Adaptive Control for Robotic Manipulators

Editors

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Preface

The robotic mechanism and its controller make a complete system. As the robotic mechanism is reconfigured, the control system has to be adapted accordingly. The need for the reconfiguration usually arises from the changing functional requirements. This book will focus on the adaptive control of robotic manipulators to address the changed conditions. The aim of the book is to introduce the state-of-the-art technologies in the field of adaptive control of robotic manipulators in order to further summarize and improve the methodologies on the adaptive control of robotic manipulators. This will be the first book that systematically and thoroughly deals with adaptive control of robotic manipulators. Advances made in the past decades are well described in this book, including adaptive control theories and design, and application of adaptive control to robotic manipulators.

We would like to thank all the authors for their contributions to the book. We are also grateful to the publisher for supporting this project. We hope the readers find this book informative and useful.

This book consists of 16 chapters. Chapter 1 discusses the role of machine learning in control from a modelling perspective and some of the most common machine learning models such as NN and mixture models were presented. Particularly, the idea of motor primitives was introduced as a biologically inspired method for motion representation and learning. Chapter 2 reviews the model reference adaptive control of robotic manipulators. Some issues of model reference adaptive control (MRAC) for robotic manipulators are covered. Very few recent papers can be found in the area of model reference adaptive control of robotic manipulators. This chapter will provide a guideline for future research in the direction of model reference adaptive control for robotic arms. Chapter 3 develops a concurrent learning based adaptation law for general Euler-Lagrange systems. Chapter 4 discusses an autonomous space robot for a truss structure assembly using some reinforcement learning. An autonomous space robot able to obtain proficient and robust skills by overcoming errors to complete a proposed task. Chapter 5 describes adaptive control for object manipulation in accordance with the contact condition and reviews the mathematical models and basic theory for stability analysis. Chapter 6 studies the networked control for a class of uncertain dynamical systems, where the control signals are computed via processors that are not attached to the dynamical systems and the feedback loops are closed over wireless networks. Chapter 7 presents the design of adaptive control of both robot manipulator and consensus-based formation of networked mobile robots in the presence of uncertain robot dynamics and with event-based feedback. Chapter 8 proposes a hybrid controller by combining a PID controller and a model reference adaptive controller (MRAC), and also compares the convergence performance of the PID controller, model reference adaptive controller, and PID+MRAC hybrid controller. This study will provide a guideline for future research in the direction of new controller designs for manipulators in terms of convergence speed and other performances. Chapter 9 presents an approach to using an impedance-controlled ankle exoskeleton ("anklebot") for task-oriented locomotor training after stroke. The objective is to determine the feasibility of using the anklebot as a gait training tool by increasing the contribution of the paretic ankle in walking function. Chapter 10 presents the open architecture high value added robot manufacturing cells. Chapter 11 discusses two basic control problems, namely set point and trajectory following control in application to rigid body manipulators with joint flexibility. Chapter 12 presents how robotic bipedal walking control can unify locomotion, manipulation, and force-based tasks into a single framework via quadratic programs utilizing control Lyapunov functions. Two common examples where the unification can be applied are introduced: ZMP-based pattern generation and locomotion, and nonlinear dynamics with push recovery. Chapter 13 develops a robust adaptive back stepping based controller considering a general uncertain dynamic model of a multi-link robotic manipulator. The designed dynamic controller only requires knowledge of the inertia matrix of the rigid links of the manipulator and is robust to uncertainties in joint stiffness, Coriolis and centrifugal terms, friction, gravity, load torques and disturbances, and actuator inertia. Chapter 14 proposes a new adaptive switching learning control approach, called adaptive switching learning PD control (ASL-PD), for trajectory tracking of robot manipulators in an iterative operation mode. Chapter 15 presents a method to design an adaptive robust control based on online estimation of the lumped time-varying model uncertainties for tracking control of the robot manipulators. Chapter 16 is concerned with evaluation of microgenetic and microimmune optimization algorithms for solving inverse kinematics of a redundant robotic arm as a prerequisite for adaptive trajectory planning in various manipulation tasks.

The editors would like to sincerely acknowledge all friends and colleagues who have contributed to this book.

Oshawa, Ontario, Canada **Dan Zhang**
March 2016 **Bin Wei**

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From MRAC to Learning-Based MPC

The Emerging Importance of Machine Learning for Control of Robot Manipulators

*K. Soltani Naveh** and *P. R. McAree*

ABSTRACT

The increasing importance of machine learning in manipulator control is reviewed from two main perspectives: modeling and learning control. The chapter starts with an introduction to history and theory of Model Reference Adaptive Control (MRAC) and its application to manipulator control. Least Square Minimization (LSM) regression is highlighted as the machine learning element in indirect MRAC that seeks to find unknown parameters from a number of data points. The limitations of indirect and direct MRAC are identified. Specifically, indirect MRAC is limited by the need for persistent excitation and the use of simple modeling assumptions that lead to undesirable control performance. Direct MRAC is limited by reference model mismatch, adaptation rate, choice of control law, and relying on error feedback correction with the frequent consequence of stability and adaptation problems. Moreover, neither direct nor indirect MRAC can handle state and input constraints which are important in the control of robotic manipulators. Machine learning techniques offer the promise of overcoming these limitations. Recent developments include Concurrent-MRAC which is able to alleviate the persistent excitation requirement for LTI systems and may have application to the control of robotic manipulators. The chapter covers the broader contributions of machine learning

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to recent developments in manipulator control that combine learning and adaptation. This includes: (i) the use of advanced modeling methods such as Mixture Models and Neural Networks and (ii) learning control methods such as Iterative Learning Control and modern Reinforcement Learning and their relation to adaptive control. State and input constraints are identified to be one of the significant remaining challenges in manipulator control. Model Predictive Control (MPC) is introduced as the control method that can handle state and input constraints in its formulation. A number of recent attempts to incorporate learning capabilities in MPC are discussed.

Keywords: Machine learning, learning control, neural networks, model predictive control

INTRODUCTION

Model Reference Adaptive Control (MRAC) was first employed for robot manipulator control in 1980s as researchers explored direct drive technology to eliminate gearboxes with the aim of faster more accurate manipulation. Before that, manipulators were generally controlled using PD controllers at each joint. Although manipulator dynamics is nonlinear, this approach worked in practice due to large gear ratios that made nonlinear inertia effects negligible from the view of the control system. In this case, it has been proven that PD controllers successfully accomplish trajectory tracking (Slotine and Li 1991). Once direct drive technology was adopted and gearboxes were eliminated faster operations became possible and undesirable gear effects such as deadzone and backlash were eliminated. However, nonlinear rigid body effects became no longer negligible and the conventional joint based PD controllers were unable to deliver required levels of performance. A MIMO control design approach that took into account the coupling and nonlinear effects was required.

Early interest in the application of MRAC to manipulator control was also motivated by model mismatch or model uncertainty. The emergence of direct drive manipulators and the push for faster operation speed, led to the development of the so-called 'computed torque' control method, which uses a rigid body model of the manipulator to compute feedforward terms which are typically applied in conjunction with a feedback strategy. Modeling errors were, however, found to be significant enough to impact on trajectory tracking. This necessitated the development of control systems that were able to estimate the unknown model parameters and compute the inputs according to the estimated plant model.

As robot manipulators were increasingly applied to problems that required interaction with changing environment, manipulator controllers had to adapt to those changes in order to maintain acceptable performance. These applications involve changing loads, varying compliance and geometry.

MRAC was introduced as a control methodology in 1961 (Astrom and Wittenmark 2013, Osburn et al. 1961). It was proposed for manipulator control in 1979 by Dubowsky (Dubowsky and Deforges 1979). The adaptation mechanism in MRAC, in principle, allows the control system to maintain a desired response robust to model mismatch and changes to the plant. One way to specify the desired behavior is through defining a reference model. This is the basis for so-called ***direct MRAC***.

Alternatively, a desired behavior can be achieved by estimating the unknown or changing parameters of the system and adapting the control input based on those estimations. This approach is known as ***indirect MRAC*** and it is the main focus of this chapter.

Indirect MRAC uses data to "learn" unknown parameters and this places it as essentially a machine learning algorithm. Machine learning is a well-studied and established area that focuses on algorithms that enable machines to adapt and change without the need to be reprogrammed. The advances that have been made in machine learning over the last 30 years offer the prospect for superior modeling approaches than classic adaptive control methods. For instance modern machine learning approaches now allow modeling a particular task such as playing table tennis (Muelling et al. 2010) while classic control approaches only work with plant models.

While MRAC was able to address most of the challenges arising from nonlinear effects, model mismatch and parameter variations, some other issues remained. As gearboxes were removed and direct drive technology replaced them, another issue appeared. The actuators would saturate when high performance was desired and this would lead to poor performance. In order to avoid constraint violation in systems, it is necessary to be able to predict or know in advance the response of the system to an input. This is the fundamental idea behind Model Predictive Control (Maciejowski 2001) which is capable of handling state and input constraints while optimizing the performance with respect to a defined metric. With the increase in computational power MPC is now applied to many systems including robotic manipulators for trajectory tracking applications (Verscheure et al. 2009). The success of MPC predominantly relies on the extent to which an analytical and linear model represents the behavior of the plant and allows predictions to be made. If the response of the plant includes unknown elements or elements that are very difficult to represent analytically or are highly nonlinear then predictions will be inaccurate or even impossible.

Just as machine learning allowed indirect MRAC to model a system with unknown parameters, it seems plausible it can help obtain better models for MPC. The composition of such models based on classical methods and modern machine learning methods will dominate part of the discussion in this chapter.

The structure of this chapter is as follows. The underlying theory of MRAC and its capabilities and limitations are explained as background to the work. Then we look at adaptation from a machine learning perspective and discuss how machine learning has the potential to improve control capabilities, especially for nonlinear systems. Following that we give a number of machine learning methods and include examples of their robotic applications. Then learning control is briefly introduced and compared to adaptive control. Finally we present MPC as the method of choice for optimal control of constrained systems and how machine learning methods are being used to combine MPC with learning and adaptive control.

FOUNDATIONS OF MRAC

Adaptive control is centered around the notion that it is necessary to adjust the parameters of a control law when faced with uncertainties that change the dynamic of a process. This uncertainty could be in the form of operational variations or structural unknowns. One particularly important application for adaptive control is in manipulator robots. For example, in manipulator control end-effector load variations can be significant. The load variations arise from interacting with a changing environment or time varying dynamics.

MRAC has been one of the well-known adaptive control methods for handling such problems. Direct MRAC allows for a desired system behavior to be specified as a reference model. This is particularly useful for MIMO systems such as robotic manipulators. Direct MRAC attempts to drive the plant response to reference model response by parametrizing the controller and estimating the "controller" parameters online. Indirect MRAC has as its objective obtaining the best estimates of "plant" parameters first and then using them to produce a suitable control input. This is why the adaptation process occurs indirectly. Common parameter estimation methods are based around Recursive Least Square (RLS) (Astrom and Wittenmark 2013), which can also be interpreted as Kalman filtering and Lyapunov based estimation (P. and Sun 1996, Slotine and Li 1987).

Direct MRAC

Direct MRAC is commonly formulated using Lyapunov methods (Astrom and Wittenmark 2013) with the advantage that such formulations are based in considerations of stability. However this approach requires identification of an appropriate Lyapunov function.

By way of introduction to the approach, consider the following first order system based off an example given in (Astrom and Wittenmark 2013):

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c$$

where $a_m > 0$ and the reference signal is bounded. Assume that the plant is described by

$$\frac{dy}{dt} = -ay + bu,$$

and the control law is defined as

$$u = \theta_1 u_c - \theta_2 y.$$

The error is then

$$e = y - y_m$$

$$\frac{de}{dt} = -ay + b(\theta_1 u_c - \theta_2 y) + a_m y_m - b_m u_c$$

$$\frac{de}{dt} = (b\theta_1 - b_m)u_c + (-a - b\theta_2)y + a_m y_m$$

$$\frac{de}{dt} = -a_m e + (b\theta_1 - b_m)u_c + (-a - b\theta_2 + a_m)y$$

Knowing the error dynamics helps in creating the Lyapunov function. Since the aim is to drive the error to zero a good candidate Lyapunov function is

$$V(e, \theta_1, \theta_2) = \frac{1}{2}e^2 + \frac{1}{2b\gamma}(b\theta_1 - b_m)^2 + \frac{1}{2b\gamma}(b\theta_2 + a - a_m)^2.$$

This function adapts the control law while driving the error to zero.

Now evaluating $\frac{dV}{dt}$ and enforcing it to be negative definite would lead to the adaptation equations as demonstrated below:

$$\frac{dV}{dt} = e\dot{e} + \frac{1}{\gamma}(b\theta_1 - b_m)\dot{\theta}_1 + \frac{1}{\gamma}(b\theta_2 + a - a_m)\dot{\theta}_2,$$

$$\frac{dV}{dt} = e(-a_m e + (b\theta_1 - b_m)u_c + (-a - b\theta_2 + a_m)y) + \frac{1}{\gamma}(b\theta_1 - b_m)\dot{\theta}_1 + \frac{1}{\gamma}(b\theta_2 + a - a_m)\dot{\theta}_2,$$

$$\frac{dV}{dt} = -a_m e^2 + (b\theta_1 - b_m)(\frac{1}{\gamma}\dot{\theta}_1 + u_c e) + (b\theta_2 + a - a_m)(\frac{1}{\gamma}\dot{\theta}_2 - ye),$$

To ensure that $\dot{V} <= 0$ we need to set $\frac{1}{\gamma}\ \theta_1 + u_c e$ and $\frac{1}{\gamma}\ \theta_2 - ye$ to zero leading us to the following adaptation equations:

$$\dot{\theta}_1 = -\gamma u_c e$$
$$\dot{\theta}_2 = \gamma y e.$$

This approach extends to general linear dynamic systems. Choosing a notation similar to that of Chowdhary et al. (Chowdhary et al. 2013), consider a linear system with state $x \in R^n$ and input $u \in R^m$:

$$\dot{x}(t) = Ax(t) + Bu(t).$$

Assume that the reference model that represents the desired close loop response is:

$$\dot{x}_{rm}(t) = A_{rm}x_{rm} + B_{rm}\mathrm{r}(t)$$

where A_{rm} is Hurwitz. A control law can be defined as a combination of feedforward and feedback terms:

$$u(t) = K_x^T(t)x(t) + K_r^T(t)r(t).$$

Substituting the control law into the state space equation of the system gives:

$$\dot{x}(t) = (A + BK_x^T(t))x(t) + BK_r^T(t)r(t).$$

If it is assumed that K_x^* and K_r^* exist such that:

$$A + BK_x^{*T} = A_{rm}$$
$$BK_r^{*T} = B_{rm}$$

then the tracking error dynamics is:

$$\dot{e}(t) = \dot{x}(t) - \dot{x}_{rm}(t) = [(A + BK_x^T(t))x(t) + BK_r^T(t)r(t)] - [(A + BK_x^{*T})x_{rm} + BK_r^{*T}r(t)].$$

Defining $\tilde{K} = K - K^*$ the equation can be reduced to

$$\dot{e}(t) = A_{rm}\,\mathrm{e}(t) + B\tilde{K}_x^T(t)\,\mathrm{x}(t) + B\tilde{K}_r^T(t)r(t)$$

Using Lyapunov theory it is possible to find the update law for gains K_x and K_r such that error converges to zero. To do that the well known Lyapunov equation for linear systems is used to find the positive definite matrix $P \in R^{n \times n}$ such that $\dot{V}(t) \leq 0$ for Lyapunov function $V(t) = \frac{1}{2} e^T(t)Pe(t)$:

$$A_{rm}^T P + PA_{rm} + Q = 0.$$

This leads to the following update equations:

$$\dot{K}_x(t) = -\Gamma_x x(t)e^T(t)PB$$

$$\dot{K}_r(t) = -\Gamma_r r(t)e^T(t)PB.$$

In general, the procedure involves:

1. Coming up with a controller structure.
2. Deriving the error dynamics equation based on the chosen controller.
3. Finding a Lyapunov equation and then using it to find the adaptation equations necessary to drive the error to zero.

An interesting property of the Lyapunov method is that the system response converges although the parameters are not guaranteed to converge. The fact that parameters can grow unboundedly can be problematic. This will be discussed later in the chapter.

Indirect MRAC

Indirect MRAC relies on estimating unknown system parameters to adapt control input accordingly. The well-known Least Square Minimization (LSM) forms the foundations of Indirect MRAC and it is where MRAC displays its machine learning features. LSM is sometimes implemented through a Kalman filter.

LSM has appeared in many areas of mathematics and engineering. It is simply defined as:

$$\text{minimize } \|Y - \Phi\Theta\|_2^2$$

There are different ways to interpret LSM depending on how it is written. Assume that $R(\Phi)$ is the range of matrix Φ that is the vector space formed by the columns of matrix Φ. Then $\Phi\Theta$ is a point in the space defined by $R(\Phi)$. The above definition seeks the value of Θ for which $\Phi\Theta \in R(\Phi)$ is the closest point to Y. If Y is a point somewhere in the space and $\Phi\Theta$ is a hyperplane, the solution projects Y on to $R(\Phi)$.

A well-known applications of LSM is in regression and model fitting. The aim of regression is to fit a parametric model, usually termed a hypothesis in the machine learning literature to data points. Given data points $y_i \in R$ a function:

$$y(x) = \sum_{j=1}^{N} \theta_j \phi_j(x) = \Phi^T(x)\Theta$$

is sought where $\phi_i(x)$ are basis functions and θ_i are the model parameters that combine these basis functions so that the model best fits the data. This means that the model parameters have to be obtained through some optimization process and LSM is commonly used for this, i.e.,

$$\text{minimize } \sum_{i=1}^{N} (y_i - \phi^T(x_i)\Theta)^2$$

This can be expressed in matrix form as

$$J=\sum_{i=1}^{M}(y_i-\Phi^T(x_i)\Theta)^2=\begin{bmatrix} y_1-\phi^T(x_1)\Theta \\ y_2-\phi^T(x_2)\Theta \\ \vdots \\ y_M-\phi^T(x_M)\Theta \end{bmatrix}=\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix}-\begin{bmatrix} \phi^T(x_1) \\ \phi^T(x_2) \\ \vdots \\ \phi^T(x_M) \end{bmatrix}\Theta$$

$$=\left|\left| Y-\begin{bmatrix} \phi_1(x_1) & \phi_2(x_1) & \dots & \phi_N(x_1) \\ \phi_1(x_2) & \phi_2(x_2) & \dots & \phi_N(x_2) \\ \vdots & \vdots & \vdots & \vdots \\ \phi_1(x_M) & \phi_2(x_M) & \dots & \phi_N(x_M) \end{bmatrix}\Theta \right|\right|_2^2=\| Y-\Phi\Theta \|_2^2$$

Here the Φ matrix is the regression matrix. If $\phi_j(x)=x^{(j)}$ where $x^{(j)}$ is the j-th element of x then the above expression reduces to the linear regression problem.

The above regression equation is sometimes called parametric estimation. The use of the regression equation is common in adaptive control whenever varying components of the system are modeled as a linear function of the unknown parameters. Recursive implementations also exist that are computational efficiency.

When using least square for parameter estimation, the Φ matrix often contains the **observations** either in its rows or columns depending on the problem description. For a good estimation, one would require that the observations provide as many independent information as possible in other words one would expect Φ to have complete rank. If some of these observations are not providing new information, i.e., the columns or rows are not independent the parameter estimation would be poor and in some cases impossible. This leads to a concept called **persistent excitation** and it will be discussed later.

Applications of MRAC to Manipulator Control

Both direct and indirect MRAC have been applied to manipulator control in order to deal with the varying load or unknown parameters (Craig et al. 1987, Maliotis 1991, Ham 1993, Dubowsky and Deforges 1979, Slotine and Li 1987, Hsia 1984, Tung et al. 2000). In both approaches, the known part of the model is used in a direct torque control or feedforward framework and only the unknown or varying part is handled by an adaptive controller. In case of direct MRAC, the adaptive controller can be a PD controller with adaptive K_p and K_d gains (Maliotis 1991) while in the case of indirect MRAC it usually takes the form of a computed torque control based on estimated parameters (Craig et al. 1987, Slotine and Li 1987). In both approaches, MRAC design is predominantly based on Lyapunov method.

The common approach in applying direct MRAC to manipulators is to combine feedforward and feedback as shown by (Maliotis 1991), who shows, in this approach the feedforward component makes use of the known portion parameters of the plant such as inertia to calculate the likely input. The adaptive feedback aims to compensate for the varying nonlinear effects by using n linear decoupled second order systems as the reference model. The feedforward component could also be interpreted as the result of feedback linearization of the plant.

To make this explicit, assume that the dynamics of a robotic manipulator is expressed by

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q)q = \tau$$

where the inertia matrix M, Coriolis and gravity terms C and G are all assumed to be composed of known and unknown parts as defined below.

$$M = M_k + M_u^* = M_k - I + M_u^* + I = M_k - I + M_u$$

$$C = C_k + C_u$$

$$G = G_k + G_u$$

The subscripts k and u define the known and unknown components. Note that M has been decomposed to known and unknown parts in a way that supports the construction of invertible matrices that are positive definite. When the above equations are substituted in the dynamic equations the following is obtained:

$$(M_k - I + M_u)\ddot{q} + (C_k + C_u)\dot{q} + (G_k + G_u)q = \tau,$$
$$M_u\ddot{q} + C_u\dot{q} + G_u q = \tau - (M_k - I)\ddot{q} - C_k\dot{q} - G_k q = u,$$
$$\tau = u + (M_k - I)\ddot{q} + C_k\dot{q} + G_k q.$$

And, if all parameters are known then $M_u = I$, $G_u = 0$, $C_u = 0$ and therefore $u = \ddot{q}$ and τ will only include the feedforward terms. However if there are unknown parameters, given a reference model then u can be calculated using adaptive terms in order to make the system behave like the reference model.

The reference model is chosen as:

$$\dot{x}_d = A_m x_d + B_m v = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix} x_d + \begin{bmatrix} 0 \\ I \end{bmatrix} v,$$

where $x_d = [q_d \;\; \dot{q}_d]^T$. This model essentially forces the manipulator to behave like a number of uncoupled second order systems. The input u is chosen to be:

$$u = u_a + u_k,$$

$$u_k = -[K_p \;\; K_v]\, x + v$$

$$u_a = -[\Delta_1 \;\; \Delta_2]\, x + \Delta_v v$$

The first term is a state feedback and feedforward and the second term is the adaptive input that deals with the unknown variations. Using the above control law, the adaptation law can be derived using Lyapunov method:

$$\dot{\Delta}_1 = -a\omega q^T$$

$$\dot{\Delta}_2 = a\omega\dot{q}^T$$

$$\dot{\Delta}_v = b\omega v^T$$

where a and b are positive scalar gains.

Computed torque control is also used in indirect MRAC where adaptation occurs through estimation of unknown parameters (Craig et al. 1987, Slotine and Li 1987).

The parameter estimation law is derived using Lyapunov method. The process is shown below:

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q)q.$$

The control law is then defined as;

$$\tau = \hat{M}\ddot{q}_d + \hat{C}(q,\dot{q})\dot{q}_d + \hat{G}(q) - K_p\tilde{q} - K_d\dot{\tilde{q}},$$

where $\widehat{(.)}$ indicates associated quantity is an estimate, and:

$$\tilde{q} = q - q_d$$
$$\dot{\tilde{q}} = \dot{q} - \dot{q}_d.$$

A Lyapunov function is defined as $V = \frac{1}{2}(\dot{\tilde{q}}^T M(q)\dot{\tilde{q}} + \tilde{a}\Gamma\tilde{a} + \tilde{q}^T K_p \tilde{q})$ where a is the vector of unknown parameters and $\tilde{a} = \hat{a} - a$ is the estimation error. In order to make sure that $\dot{V} \leq 0$ the estimation law is chosen as:

$$\dot{\hat{a}} = -\Gamma^{-1} Y^T(q,\dot{q},\dot{q}_d,\ddot{q}_d)\dot{\tilde{q}}$$

with Y coming from

$$\tilde{M}\ddot{q}_d + \tilde{C}(q,\dot{q})\dot{q}_d + \tilde{G}(q) = Y\tilde{a}.$$

This process is more or less followed in all Lyapunov-based indirect MRAC applications in manipulator control. See for example (Tsai and Tomizuka 1989, Rong 2012, Mohan and Kim 2012).

Researchers have also used indirect MRAC in combination with direct MRAC for manipulator control. This method utilizes both tracking error and prediction error for better adaptation (Yu et al. 1993, Ciliz and Cezayirli 2004).

Limitations of MRAC and the Adoption of Machine Learning

Traditional MRAC as presented in the previous section has limitations that arise from the assumptions, control architecture and the computational method. Direct and indirect MRAC have different limitations.

In traditional MRAC, it is assumed that the unknown parameters are either constant or change very slowly (Astrom and Wittenmark 2013, Slotine and Li 1987). This assumption is true in some circumstances but not in all and it is arguably predicated on mathematical convenience rather than its appropriateness for manipulators and other high performing mechanical systems. Because of the improvements in actuator technology direct drive manipulators are capable of performing high speed high torque motions which means that variations and nonlinearities can occur at a faster rate. Therefore there is a need to account for such abrupt variations that do not fit in the classic MRAC framework.

Both direct and indirect MRAC are reliant on the so-called *persistent excitation* condition in order to guarantee stability and convergence for states and parameters (Narendra and Annaswamy 1987, Chowdhary et al. 2013). This requirement ensures that the input to the system meets the conditions for parameters to be bounded. If persistent excitation is not satisfied it can lead to robustness and stability problems due