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The Rotation of the Earth.

By

Sir HAROLD SPENCER-JONES.

With 3 Figures.

I. The Unit of Time.

1. The three fundamental units in physics, which form the basis of the c.g.s. system, are the centimetre, the unit of length; the gram, the unit of mass; and the second, the unit of time. Material standards of length and mass are provided by the International Prototype Metre and Kilogram, preserved at the International Bureau of Weights and Measures at Sèvres, near Paris. Any length or mass can be compared either directly or indirectly with sub-standards, which have themselves been compared with the actual prototype standards.

But with the unit of time it is otherwise. There is no material standard of time which can be used as an invariable standard of reference. When an atomic clock has been successfully developed, it may provide an absolute standard of time; but as no clock will function perpetually without stopping, it would be necessary to employ several atomic clocks, regularly intercompared with one another, so as to carry on the measure of time in the event of a stoppage of any one clock.

Lacking any material absolute standard of time, the rotation of the Earth has been adopted as a standard. The second, the unit of time in both the metric and the imperial (f.p.s.) system of measurement, is the mean solar second, which is the 1/86,400th part of the mean solar day. The mean solar day is a day of a length equal to the average of the true or apparent solar days throughout the year. Astronomers use the sidereal day, the time interval required for the Earth to make one complete rotation through 360°. It is the interval provided by two consecutive transits of a star across any given meridian. Sidereal time is determined by relating the instants at which stars of accurately known right ascension transit across the meridian with the corresponding time by the standard clock of the observatory. By a sequence of observations, the error and rate of the clock are controlled, so that the true sidereal time corresponding to any clock time is known.

2. The use of the Earth as a clock tacitly assumes that its rotation is uniform. If this assumption is not correct, so that the length of the day is variable, discordances will be produced between the observed and ephemeris positions of bodies in the solar system. The ephemeris positions of the Sun, Moon and planets which are tabulated in the various national ephemerides (*Nautical Almanac*, *Astronomisches Jahrbuch*, *Connaissance des Temps* etc.) are based on gravitational theory with the assumption that the length of the day is constant. Differences between observation and theory would be produced by errors of observation, by errors or incompleteness of the theory, or by variations in the length of the day. The theories of the motions of the Moon, Sun, Mercury and Venus, which are the four bodies of main interest in the investigation of variations in the length of the day, are sufficiently complete and exact for this purpose.

The discordances between the observed and theoretical positions caused by variations in the length of the day will be greater the more rapid the geocentric motion of the body, for they are attributable to a time error on the part of the Earth; the discordances in position represent the motions in longitude during that time. The Moon, which has the most rapid motion in longitude ($0''.55$ in one second of time) is consequently the most favourable object for detecting changes in the Earth's rotation; Mercury is the next most favourable; the Sun and Venus can also be used. Each of these four bodies can serve as a clock; their ephemerides are based on a uniform time which we shall term *ephemeris time*. To each observed position, an ephemeris time can be found, by interpolation in the ephemeris, at which the ephemeris position agrees with the observed position. This Ephemeris Time can be compared with the Universal Time (or G.M.T.) of the observation. The discordances between the two times must be due to a lack of uniformity in Universal Time, assuming that there are no defects in the theory of the motion of the body. Thus each of the four bodies, the Moon, Sun, Mercury and Venus, can be used as a clock, against which the rotation of the Earth can be checked. If the four bodies agree with each other but not with the Earth, it can safely be concluded that the Earth is at fault and that its rotation is not uniform or that, in other words, our adopted standard of time is variable.

The errors inherent in the astronomical observations of position of the four bodies make it impossible to detect in this way small discordances in position which are of short period or which fluctuate rapidly. The astronomical observations of position are suitable, however, for detecting *slow secular changes* in the Earth's rotation or the cumulative effect over a long time interval of small changes. Short period or *rapid changes* occurring within a year or so can best be investigated by comparing the time provided by the Earth's rotation with the time given by modern precision quartz-crystal oscillators. These oscillators are capable of very high precision over short time intervals, but because of ageing effects, and the possibility of involuntary stoppages, they are not suitable for the control of slow secular changes or of long period changes in the Earth's rotation.

3. It has been established within recent years that the rotation of the Earth is not strictly uniform and that the departures from uniformity are of three different types, which are due to different causes. They are:

a) A slow secular increase in the length of the day.

b) Irregular fluctuations, the length of the day sometimes increasing and sometimes decreasing.

c) A seasonal variation in length.

Of these, a) and b) have been detected by observations of discordances between observations and theory in the positions of the Moon, Mercury, Venus and the Sun; c) has been detected through the high precision of modern clocks. The first two of these effects are somewhat inter-related and must be considered together.

II. The Secular Acceleration of the Motions of the Sun and Moon.

4. HALLEY[1], in 1695, by comparing the positions of the Moon derived from early observations of eclipses with observations in his time, concluded that the motion of the Moon was being accelerated. This was confirmed by DUNTHORNE in 1749, by MAYER in 1753, and by LALANDE in 1757. An acceleration of the mean

[1] E. HALLEY: Phil. Trans. **19**, 174, (1695).

motion of the Moon implies that its mean longitude (which differs from the true longitude by the omission of all periodic terms) can be represented by an expression of the form

$$L = a + bT + cT^2 \tag{4.1}$$

in which T denotes the time (which is usually expressed in Julian centuries of 36525 days). The longitude at epoch $T=0$ is a; the mean motion at that epoch (dL/dT) is b. The mean motion at any epoch T is represented by $(b+2cT)$. The acceleration of the mean motion in unit time is thus $2c$. But from long usage it has become customary to designate the coefficient c of the term in T^2 in the mean longitude as *the secular acceleration of the mean motion*. The investigations of DUNTHORNE, MAYER and LALANDE indicated that the secular acceleration of the Moon's mean motion was about 10″ a century.

In the latter half of the 18th century much attention was being given by mathematicians to the detailed explanation of the motions of the bodies in the solar system under the action of NEWTON's law of gravitation. EULER and LAGRANGE both attempted, without success, to account for the secular acceleration of the Moon's motion. At length in 1787 LAPLACE[1] announced that he had discovered the explanation, which may be briefly stated as follows. The mean action of the Sun upon the Moon tends to diminish the Moon's gravity towards the Earth, and thereby to cause her angular velocity to decrease. The diminution being once supposed to have occurred, the angular velocity will thereafter remain constant, if the mean solar action remains constant. But the mean action of the Sun depends to a certain extent upon the eccentricity of the orbit of the Earth, and this eccentricity is decreasing secularly as a result of the action of the planets on the Earth. This gradual decrease of the eccentricity will cause a gradual decrease in the mean action of the Sun, which will result in a slow increase in the Moon's mean motion.

On the basis of his mathematical investigation, LAPLACE computed the amount of the acceleration as 10.″18 per century, in close agreement with the observed value. But in 1853 J. C. ADAMS[2] found that the calculations of LAPLACE were not complete. LAPLACE had assumed that the areal velocity of the Moon remained unaltered, so that the tangential disturbing forces produced no permanent effect. ADAMS pointed out that this was correct if the eccentricity were constant but that, as a result of the gradual change of shape of the orbit, there was an uncompensated effect. Allowing for this, the theoretical value of the secular acceleration of the Moon's motion was reduced to 5.″70, little more than half the observed value.

5. As there seemed to be no way in which the residual acceleration could be accounted for by gravitational action, it was concluded that it must be an effect arising from a secular retardation of the rotation of the Earth. In 1754 KANT had published an essay entitled "Untersuchung der Frage, ob die Erde in ihrer Umdrehung um die Achse, wodurch sie die Abwechselung des Tages und der Nacht hervorbringt, eine Veränderung seit den ersten Zeiten ihres Ursprunges erlitten habe, und woraus man sich ihrer versichern könne". In this essay he pointed out that the action of the Moon in raising tides in the oceans must have a secondary effect in a slight retardation of the Earth's motion by tidal friction, and explained the fact the Moon always turns the same face to the Earth as a

[1] P. S. DE LAPLACE: Mém. Acad. Sci. **235**, 1788 (1786). See also Mécanique Celeste, Book 7, Chapter I, § 16 and Chapter IV, § 23.
[2] J. C. ADAMS: Phil. Trans. A **143**, 397 (1853).

consequence of the retardation of the Moon's rotation in an early period of existence by bodily tides raised by the Earth on the Moon. LAPLACE had considered and rejected this suggestion on the ground that, if such an effect existed, accelerations of the mean motions of the planets, as well as of the Moon, should be observed, whereas no such accelerations had been detected by observation. DELAUNAY in 1865 revived the hypothesis of tidal friction to account for the acceleration of the mean motion of the Moon. It was not until 1905, however, that COWELL[1] found that there was a small secular acceleration of the orbital motion of the Earth or, otherwise expressed, that the mean motion of the Sun was being accelerated. Secular accelerations of the mean motions of Mercury and Venus have since been established. Tidal friction appears qualitatively to be a *vera causa* for the secular retardation of the rotation of the Earth.

6. The rate of dissipation of energy required to account for the observed secular accelerations of the Moon and Sun can be estimated. The following discussion is based on the investigation by JEFFREYS [*1*].

M, m, m'	= denote the masses of the Earth, Moon and Sun;
n, n'	= are the mean angular velocities of the Moon and Sun about the Earth;
c, c'	= are the distances of the Moon and Sun from the Earth;
$-N, -N'$	= are the couples acting on the Earth due to lunar and solar tides;
ω	= is the angular velocity of the Earth;
C	= is the moment of inertia of the Earth about its axis of rotation;
f	= denotes the constant of gravitation.

Then

$$n^2 c^3 = f(M+m), \qquad n'^2 c'^3 = f(M+m'). \tag{6.1}$$

Put

$$c = c_0 \xi^2, \qquad n = n_0 \xi^{-3}, \qquad c' = c'_0 \xi'^2, \qquad n' = n'_0 \xi'^{-3} \tag{6.2}$$

the suffix 0 denoting the present value.

The angular momentum of the orbital motion of the Moon and Earth about their centre of mass is

$$\frac{M m c^2 n}{M+m} = \frac{M m c_0^2 n_0}{M+m} \xi. \tag{6.3}$$

To the couple $-N$ acting on the Earth's rotation must correspond an equal and opposite couple $+N$ tending to increase the orbital angular momentum. Because of the difference in periods of the lunar and solar tides, the solar tides will have no secular effect on the Moon and the lunar tides will have no secular effect on the Sun. Thus we must have

$$\frac{M m c_0^2 n_0}{M+m} \frac{d\xi}{dt} = N, \tag{6.4}$$

$$\frac{M m' c_0'^2 n'_0}{M+m'} \frac{d\xi'}{dt} = N', \tag{6.5}$$

$$C \frac{d\omega}{dt} = -N - N'. \tag{6.6}$$

[1] P. H. COWELL: M. N. **66**, 13 (1905). — M. N. = Monthly Notices Roy. Astron. Soc.

If E is the total mechanical energy in the system, its rate of decrease must be equal to the rate of performance of work by the angular motions in overcoming the couples. Consequently

$$-\frac{dE}{dt} = (N + N')\,\omega - N\,n - N'\,n'. \tag{6.7}$$

The part of the right-hand side of this equation due to the lunar tides is $N(\omega - n)$, and must be positive, since the couples arise from dissipation of energy. As $(\omega - n)$ is positive, N must be positive. Similarly N' must be positive. Hence $d\xi/dt$, $d\xi'/dt$ must both be positive, from which it follows that c, c' must be increasing and accordingly the mean motions of the Sun and Moon must be decreasing. The rate of rotation of the Earth must be decreasing, since $d\omega/dt$ must be negative.

7. We must now consider the effect of the changes in the angular velocities on observation. If ω, n, n' are assumed to be derived from observations at time zero, then the effect of the variation in the Earth's rotation is to put the Earth ahead in time T by an angular amount $\frac{1}{2}T^2\frac{d\omega}{dt}$. The time of transit of a fixed star is earlier by $\frac{T^2}{2\omega}\frac{d\omega}{dt}$. As the mean angular velocity of the Moon relative to the stars is n, the alteration of the time of observation results in the Moon being behind its calculated position when the star transits by an angle $\frac{nT^2}{2\omega}\frac{d\omega}{dt}$. But the change in the Moon's angular velocity puts it ahead by an angle $\frac{1}{2}T^2\frac{dn}{dt}$. The combined effect is for the Moon to appear to have gained on the stars by $\frac{1}{2}T^2\left(\frac{dn}{dt} - \frac{n}{\omega}\frac{d\omega}{dt}\right)$. If ν, ν' denote the secular accelerations of the Moon and Sun, we therefore have

$$\nu = \frac{1}{2}\left(\frac{dn}{dt} - \frac{n}{\omega}\frac{d\omega}{dt}\right) \tag{7.1}$$

and similarly

$$\nu' = \frac{1}{2}\left(\frac{dn'}{dt} - \frac{n'}{\omega}\frac{d\omega}{dt}\right). \tag{7.2}$$

But from (6.2)

$$\frac{dn}{dt} = -3n_0\,\xi^{-4}\frac{d\xi}{dt}, \quad \frac{dn'}{dt} = -3n'_0\,\xi'^{-4}\frac{d\xi'}{dt} \tag{7.3}$$

whence, using (6.4) to (6.6), and assuming c is constant

$$\nu = -\frac{3}{2}\,\frac{M+m}{M\,m}\,\frac{N\xi^{-4}}{c_0^2} + \frac{N+N'}{2C\omega}\,n_0\,\xi^{-3}, \tag{7.4}$$

$$\nu' = -\frac{3}{2}\,\frac{M+m'}{M\,m'}\,\frac{N'\xi'^{-4}}{c_0'^2} + \frac{N+N'}{2C\omega}\,n'_0\,\xi'^{-3}. \tag{7.5}$$

For intervals of time of a few thousand years, since the earliest observations of eclipses, it is sufficient to take ξ and ξ' both equal to 1.

If we denote by $\varkappa$ the present ratio of the orbital angular momentum to the angular momentum of the Earth's rotation

$$\varkappa = \frac{M\,m}{M+m}\,\frac{c_0^2\,n_0}{C\,\omega_0}, \tag{7.6}$$

we obtain

$$\nu = \frac{M+m}{2M\,m\,c^2}\{\varkappa(N+N') - 3N\}. \tag{7.7}$$

The ratio of the first term in ν' to the second term is of the order of 10^{-6}, and the first term can consequently be neglected, whence

$$\nu' = \frac{M+m}{2Mmc^2}\varkappa(N+N')\frac{n'}{n}. \tag{7.8}$$

The present value of $\varkappa$ is 4.82 and of n/n' is 13.4.

8. The precise ratio of the retarding couples due to the lunar and solar tides, N/N', can not be calculated, because a detailed knowledge of the forces that are acting is not available. On the assumption that the equations of motion are linear and that the system is far from resonance, JEFFREYS [*1*] obtained a value of 5.1 for the ratio. But the equations are much more likely to be non-linear; on the assumption that the friction is proportional to the square of the velocity, he obtained a value of 3.4. Using this estimate and the observed value of the secular acceleration of the Moon, he computed the rate of dissipation of energy to be about 1.4×10^{19} ergs/sec.

Several discussions of the tides in mid-ocean have been made, by G. H. DARWIN[1], JEFFREYS [*1*] and others[2]. It appears that neither viscosity nor turbulence in the oceans can cause sufficient friction to account for the required rate of dissipation of energy. JEFFREYS, for instance, estimates the dissipation of energy in the open oceans to be of the order of 10^{16} ergs/sec. The reason for this low value is that the rate of dissipation is proportional to the third power of the velocity of the water, while the velocities in the open oceans are small, of the order of 1 cm./sec.

In the shallow seas, however, the tidal currents can have much greater velocities and the rate of dissipation of energy can be considerable. But the areas of the shallow seas are relatively small, and detailed computation is needed to decide whether the total rate of dissipation of energy in these seas is adequate to account for the Moon's secular acceleration.

The rate of dissipation by the tides in the Irish Sea was determined by G. I. TAYLOR[3] in 1919 by two different methods. The mean rate of dissipation in this sea was found to be about 30 times the total rate of dissipation for the whole of the open oceans and about 2% of the total rate needed to account for the lunar secular acceleration.

TAYLOR's method was extended by JEFFREYS [*1*] to include most of the shallow seas of the globe, using the tide ranges and currents given in the *Admiralty Pilots*. The Bering Sea was found to be much the most important, this one sea accounting for about 70% of the total dissipation for the whole of the shallow seas. The total dissipation computed by JEFFREYS amounted to 2.2×10^{19} ergs/sec. at spring tides, the average rate being 1.1×10^{19} ergs/sec., which is about 80% of the amount needed to account for the secular acceleration of the Moon as determined by FOTHERINGHAM.

9. There is, however, another effect which has to be considered, to which attention has been called by HOLMBERG[4]. It is well known that there is a large semi-diurnal variation in the height of the barometer; the phase of this variation is remarkably constant over the globe. In 1882 Lord KELVIN[5] suggested that the close coincidence between the period of the Earth's rotation and the natural

[1] G. H. DARWIN: Collected Papers. Cambridge 1907.1916. (See Vol. 2.) Also Phil. Trans. A **170**, 447 (1879).

[2] See DEFANT's article on Ocean Tides in Vol. XLVIII of this Encyclopedia.

[3] G. I. TAYLOR: Phil. Trans. A **220**, 1 (1919).

[4] E. R. R. HOLMBERG: M. N. Geophys. Suppl. **6**, 325 (1952).

[5] W. THOMSON (Lord KELVIN): Proc. Roy. Soc. Edinburgh **11**, 396 (1882).

period of resonance of the atmosphere might provide the explanation of the large amplitude of the semi-diurnal barometric variation. He pointed out that the phase of the variation (with maxima occurring in the second and fourth quadrants after midnight) is such that the gravitational attraction of the Sun on the atmospheric tides exerts an accelerating couple on the Earth, whose amount he was able to estimate. This paper by KELVIN has been generally overlooked and the effect of the accelerating couple has been neglected in the discussions of the lunar secular acceleration.

More recently the resonance between the natural period of the atmosphere and the period of the rotation of the Earth has been called upon to account for the magnitude of the diurnal variation of the Earth's magnetic field. This requires the resonance to be very sharp, the periods not differing by more than a few minutes[1].

The acceleration of the rotation of the Earth by the atmospheric tidal couple requires a flow of angular momentum from the Earth's heliocentric orbit, which increases the mechanical energy of the system. HOLMBERG suggests that this energy is extracted from the solar energy falling on the Earth's surface by a heat-engine effect. His investigation of this effect shows that the maxima of the diurnal variations of temperature and pressure—the temperature maximum tending to occur in the afternoon and the maximum of the 24-hour pressure component in the forenoon—are such that work is done by the atmosphere. The mechanical energy produced in the atmosphere maintains a back pressure, which keeps the semi-diurnal maxima in the barometric height from slipping round into the first and third quadrants under the action of frictional drag from the Earth's surface.

A very large number of determinations of the semi-diurnal pressure variation have been made. Using SIMPSON'S[2] data for this variation

$$\Delta P = 1.25 \times 10^3 \sin^3\vartheta \cos(2t + 64°) \text{ dyn./cm.}^2 \qquad (9.1)$$

in which ϑ denotes the colatitude and t the local time, and following Lord KELVIN's argument, the total couple is found by HOLMBERG to be 3.7×10^{22} dyn./cm., working at a rate of 2.7×10^{18} erg./sec., which can be compared with the rate of working calculated by JEFFREYS for the oceanic tidal couple of 1.1×10^{19} erg./sec.

10. The value of the atmospheric couple is unlikely to be in error by more than a few per cent, whereas JEFFREYS considered that his computation of the tidal couple might be in error by half its amount. HOLMBERG has raised the question whether the two couples might not actually be equal in their rate of working. This possibility is related to the question whether the present close agreement between the rate of rotation of the Earth and the natural period of resonance of the atmosphere is fortuitous or not. He suggests that the period of rotation of the Earth was progressively slowed down by oceanic friction until it approached the resonance period of the semi-diurnal atmospheric tide. This approach was accompanied by a progressive increase in the amplitude of the atmospheric tide, which resulted in a progressive increase in the atmospheric accelerating couple. The retardation of the Earth's rotation continued until the two couples were brought into quasi-equilibrium, with the rotational and vibrational periods closely coincident.

[1] The sharpness of this resonance has recently been questioned by M. SIEBERT: Naturwiss. **41**, 446 (1954). See the article by W. KERTZ on Atmospheric Tides in Vol. XLVIII of this Encyclopedia.

[2] G. C. SIMPSON: Quart J. Roy. Met. Soc. **44**, 1 (1918).

This possibility will be discussed further when the determinations of the secular accelerations of the Sun and Moon have been considered. But it is necessary first to discuss another aspect of the Earth's rotation.

III. Fluctuations of the Motion of the Moon.

11. In 1870 NEWCOMB called attention to the existence of fluctuations in the motion of the Moon which were either of long period or of an irregular nature, and which did not seem to be accounted for by current theories. The reason that such fluctuations had not been detected earlier was that the theories of the motion of the Moon were not sufficiently complete. The theory of the motion of the Moon is of extreme complexity and has been gradually developed and refined by the researches of many eminent investigators. In 1857 the British Admiralty had published new tables of the Moon, based upon the investigations of the Danish astronomer P. A. HANSEN, to whom the British Government made a grant of £ 1,000 in recognition of his work. The Astronomer Royal, AIRY, said of HANSEN'S Tables that "probably in no recorded instance has practical science ever advanced so far by a single stride". HANSEN'S Tables were adopted as the basis for the computation of the ephemerides of the Moon published in the *Nautical Almanac*. They were believed to represent closely all observations of the Moon since 1750 and it was expected that they would continue to represent the motion of the Moon for many years to come. By the year 1870 this hope had been destroyed; the Moon had deviated from the positions computed on the basis of HANSEN'S Tables by an amount that could not be attributed to errors of observation. The departure between the tabular and observed positions went on increasing year by year.

In 1878 NEWCOMB [2] published a large memoir in which he discussed all available observations of the Moon, particularly those before 1750, and compared them with HANSEN'S Tables. He found that there were many observations of occultations of stars by the Moon recorded in the observation books of the Paris Observatory, which had not hitherto been published. He was able to extend the history of the Moon's motion back from 1750 to 1675 and, with a lesser degree of accuracy, thirty years farther still. He found that these earlier observations also showed an increasingly large deviation from the tabular positions. He concluded that either a) HANSEN'S theory was inadequate and that some important terms of long period had been omitted, which a careful revision of the theory should reveal; or b) the rotation of the Earth on its axis is subject to fluctuations of irregular character. He proposed the following criterion by which the two possibilities could be separated: if other celestial phenomena present fluctuations of the same general type, we must suspect the rotation of the Earth; if they do not, there must be some error in HANSEN'S theory or in his calculations.

NEWCOMB found that the fluctuations in the motion of the Moon from about 1650 onwards could be approximately represented by an empirical periodic term, with a period of 260 years, for which there was no gravitational explanation, and that superposed on it were irregular fluctuations of minor extent. His separation of the fluctuations into what became known as the *Great Empirical Term*, and the minor fluctuations, was followed by other investigators.

12. Though NEWCOMB spent many years in the attempt to account for the fluctuations, their origin was still undecided at the time of his death in 1909. The first essential question for their further elucidation was a decision whether the existing theories of the Moon (HANSEN'S and DELAUNAY'S) were inadequate. A complete determination of the perturbations in the motion of the Moon was needed. This work was undertaken by E. W. BROWN at the suggestion of Sir GEORGE DARWIN and the new theory of the motion of the Moon was published by him in five important memoirs[1]. On the completion of the theoretical investigations, the construction of new lunar tables, based on the theory, was undertaken. The tables were published in three large volumes in 1919 and have been used for predicting the place of the Moon in the *Nautical Almanac* from 1923 onwards. The accuracy of the tables has recently been checked by numerical integration with an electronic computing machine; they have been found to agree satisfactorily with the numerical integration, except for a few minor errors which are immaterial for the purposes of this discussion.

In his Tables BROWN included that portion of the total secular acceleration which is attributable to the gravitational attraction of the planets as computed by ADAMS, but not the portion which is attributable to the retardation of the rotation of the Earth by tidal friction. He incorporated NEWCOMB's great empirical term, though it remained without any theoretical justification, in order to obtain an approximate representation of the observations from about 1650 onwards. Observations since 1923 have shown a changing difference between the tables and observation, whether the great empirical term is included or excluded. As it is now certain that there is no defect in theory to which the fluctuations can be attributed, the presumption is that they are caused by fluctuations in the speed of rotation of the Earth, and therefore in our measure of time.

The decision whether this is the correct explanation depends upon whether similar fluctuations are present in the observed motions of Mercury, Venus and the Sun. Several investigations were devoted to this problem, by BROWN[2], DE SITTER[3] and SPENCER JONES [*3*], the question being finally settled by the latter.

13. It is necessary to adopt a value for the secular acceleration of the Moon due to tidal action, from the discussion of early observations of solar and lunar eclipses and of occultations. The ancient observations were very thoroughly discussed by FOTHERINGHAM in a series of papers and by SCHOCH. These discussions were co-ordinated by DE SITTER[3] and from a least squares solution, the following values of the secular accelerations were derived:

$$\left.\begin{array}{ll}\text{for the Sun} & +1''\!.80 \pm 0''\!.16\\ \text{for the Moon} & +5''\!.22 \pm 0''\!.30.\end{array}\right\} \tag{13.1}$$

In the investigation by SPENCER JONES [*3*] the fluctuation in the position of the Moon is denoted by B, defined as

$$B = \text{Observed Longitude} - C \tag{13.2}$$

where $C = \text{BROWN's Tables} - 10''\!.71 \sin(140^\circ\!.0\, T + 240^\circ\!.7)$

$$+ 5''\!.22\, T^2 + 12''\!.96\, T + 4''\!.65 \tag{13.3}$$

in which T denotes centuries from 1900.0. The sine term removes the great empirical term, the term in T^2 incorporates the effect of the secular retardation of the Earth's rotation, and the other terms represent the consequential

[1] E. W. BROWN: Mem. Roy. Astronom. Soc. **53**, 39, 163 (1899); **54**, 1 (1904); **57**, 51 (1905); **59**, 1 (1908).

[2] E. W. BROWN: Trans. Yale U. Obs. **3**, pt. 6 (1926).

[3] W. DE SITTER: Bull. Astronom. Inst. Netherlands **4**, 21 (1927).

corrections to the mean motion and longitude at epoch in order to secure close agreement with modern observations.

It is assumed provisionally that the fluctuations in the Moon's position are due to some cause or causes affecting the Earth only, such as changes in its moment of inertia, or changes in its angular momentum which are compensated by changes in the angular momentum of the atmosphere or the oceans. The effects on the mean longitudes of other bodies will then be similar to the effects on the mean longitude of the Moon, but reduced in proportion to the ratio of their mean motions in longitude.

When we come to consider the effects of tidal friction, there is an important difference. Tidal friction does not concern the Earth alone; it has an influence on the motion of the Moon. But in order to calculate the exact quantitative effects produced on the motion of the Moon, it would be necessary to have a detailed knowledge of the forces that are acting, which is lacking. The apparent discordances in the positions of the Sun and planets, consequent upon the retardation of the Earth's rotation by tidal friction, will have the same value when expressed in time and will therefore be proportional, when expressed as differences of longitude, to the mean motions. In this respect tidal friction behaves in a similar way to change of moment of inertia of the Earth. The discordance in position of the Moon, due to tidal friction, is not proportional to its mean motion, however, and in this respect tidal friction behaves differently from change of moment of inertia.

14. Accordingly it is to be expected that the observed minus tabular differences in longitude of the Moon, Sun, Mercury and Venus should be capable of representation by the following expressions:

$$\text{Moon} \qquad \Delta L = a + b\,T + 5''.22\,T^2 + B, \tag{14.1}$$

$$\text{Sun} \qquad \Delta l' = a' + b'\,T + c'\,T^2 + \frac{n'}{n} B, \tag{14.2}$$

$$\text{Mercury} \qquad \Delta l'' = a'' + b''\,T + \frac{n''}{n'} c'\,T^2 + \frac{n''}{n} B, \tag{14.3}$$

$$\text{Venus} \qquad \Delta l''' = a''' + b'''\,T + \frac{n'''}{n'} c'\,T^2 + \frac{n'''}{n} B, \tag{14.4}$$

where n, n', n'', n''' are the mean motions of the Moon, Sun, Mercury and Venus respectively; $5''.22$ is adopted as the secular acceleration of the Moon; c' denotes the secular acceleration of the Sun; and B denotes the fluctuation in the motion of the Moon, defined as above.

The observations of the transits of Mercury across the Sun, which are available from the transit of November 1677, provide the most complete data. Observations of the Sun's declination are more reliable for the present purpose than observations of its right ascension, being less affected by personal errors of observation and by changes in methods. Observations of the Sun's declination from 1760 can be used, and of its right ascension from 1835. Observations of the right ascension of Venus from 1835 can be used.

The data were analysed by SPENCER JONES [*3*] and the constants in the above formulae were determined. If the observational data are satisfactorily represented by the formulae, the values of B derived for each body on substituting the values of $a, a', a'', a''', b, b', b'', b'''$ and c' should be in satisfactory agreement. That this is in fact the case is shown by Figs. 1 and 2. The first