$Lomax(\cdot)$
Normal (Gaussian)	$Normal(\cdot)$
Normal Inverse Gaussian	$NIG(\cdot)$
Singh–Maddala or Burr Type III	$BurrIII(\cdot)$
Standard Normal	$\Phi(\cdot)$
Tempered stable	$TS(\cdot)$

Contents in Brief

Contents