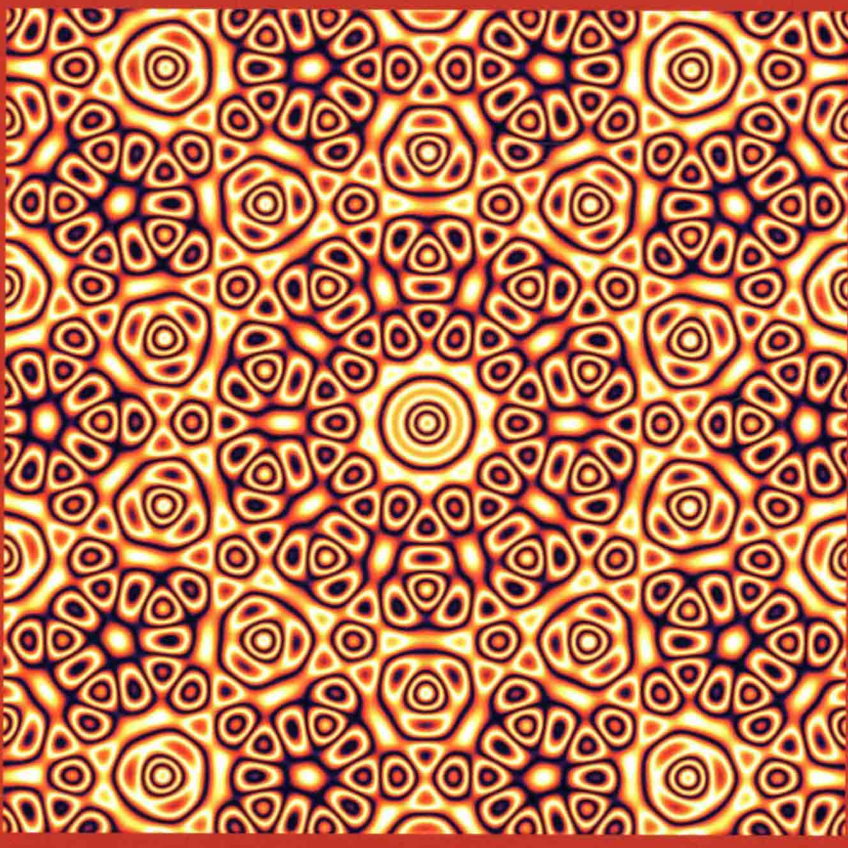
Collective Classical and Quantum Fields

in Plasmas, Superconductors, Superfluid ^{3}He, and Liquid Crystals

Hagen Kleinert



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This is an introductory book dealing with collective phenomena in many-body systems. A gas of bosons or fermions can show oscillations of various types of density. These are described by different combinations of field variables. Especially delicate is the competition of these variables. In superfluid ^{3}He, for example, the atoms can be attracted to each other by molecular forces, whereas they are repelled from each other at short distance due to a hardcore repulsion. The attraction gives rise to Cooper pairs, and the repulsion is overcome by paramagnon oscillations. The combination is what finally led to the discovery of superfluidity in ^{3}He. In general, the competition between various channels can most efficiently be studied by means of a classical version of the Hubbard-Stratonovich transformation.

A gas of electrons is controlled by the interplay of plasma oscillations and pair formation. In a system of rod- or disc-like molecules, liquid crystals are observed with directional orientations that behave with unusual five-fold or seven-fold symmetry patterns. The existence of such a symmetry was postulated in 1975 by the author and K. Maki. An aluminium material of this type was later manufactured by Dan Shechtman which won him the 2014 Nobel prize. The last chapter presents some solvable models, of which one of them was the first to illustrate the existence of broken supersymmetry in nuclei.

Hagen Kleinert is professor of physics at the Freie Universität Berlin, Germany. As a visiting scientist, he has spent extended periods of time at CERN, the European Organization for Nuclear Research in Geneva; at Caltech in Pasadena; at the Universities of California in Berkeley, Santa Barbara, and San Diego; at the Los Alamos National Laboratory in New Mexico; and at Princeton University, New Jersey. He has made numerous contributions to the understanding of particle physics, mathematical physics, condensed matter physics, chemical physics, and nuclear physics. His two-volume book *Gauge Fields in Condensed Matter* (World Scientific, 1989) develops a new quantum field-theory of phase transitions on the basis of *disorder fields*. Such fields have meanwhile become a powerful tool supplementing the *order fields* introduced by the Landau school. Another book on *Path Integrals* appeared with World Scientific in 1990 (5th edition, 2009). It is so far the most comprehensive text on this subject. He also published with World Scientific in 2001, in collaboration with V. Schulte-Frohlinde, a thorough review book on the field-theoretic renormalization group approach to critical phenomena. The title is *Critical Properties of ϕ^4-Theories*. The divergent perturbation expansions are evaluated with exponentially fast convergence using a new technique called VPT. This evolved from a paper by the author with Richard Feynman. The list of books by the author continues with *Multivalued Fields in Condensed Matter, Electromagnetism, and Gravitation* (World Scientific, 2008). There he shows that many phenomena in physics, in particular all those commonly explained by vortices and by gauge theories, can be understood as consequences of a Riemann-sheet nature of fields. His latest book, *Particles and Quantum Fields* (World Scientific, 2016), explains the presently known elementary particles and studies their interactions using the modern techniques of quantum field theory.

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To Annemarie and Hagen II

Preface

Strongly interacting many-body systems behave often like a system of weakly interacting collective excitations. When this happens, it is theoretically advantageous to replace the original action involving the fundamental fields (electrons, nucleons, ^{3}He, ^{4}He atoms, quarks etc.) by another action in which only certain collective excitations appear as independent quantum fields. Mathematically, such replacements can be performed in many different ways without changing the physical content of the initial theory. Experimental understanding of the important processes involved can help theorists to identify the *dominant* collective excitations. If they possess only weak residual interactions, these can be treated perturbatively. The associated collective field theory greatly simplifies the approximate description of the physical system.

It is the purpose of this book to discuss some basic techniques for deriving such collective field theories. They are based on Feynman's functional integral formulation of quantum field theory. In this formulation, the transformation to collective fields amounts to mere changes of integration variables in functional integrals.

Systems of charged particles may show excitations of a type whose quanta are called *plasmons*. For their description, a real field depending on one space and one time variable is most convenient. If the particles form bound states, a complex field depending on two spacetime coordinates renders the most economic description. Such fields are *bilocal*, and are referred to as *pair fields*. If the attractive potential is of short range, the bilocal field simplifies to a local field. This has led to the field theory of superconductivity by Ginzburg and Landau. A bilocal theory of this type has been used in elementary-particle physics to explain the observable properties of strongly interacting mesons.

The change of integration variables in path integrals will be shown to correspond to an exact resummation of the perturbation series, thereby accounting for phenomena which cannot be described perturbatively in terms of fundamental particles. The path formulation has the great advantage of translating all quantum effects among the fundamental particles completely into the field language of collective excitations. All fluctuation corrections may be computed using only propagators and interaction vertices of the collective fields.

The method becomes unreliable if several collective effects compete with each other. An example is a gas of electrons and protons at low density where the attractive forces can produce hydrogen atoms. They are absent in a description involving a plasmon field. A mixture of plasmon and pair effects is needed to describe these.

Another example is superfluid ^{3}He, where pairing forces are necessary to produce the superfluid phase transition. Here plasma-like magnetic excitations called *paramagnons* provide strong corrections. In particular, they are necessary to obtain the pairing in the first place. If we want to tackle such mixed phenomena, another technique must be used called *variational perturbation theory*.

In Chapter 1, I explain the mathematical method of changing from one field description to another by going over to *collective fields* representing the dominant collective excitations. In Chapters 2 and 3, I illustrate this method by discussing simple systems such as an electron gas or a superconductor. At the end of Chapter 3, I had good help from my collaborator S.-S. Xue, with whom I wrote the basic strong-coupling paper (`arxiv:cond-mat/1708.04023`), that is cited as Ref. [89] on page 143. In Chapter 4, I apply the technique to superfluid ^{3}He. In Chapter 5, I use the field theoretic methods to study physically observable phenomena in liquid crystals. In Chapter 6, finally, I illustrate the working of the theory by treating some simple solvable models.

I want to thank my wife Dr. Annemarie Kleinert for her great patience with me while writing this book. Although her field of interest is French Literature and History (her homepage https://a.klnrt.de), and thus completely different from mine, her careful reading detected many errors. Without her permanent reminding me of the still missing explanations of certain questions I could never have completed this work. My son Michael, who just received his PhD in experimental physics, deserves the credit of asking many relevant questions and making me improve my sometimes too formal manuscript.

Berlin, December 2017
H. Kleinert

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