AN INTRODUCTION TO THE PHYSICS OF NUCLEAR MEDICINE

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This text will assist physicians who utilize nuclear medicine in their practices and technologists who perform related procedures to understand the physical principles involved in this field. It will provide them with the ability to define the limitations of their methods, select appropriate studies and properly use, maintain and calibrate their instruments. Whether used in conjunction with lecture material or as a self-study text, this volume clearly answers the need for an introduction to the physics of nuclear medicine for students, residents, physicians and technologists.

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By

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The physics of nuclear medicine

Preface

The physician who enters the field of nuclear medicine often finds that he has forgotten most of the physics to which he was exposed in college. Yet, in order to appreciate both the advantages and the limitations of nuclear imaging, he must understand the basic principles of radioactivity and radiation detection. This book presents those principles in as elementary a way as possible. Thus no attempt has been made to be comprehensive, but rather only that material is presented which will enable the physician to understand the equipment and the techniques used in his specialty. In addition, brief descriptions are given of some of the newer types of detectors and gamma cameras. We believe that these instruments, while not yet fully developed or widely available, will soon have a major impact on the field of nuclear medicine and that the physician who wishes to keep abreast of his specialty should understand the physical principles behind them.

The presentation is based on our experience in teaching physics to radiology residents, other M.D.'s, and nuclear medicine technologists. As an aid to self-study, problems and questions are included at the end of each chapter. These questions are of the multiple-choice type, so that the reader may determine if he has understood the subject matter.

Nuclear medicine technologists should find that this book covers most of the physics material for which they are held responsible on their certification exams. Also, this book may be of use to medical students who have the opportunity to take an elective in nuclear medicine, since an understanding of the basic principles and equipment will make their exposure to the field more interesting and profitable.

The authors would like to acknowledge advice and assistance from a number of persons who helped in preparing this book. In particular, we thank Professor K.S.R. Sastry, University of Massachusetts, for critical reading of the manuscript. Dr. Rao thanks Dr. M. Frendlich for his valuable criticism, Mrs. Vera Korda for her excellent typing, and his wife for her assistance. Dr. Goodwin thanks Dr. Leonard Freeman for helpful suggestions, Edith Ross, Joyce Rush, and particularly his wife, for secretarial assistance.

P.N.G.
D.V.R.

Foreword

There are many quotations which have become so cliched due to repetition that they lose all impact. A few, however, express so much truth that they still serve a purpose. One of these is the old adage that one must learn to walk before one can run. To do other than this is just about impossible. This is particularly applicable to the field of Nuclear Medicine, where persons who attempt to practice it as physicians or to perform procedures as technicians without understanding the physical principles involved could not expect to be able to define the limitations of their methods, select the appropriate studies, or to be able to properly use, care for, and calibrate their instrumentation. Without the knowledge of the basic physics, one would have to function by rote methodology, depending upon "cook book" procedures and interpreting studies through empiric recognition. Such a situation shortchanges the patient and places the physician treating that patient in jeopardy.

This book presents, in a simple and concise form, the physics necessary "to learn how to walk." No attempt has been made to inform the reader of the facts that do not relate directly to the studies or to the instruments which the practitioner will have to use. Some of the mathematics may seem detailed but working through the examples is not difficult, and the reader should at least know how his or her consulting or visiting physicist comes up with the magic numbers needed for new procedures or for health safety purposes. In addition, these types of calculations turn up with disturbing frequency on Board and Certification examinations.

Having spent considerable time teaching Nuclear Medicine principles to physicians, technicians, and students, I welcome this book. It clearly answers a need as a text for the physics lectures for students, residents, and technologists. In addition, it stands by itself as a self-study text for those in the field who need to walk leisurely and carefully through its pages in order to better compete in the Nuclear Medicine race.

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A review of elementary mathematics

To clearly explain physics without mathematical ideas is difficult. Although every attempt has been made to minimize the mathematical formalism, inescapably exponents and logarithms will be dealt with throughout this book. Therefore these mathematical ideas need to be discussed before going on to the physics of nuclear medicine.

POWERS AND EXPONENTS

WHOLE NUMBERS. Supposing y is any number and m is its exponent which can be any positive whole number, then

$$y^m = y \times y \times y \ldots\ldots\ldots\ldots m \text{ times.} \quad (1.1)$$

Thus, $3^4 = 3 \times 3 \times 3 \times 3$ or $5^3 = 5 \times 5 \times 5$.
Also,

$$y^m \times y^n = y^{(m+n)}. \quad (1.2)$$

As an example, take

$$y = 3,\ m = 4,\ n = 2, \text{ then}$$

$$3^4 \times 3^2 = 3^6.$$

The division rule is

$$\frac{y^m}{y^n} = y^{(m-n)}. \quad (1.3)$$

Again, if $m = 4$, $n = 2$, and $y = 3$,

$$\frac{3^4}{3^2} = 3^{(4-2)} = 3^2.$$

In equation (1.3), if $n > m$, then the exponent is negative. For example, take $m = 2$, $n = 4$, $y = 3$, then

$$\frac{3 \times 3}{3 \times 3 \times 3 \times 3} = \frac{1}{3 \times 3} = \frac{1}{3^2}.$$

Use of equation (1.3) gives

$$\frac{3^2}{3^4} = 3^{(2-4)} = 3^{-2},$$

suggesting

$$3^{-2} = \frac{1}{3^2}.$$

In general,

$$y^{-m} = \frac{1}{y^m}. \tag{1.4}$$

If $m = n$ in equation (1.3), then

$$\frac{y^m}{y^m} = y^0 = y^{(m-m)}$$

But it is evident $y^m/y^m = 1$, therefore for any value of y,

$$y^0 = 1. \tag{1.5}$$

If y^m is raised to power n, then

$$(y^m)^n = y^{(m \times n)}. \tag{1.6}$$

For $m = 4$, $n = 2$, and $y = 3$,

$$(3^4)^2 = 3^{(4\times 2)} = 3^8.$$

FRACTIONS. The equations (1.2), (1.3), (1.4), and (1.6) are valid even though the exponents are fractions. Supposing m is still a whole number, then $1/m$ is a fraction and

$$y^{\frac{1}{m}} = \sqrt[m]{y}. \tag{1.7}$$

If $m = 2$, then

$$y^{1/2} = \sqrt{y};$$

or $m = 4$,

$$y^{1/4} = \sqrt[4]{y}.$$

Also,

$$y^{\frac{m}{n}} = \sqrt[n]{y^m}. \tag{1.8}$$

For example, if $m = 2$, $n = 3$, then

$$y^{2/3} = \sqrt[3]{y^2}.$$

It also follows from above equations,

$$y^{1/2} = y^{0.5} = y^{5/10} = \sqrt[10]{y^5};$$

but

$$y^{1/2} = \sqrt{y},$$

which suggests that

$$\sqrt{y} = \sqrt[10]{y^5}.$$

Large and Small Numbers. This book will deal with extremely large and small numbers. Therefore it is necessary to have an idea as to their magnitude. The velocity of light is a very large number, while the wavelength of yellow light is a very small number. How small or large are they? The velocity of light is known to be 30,000,000,000 cm/sec, i.e. 30 billion centimeters per second. The wavelength of yellow light is 0.000,058 cm, i.e. 58 times one millionth of a centimeter. Obviously these numbers are hard to deal with in this manner. A more convenient method using powers of ten is necessary.

To write the large numbers, positive whole-number powers of 10 are used.

$$10^m = \text{one followed by } m \text{ zeros.} \tag{1.9}$$

If $m = 0$, $10^0 = 1$ in agreement with equation (1.5),

$$10^1 = 10;\ 10^2 = 100;\ 10^6 = 1{,}000{,}000;$$
$$10^{12} = 1{,}000{,}000{,}000{,}000 \text{ and so on.}$$

Similarly, to write small numbers, negative whole-number powers of 10 are used.

$$10^{-m} = \text{A decimal point and } (m-1) \text{ zeros followed by } 1 \tag{1.10}$$

The value of 10^{-m} for $m = 0$ is still 1; $10^{-1} = 0.1$

$$10^{-2} = 0.01;\ 10^{-6} = 0.000{,}001,$$
$$10^{-12} = 0.000{,}000{,}000{,}001 \text{ and so on.}$$

Now the velocity of light and the wavelength of yellow light can be written in the above notation:

$$\begin{aligned}\text{velocity of light} &= 30{,}000{,}000{,}000 \text{ cm/sec}\\ &= 3 \times 10{,}000{,}000{,}000 \text{ cm/sec} = 3 \times 10^{10} \text{ cm/sec},\\ \text{wavelength of yellow light} &= 0.000{,}058 \text{ cm}\\ &= 5.8 \times 10^{-5} \text{ cm.}\end{aligned}$$

This notation also simplifies multiplication and division of numbers. For example, divide the velocity of light by the wavelength of the yellow light. If these numbers are not written in exponential form, it would be very difficult to carry out this operation.

$$\begin{aligned}\frac{\text{velocity of light}}{\text{wavelength of yellow light}} &= \frac{3 \times 10^{10} \text{ cm/sec}}{5.8 \times 10^{-5} \text{ cm}}\\ &= \frac{3 \times 10^{10} \times 10^5}{5.8} \text{ sec}^{-1}\\ &= \frac{3}{5.8} \times 10^{15} \text{ sec}^{-1}\\ &= 0.52 \times 10^{15} \text{ sec}^{-1}\\ &= 5.2 \times 10^{14} \text{ sec}^{-1}.\end{aligned}$$

Similarly the square of the velocity of light is

$$\left(3 \times 10^{10} \frac{\text{cm}}{\text{sec}}\right)^2 = 3^2 \times (10^{10})^2 \text{ cm}^2 \text{ sec}^{-2}$$
$$= 9 \times 10^{20} \text{ cm}^2 \text{ sec}^{-2}.$$

LOGARITHMS

Consider the equation

$$z = y^m.$$

Sometimes it is necessary to find the value of m, given y and z. If $z = 100$ and $y = 10$, then

$$100 = 10^m.$$

In this case $m = 2$, which is not difficult to figure out. However, the exponent m may not be a whole number, in which case it would be difficult to find the value of m.

For example, consider

$$12 = 10^m.$$

Since $10^1 = 10$ and $10^2 = 100$, the value of m is somewhere between 1 and 2. In order to solve this kind of problem, the logarithm is very helpful. In general, the number m in the equation:

$$m = \log_y z \qquad (1.11)$$

is called the logarithm to the base y of z. Here y and z can be any positive number, whole or fractional, except 0 and 1. In the example, $12 = 10^m$, the value of $m = \log_{10} 12$. Then the logarithm of 12 to the base 10 can be found.

The *common logarithms* tabulated in mathematical handbooks are usually to the base 10, which is found to be convenient to many calculations. The logarithm taken to the base e is also useful sometimes. The value of e is 2.71828. The logarithm taken to the base e is called the *natural logarithm*, represented by ln. Which base should be used: 10 or e? For many calculations, base e may be more convenient.

Consider the equation

$$z = e^m, \qquad (1.12)$$

where m is any positive or negative number. The value of m that satisfies this equation can be obtained by taking logarithms on both sides:

$$\ln z = m \ln e.$$

But $\ln e = 1$, therefore

$$m = \ln z.$$

The natural logarithms and the values of the functions e^{-m} and e^m are tabulated in many mathematical handbooks. The availabilty of hand and desk calculators in many institutions makes these calculations even simpler.

The functions of the kind e^{-m} and e^m are important for understanding radioactive decay and absorption of radiation in matter. The value of the function e^{-m} decreases from 1.0 when $m = 0$ very rapidly in the beginning