MARCO L. BITTENCOURT

COMPUTATIONAL SOLID MECHANICS

Variational Formulation and High Order Approximation

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Dedication

I dedicate this book to my wife Maristela, my son Felipe, and my daughter Ana Luísa. I would like to express my gratitude to my parents, Fábio and Terezinha, who have provided me with very good educational opportunities.

Preface

Nowadays computational simulation is a basic tool in the development of engineering projects. Due to the increase of computing resources, it has become possible to consider together many aspects of the mechanical models.

Simulation software, such as ANSYS, ABAQUS, and NASTRAN, implement a discrete version of continuous mathematical models. As the numerical models become more complex, engineers require a strong and fundamental background to confidently use the software features. Learning the fundamental concepts of mechanics and approximation in a general way should be the starting point for the application of computers in solving real engineering applications. The impossibility of understanding clearly the fundamental assumptions and limitations of the mechanical and numerical models makes it highly likely that engineers will obtain computational solutions which do not represent the actual behavior of mechanical components.

There have been many books presenting the formulation and finite element approximation of solid mechanical models. Relative to the books on formulation of mechanical models, particular cases in nonpedagogical and/or very formal approaches are presented. Books on finite elements do not make clear the boundary between the mechanical models and their approximations. These aspects make the learning process of engineering students difficult. Another aspect is that in the computer era, it is crucial to organize the way that the information is supplied to students, as they have access to many information sources from the Internet. Using standard procedures to formulate and approximate models and at the same time illustrating their application by software are very important aspects. This book intends to address these points.

In terms of formulation of mechanical models, the basic tool considered here is the variational formulation based on the principle of virtual work (PVW). All models are presented following the same sequence of steps, which includes kinematics, strain measure, rigid body deformation, internal loads, external loads, equilibrium, constitutive equations, and structural design. This sequence allows the reader to establish a logical reasoning for the treatment of any mechanical model. In addition, all aspects of a mechanical model are presented in each chapter and not spread out in many chapters, as is common in many books on solid mechanics. Mechanical models for plates and solids models are also considered using the same approach.

In terms of finite element approximation, the book starts with simple applications of low-order approximation to bars and shafts elements. The main concepts are introduced gradually in the others chapters, including high-order approximations. As in the formulation, all approximations are presented following the same sequence of steps which includes the definition of strong form, weak form, global and local approximations, finite elements, and applications.

In terms of software, MATLAB scripts and an object-oriented high-order program are supplied with the book to run examples.

Taking into consideration these three main aspects, readers should learn the limitations and strengths of the considered mechanical models, their approximations, and how they are implemented in computer software.

The book is intended for the use by undergraduate and beginning graduate students in engineering. Most of the chapters include many examples, problems, and software applications. This edition will be limited to models with small deformation and linear material behavior. The book is organized in 10 chapters. Chapter 1 provides a general introduction to variational formulation and an overview of the mechanical models to be presented in the other chapters. Chapter 2 presents a review about the Newton and variational formulations, the principle of virtual work, and the equilibrium of particles and rigid bodies using the PVW. The main idea is to use the concepts on equilibrium that readers should already have to introduce basic notions on kinematics, virtual work, and the PVW.

Chapters 3 to 10 present mechanical models, approximations, and applications to bars, shafts, beams, beams with shear, general two- and three-dimensional beams, solids, plane models, and general torsion, and plates. In particular, Chapter 8 presents the most general case of solids using two approaches. The first one follows the basic idea of the other chapters. In the second approach, the concept of second-order tensor is introduced using a Taylor expansion, and the solid model is reformulated using again the same previously formulation steps. In this case, small and large deformations are considered. After the presentation of elastic solids, the kinematical hypotheses of the previously considered problems are introduced in this model. It is then possible to observe where the simplifications are introduced in the solids to formulate the previous cases. Chapter 9 presents a more formal introduction to variational formulation based on the general steps applied to the other chapters.

I believe that the main features of the book are: the systematic and pedagogical approaches to formulate and approximate solid mechanical models, starting from simple cases and going to more complex models; a clear separation of formulation and finite element approximation; and the user-friendly MATLAB software.

I have used this material at the University of Campinas in Brazil for about 15 years. We have two one-semester courses on solid mechanics and another one-semester course on numerical methods in engineering for undergraduate students. I have also used the material included in this book for a one-semester course for beginner graduate students. I used also part of this material for a graduate course at the Division of Applied Mathematics at Brown University in 2010.

I would like to express my invaluable gratitude to my PhD advisor, Professor Raúl A. Feijóo of the National Laboratory of Scientific Computation at Petropólis in Brazil, who introduced me the fundamentals of mechanics and variational formulation. I would like also to thank Professor George Em Karniadakis of the Division of Applied Mathematics at Brown University for his careful review of the book proposal and suggestions. I would like to thank my students for their collaboration during the time I have written this material. In particular, I would like to thank Cláudio A. C. Silva, Rodrigo A. Augusto, Jorge L. Suzuki, and Allan P. C. Dias for their invaluable help in the preparation of the many versions that led to this book. Finally, I would like to thank CRC Press for the publication of the book.

The MATLAB scripts used in this book are available online at `www.facebook.com/ComputationalSolidMechanics`.

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