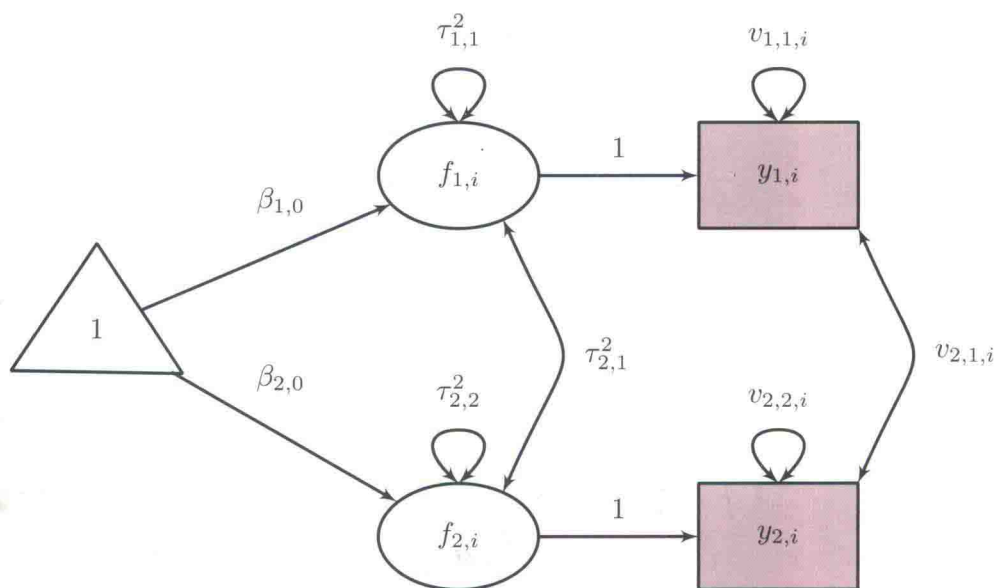


Meta-Analysis

A Structural Equation Modeling Approach



MIKE W.-L. CHEUNG



WILEY

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This edition first published 2015
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Registered office

John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom

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Library of Congress Cataloging-in-Publication Data applied for

A catalogue record for this book is available from the British Library.

ISBN: 9781119993438

Set in 10/12pt TimesRoman by Laserwords Private Limited, Chennai, India.

Printed and bound in Singapore by Markono Print Media Pte Ltd.

Meta-Analysis

*For my family—my wife Maggie,
my daughter little Ching Ching, and my parents*

Preface

"If all you have is a hammer, everything looks like a nail."

—Maslow's hammer

Purpose of This Book

There were two purposes of writing this book. One was personal and the other was more "formal." I will give the personal one first. The primary motivation for writing this book was to document my own journey in learning structural equation modeling (SEM) and meta-analysis. The journey began when I was a undergraduate student. I first learned SEM from Wai Chan, my former supervisor. After learning a bit from the giants in SEM, such as Karl Jöreskog, Peter Bentler, Bengt Muthén, Kenneth Bollen, Michael Browne, Michael Neale, and Roderick McDonald, among others, I found SEM fascinating. It seems that SEM is *the* statistical framework for all data analysis. Nearly all statistical techniques I learned can be formulated as structural equation models.

In my graduate study, I came across a different technique—meta-analysis. I learned meta-analysis by reading the classic book by Larry Hedges and Ingram Olkin. I was impressed that a simple yet elegant statistical model could be used to synthesize findings across studies. It seems that meta-analysis is the key to advance knowledge by combining results from different studies. As I was trained with the SEM background, everything looks like a structural equation model to me. I asked the question, "could a meta-analysis be a structural equation model?" This book summarized my journey to answer this question in the past one and a half decades.

Now, I will give a more formal purpose of this book. With the advances in statistics and computing, researchers have more statistical tools to answer their research questions. SEM and meta-analysis are two powerful statistical techniques in the social, educational, behavioral, and medical sciences. SEM is a popular tool to test hypothesized models by modeling the latent and observed variables in primary research, while meta-analysis is a *de facto* tool to synthesize research findings from a pool of empirical studies. These two techniques are usually treated as two unrelated topics in the literature. They have their own strengths,

weaknesses, assumptions, models, terminologies, software packages, audiences, and even journals (*Structural Equation Modeling: A Multidisciplinary Journal* and *Research Synthesis Methods*). Researchers working in one area rarely refer to the work in the other area. Advances in one area have basically no impact on the other area.

There were two primary goals for this book. The first one was to present the recent methodological advances on integrating meta-analysis and SEM—the *SEM-based meta-analysis* (using SEM to conducting meta-analysis) and meta-analytic structural equation modeling (conducting meta-analysis on correlation matrices for the purpose of fitting structural equation models on the pooled correlation matrix). It is my hope that a unified framework will be made available to researchers conducting both primary data analysis and meta-analysis. A single framework can easily translate advances from one field to the other fields. Researchers do not need to reinvent the wheels again.

The second goal was to provide accessible computational tools for researchers conducting meta-analyses. The `metaSEM` package in the R statistical environment, which is available at <http://courses.nus.edu.sg/course/psycwlm/Internet/metaSEM/>, was developed to fill this gap. Using the `OpenMx` package as the workhorse, the `metaSEM` package implemented most of the methods discussed in this book. Complete examples in R code are provided to guide readers to fit various meta-analytic models. Besides the R code, *Mplus* was also used to illustrate some of the examples in this book. R (3.1.1), `OpenMx` (2.0.0-3654), `metaSEM` (0.9-0), `metafor` (1.9-3), `lavaan` (0.5-17.698), and *Mplus* (7.2) were used in writing this book. The output format may be slightly different from the versions that you are using.

Level and Prerequisites

Readers are expected to have some basic knowledge of SEM. This level is similar to the first year of research methods covered in most graduate programs. Knowledge of meta-analysis is preferable though not required. We will go through the meta-analytic models in this book. It will also be useful if readers have some knowledge in R because R is the main statistical environment to implement the methods introduced in this book. Readers may refer to Appendix at the end of this book for a quick introduction to R. For readers who are more familiar with *Mplus*, they may use *Mplus* to implement some of the methods discussed in this book.

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Acknowledgments

I thank Wai Chan, my former supervisor, for introducing me to the exciting field of structural equation modeling (SEM). He also suggested me to explore meta-analytic structural equation modeling in my graduate studies. I acknowledge the suggestions and comments made by many people: Shu-fai Cheung, Adam Hafdahl, Suzanne Jak, Yonghao Lim, Iris Sun, and Wolfgang Viechtbauer. All remaining errors are mine. I especially thank my wife for her support and patience. My daughter was born during the preparation of this book. I enjoyed my daughter's company when I was writing this book. Part of the book was completed during my sabbatical leave supported by the Faculty of Arts & Social Sciences, the National University of Singapore. I also appreciate the funding provided by the Faculty to facilitate the production of this book. I thank Heather Kay, Richard Davies, Jo Taylor, and Prachi Sinha Sahay from Wiley. They are very supportive and professional. It has been a pleasure working with them.

The `metaSEM` package could not be written without `R` and `OpenMx`. Contributions by the `R` Development Core Team and the `OpenMx` Core Development Team are highly appreciated. Their excellent work makes it possible to implement the techniques discussed in this book. I have to specially thank the members of the `OpenMx` Core Development Team for their quick and helpful responses in addressing issues related to `OpenMx`. I also thank Yves Rosseel for answering questions related to the `lavaan` package. Finally, the preparation of this book was mainly based on the open-source software. This includes `LATEX` for typesetting this book, `R` for the analyses, `Sweave` for mixing `R` and `LATEX`, `Graphviz` and `dot2tex` for preparing the figures, `GNU make` for automatically building files, `Git` for revision control, `Emacs` for editing files, and finally, `Linux` as the platform for writing.

List of abbreviations

Abbreviation	Full name
CFA	confirmatory factor analysis
CFI	comparative fit index
CI	confidence interval
FIML	full information maximum likelihood
GLS	generalized least squares
LBCI	likelihood-based confidence interval
LL	log likelihood
LR	likelihood ratio
MASEM	meta-analytic structural equation modeling
ML	maximum likelihood
NNFI	non-normed fit index
OR	odds ratio
OLS	ordinary least squares
RAM	reticular action model
REML	restricted (or residual) maximum likelihood estimation
RMD	raw mean difference
RMSEA	root mean square error of approximation
SE	standard error
SEM	structural equation modeling
SMD	standardized mean difference
SRMS	standardized root mean square residual
TLI	Tucker–Lewis index
TSSEM	two-stage structural equation modeling
UMM	unweighted method of moments
WLS	weighted least squares
WMM	weighted method of moments

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