MODELS OF HIGH ENERGY PROCESSES

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Preface

Theoretical physics makes extensive use of models to test and develop intuition. In non-relativistic quantum mechanics the principal source of insight is provided by the study of suitably chosen potentials. However, such an approach can be of little value in relativistic quantum mechanics. Instead the Feynman integrals of perturbation theory have provided a rich testing ground for assessing dynamical conjectures. The method is unashamedly heuristic but it commands respect because Feynman perturbation theory gives a formal solution of the requirements of analyticity and unitarity. These principles are believed to provide the essential kinematic setting for relativistic quantum mechanics. It is true that recent ideas of confinement, and of the role of non-perturbative classical solutions of field theory, have suggested important aspects of relativistic quantum mechanics that are not to be seen in Feynman integrals. Nevertheless the method retains its power to act as a guide to the answer of many dynamical questions. In particular it remains an indispensable tool to investigate the fundamental interactions of quarks and gluons, a role which has been given an enhanced respectability by the elegant notion of asymptotic freedom for non-Abelian gauge theories.

While perturbation theory continues to be an important model, eliciting its guidance is sometimes a formidable analytic task. An important advance was made when Academician Gribov introduced hybrid models, based on Sudakov parameter methods. Not only are these models in many cases easier to calculate but also their largely non-perturbative character makes their conclusions stand on firmer ground. The technique pioneered by Gribov has proved a fruitful source of model making for many physical regimes. One of its most important uses has been to provide a covariant and non-perturbative formulation of the parton model. This model describes the substructure within hadrons which appears to be manifest in deep inelastic scattering reactions of all kinds. Such processes, characterised by high energy and large momentum transfer, probe the constituents out of which hadrons are made. They provide much of the detailed evidence for the quark structure of matter.

This monograph seeks to provide an introduction to these types of model making. Its aim is to explain the basic ideas in a form accessible to graduate students and other readers who have acquired a first knowledge of quantum field theory and basic particle physics, including the elements of Regge theory. I believe that it describes all major calculational techniques together with sufficient physical applications to illustrate their utility. No attempt has been made to be encyclopaedic, for an exhaustive treatment of every application would have created a volume too large for the simple pedagogic purpose intended. For example the parton model is discussed in a way which exhibits its physical structure but which avoids commitment to details which are still a matter of unresolved phenomenological debate. Similarly, on the theoretical side I have been content to illustrate the connection of the ideas presented with Reggeon field theory and with K. Wilson's operator product expansion, without developing either subject in detail since each is really an autonomous discipline in the regime it describes.

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J. C. POLKINGHORNE

Summary of analytical techniques

In this monograph we describe a number of mathematical techniques. They are employed in appropriate physical contexts but often they are capable of much wider application than can be illustrated in a book of this size. The aim of this summary is to give an indication of these techniques, and the sections in which they are developed, in the hope that this will prove useful to a reader in search of a line of attack on a problem.

The basic method for evaluating Feynman integrals is symmetric integration (section 1.2). If numerator factors are present the use of auxiliary momenta as dummy variables is often helpful (section 1.3). A technique for handling logarithmic factors is known (section 3.3, equation (3.3.28)). Sometimes it is convenient to rewrite the loop momentum integrals as integrals over invariants (section 3.7). Ways of handling θ-functions and δ-function constraints are also available (section 4.4).

The asymptotic behaviour of integrals can sometimes be determined by direct integration by means of formulae like (2.1.7) of section 2.1. This section describes the important notions of natural behaviour, end point contributions and pinch contributions.

A powerful general method for treating end point contributions is provided by Mellin transforms (section 2.2). Key ideas are scaling transformations (section 2.3), disconnected scaling sets (section 2.3), independent scaling sets (section 2.3) and singular configurations (section 2.4). Multiple Mellin transforms (section 2.9) can be used to discuss limits in several variables.

Pinch contributions are discussed in section 2.6, where it is also explained how they can be evaluated by using end point techniques. An example of behaviour governed by a mixture of end points and pinches is given in section 2.7.

The treatment of divergences by dimensional regularisation is discussed in section 2.5 and the effect of divergences on asymptotic behaviour illustrated in section 3.3.

The determination of momentum flows associated with scaling sequences is given in section 2.8, where the eikonal approximation is also worked out.

Sudakov parameters are defined in section 3.1. The importance of contour closing arguments in determining the significant range of values of Sudakov parameters in high energy regimes is illustrated in section 3.2. In section 3.3 a modified Sudakov representation with massless momenta is defined (see (3.3.13)) and in section 3.4 an alternative and universally powerful parametrisation for constituent momenta almost parallel to parent hadron momenta is written down ((3.4.3) *et seq.*).

A method by which Fourier transforms can be used to specify general analytic properties is given in section 3.6 (see (3.6.8)). The way $i\varepsilon$ prescriptions for internal invariants are specified by the $i\varepsilon$ prescriptions for external invariants is explained in section 4.2.

Contents

1
Feynman diagrams

1.1 Introduction

Relativistic quantum mechanics is a rich and intricate theory. It has defied general solution, so that its study has required recourse to models. These are chosen in the hope of reproducing the behaviour expected of the true theory in some appropriate extreme regime. Examples of such regimes are the Regge regime of high energy scattering at fixed momentum transfer (see Collins, 1977) and the deep inelastic scattering regime, in which the transfer of large momentum by weak or electromagnetic currents probes the constituents of hadronic matter (see Feynman, 1972). The results obtained from such models are of two kinds. Firstly the model may suggest that the scattering amplitude is constrained to take a restricted functional form in terms of the relevant variables. Examples drawn from the two regimes mentioned are, respectively, Regge pole behaviour $s^{\alpha(t)}$ (see section 2.1 *et seq.*) and the Bjorken scaling law $\nu^{-1}f(2\nu/-q^2)$ (see sections 2.10, 3.2). In fact more elaborate models suggest modifications to both these formulae (see sections 2.7 and 3.3) but the insight afforded by the simplest model is a valuable starting point for the discussion of each regime. Such kinematic results represent a stable component in our understanding of elementary particle physics. The second type of result sometimes extracted from the discussion of models is dynamic in character. It will try to predict the specific form of such functions as $\alpha(t)$ or $f(2\nu/-q^2)$ in terms of some precise physical mechanism. Clearly the successful accomplishment of this task is of the highest interest. However, in many regimes its attempt is somewhat ambitious in relation to current physical understanding and such results are liable to extensive revision as theory and experiment advance. The chief emphasis of this monograph will be on kinematic results of the first kind.

The oldest model used successfully to gain an insight into aspects of relativistic quantum mechanics is provided by the Feynman integrals of perturbation theory (see, for example, Bjorken & Drell, 1965). Interest in this model, and the methods associated with it, has recently been enhanced by the ideas of asymptotic freedom and the quantum chromodynamic theory of strong interactions (see Politzer, 1974). In such gauge theories the running coupling constant can be shown to become small like $(\ln \nu)^{-1}$ when all variables are large like ν. Thus perturbation theory acquires a specific role in such theories at high energy.

Of course Feynman integrals only provide a model. However small the coupling constant may be there is no proof that the series is literally convergent. Nevertheless the perturbation series is formally a solution of the analytic and unitary properties which provide a basis for relativistic quantum mechanics (see Eden, Landshoff, Olive & Polkinghorne, 1966). Even here we must enter a note of caution. Feynman integrals have analytic and unitary properties expressed in terms of the particles associated with the propagators appearing in them. For quantum chromodynamics this means that they are expressed in terms of quarks and gluons. However the particles that figure in the S-matrix of the observable world are the hadrons. The relation between these two descriptions can only properly be understood when the bound state problem and confinement have been solved. Surely the eventual mastery of these problems will take us outside perturbation theory.

A partial way round this difficulty is afforded by the second model we are about to describe. Nevertheless Feynman integrals must retain their value as a guide to the fundamental interactions between quarks and gluons themselves (see, for example, the discussion of section 3.3). Their use is frankly heuristic but they have proved so powerful and convincing a guide to intuition that Feynman integrals have sometimes been described as a 'theoretical laboratory' in which conjectures on relativistic quantum mechanics can be tested. The second chapter of this monograph is intended as a handbook to the use of the apparatus of this laboratory.

The second main class of model of high energy processes stems from the work of Gribov (1968). Its earliest application was to Regge theory but later the method was used by Landshoff, Polkinghorne & Short (1971) to give a covariant formulation of the parton model of deep inelastic processes. Its nature can be illustrated by considering the process of fig. 1.1.1. The two thick external lines represent incident hadrons. They interact through the emission and scattering of constituents (or partons) corresponding to the lines 1 and 2 of the figure. The unshaded bubbles represent amplitudes for the emission of the partons, leaving a 'core' or residue behind. The shaded bubble corresponds to the scattering amplitude for the interaction of the two partons. These three bubbles are not to be thought of in a Feynman integral way at all. They are complete

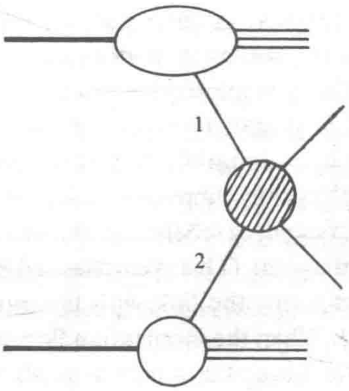

FIGURE 1.1.1 Constituent scattering.

(non-perturbative) subamplitudes for the subprocess (emission or scattering) which they represent. The kinematics of the overall process will constrain them to be evaluated in some specific regime. Many examples of this will be given in Chapter 3. Part of the specification of the model will include the stipulation of how the subamplitudes are to behave in their respective regimes. For example the shaded bubble might represent quark–quark scattering at high energy and large momentum transfer. It will be necessary then to have an *ansatz* for the behaviour of the amplitude in this regime. (Almost certainly this would be derived from applying perturbation theory analysis to quantum chromodynamics, so that the two models are closely interrelated.)

The Gribov approach is a hybrid one, for the subamplitudes are linked together by Feynman propagators associated with the interacting constituents. For example, the lines 1 and 2 of fig. 1.1.1 correspond to propagators of this type. Each high energy process modelled in this way will correspond to external momenta with large components in an appropriate frame of reference. The role of the propagators is to carry the flow of this large momentum through the interaction. The calculation of the effects of this flow is greatly facilitated by the use of Sudakov (1956) parameters. These give an expansion of all internal momenta in terms of the important external momenta. They thus facilitate a convenient covariant separation with large and small components. Full details of these techniques are given in chapter 3.

As far as possible the two main chapters, 2 and 3, are written in a self-contained way so that they are capable of being read independently of each other with the minimum of cross-reference. Chapter 2 may appear predominantly mathematical in character, for though it obtains many results of clear physical significance it does so by techniques which can be used to extract the asymptotic behaviour of functions defined by integrals, be they of Feynman form or otherwise. These methods are capable of systematic exploitation but their economic application – particularly to the physically important case of particles with spin – requires a clear understanding of the dominant pattern of momentum flow in the Feynman diagram. (This is emphasised particularly in sections 1.2 and 2.8.) This fact forms the link with the more obviously physical approach of chapter 3. When the momentum flow is properly understood the methods of chapter 3 are often easier to apply and they carry greater conviction because of their largely non-perturbative character.

Mueller (1970) has taught us how to use a generalised optical theorem to calculate cross-sections by taking discontinuities of scattering amplitudes in suitable variables. In simple cases such discontinuities are readily evaluated by elementary means. In chapter 4 we have collected together some more advanced discussion of discontinuities, making use of techniques drawn from both chapter 2 and chapter 3.

1.2 Feynman integrals and symmetric integration

An account of the origin of Feynman integrals in relativistic time-ordered perturbation theory can be found in introductory texts on quantum field theory (for example, Bjorken & Drell, 1965). In this section we shall consider only the simplest non-trivial example of a field theory, given by a real scalar field $\phi(x)$ of mass m interacting with itself through a term

$$g\phi^3(x)/3! \qquad (1.2.1)$$

in the Lagrangian. In the succeeding section we shall begin the consideration of more complicated and realistic theories involving particles with spin. These theories have additional complicating features not present in ϕ^3 theory but the latter provides a convenient illustration of many simple properties common to all expansions in terms of Feynman integrals.

The S-matrix for the scattering process may be written

$$S = 1 + \mathrm{i}R, \qquad (1.2.2)$$

and the T-matrix is then defined in terms of the R-matrix by the equation

$$\langle p_{1\mathrm{f}}\cdots p_{n\mathrm{f}}|R|p_{1\mathrm{i}}\cdots p_{m\mathrm{i}}\rangle = (2\pi)^4\delta(\textstyle\sum p_\mathrm{i} - \sum p_\mathrm{f})\langle p_{1\mathrm{f}}\cdots p_{n\mathrm{f}}|T|p_{1\mathrm{i}}\cdots p_{m\mathrm{i}}\rangle. \qquad (1.2.3)$$

The momenta $p_{1i} \ldots p_{mi}$ correspond to an initial state with m particles, and the momenta $p_{1f} \ldots p_{nf}$ to a final state with n particles. Each Feynman integral contributing to T can be associated with a Feynman diagram. Each diagram has external lines carrying the initial and final state momenta, there being one such line for each particle in the process. In addition the diagrams contain internal lines which correspond to the so-called virtual particles which mediate the interaction. Lines meet in trilinear vertices (corresponding to the trilinear interaction (1.2.1)). Internal lines must be connected to a vertex at both ends (since virtual particles are emitted and then reabsorbed) while external lines are only connected at one end (corresponding to the interaction in which the external particle is absorbed or emitted). These somewhat involved statements are illustrated by the examples of fig. 1.2.1 for the two particle–two particle scattering,

$$p_1 + p_4 \to p_2 + p_3, \tag{1.2.4}$$

(where we use the transparent notation of representing a particle by its four-momentum).

The momenta flowing in the internal lines are constrained by the requirement of four-momentum conservation at each vertex. The satisfaction of these constraint equations is analogous to a Kirchoff's law problem in electrical networks. The general solution is given in terms of l circulating loop momenta associated with the l independent loops of the diagram. For example in fig. 1.2.1a there is just one such loop with k the loop momentum, while in fig. 1.2.1b there are two independent loops with momenta k_1 and k_2 respectively.

Every topologically distinct diagram constructed in this way represents a contribution to $i(2\pi)^4\delta(\sum p_i - \sum p_f)\langle T\rangle$ given by a Feynman integral specified by the following rules:

(i) a term $i/(2\pi)^4\, 1/(q^2 - m^2 + i\varepsilon)$ for each internal line carrying momentum q. The infinitesimal $i\varepsilon$ is required to define how the pole at $q^2 = m^2$

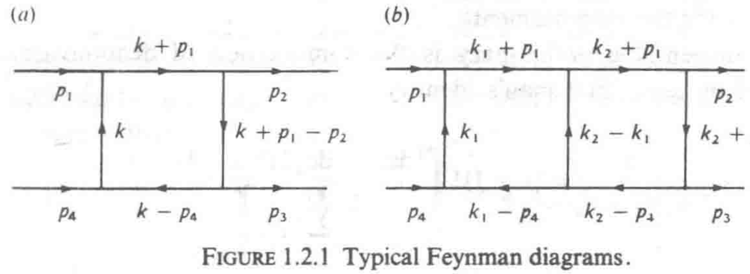

FIGURE 1.2.1 Typical Feynman diagrams.

is to be treated. The internal momenta q are linear combinations of the external momenta p and the loop momenta k, as in fig. 1.2.1.

(ii) a term $-\mathrm{i}(\sum 2\pi)^4 \delta^{(4)}(\sum q)$ for each vertex, where $\sum q$ is the algebraic sum of the momenta at the vertex. These δ-functions enforce energy–momentum conservation at each vertex and it is immediately possible to factor out from them the $(2\pi)^4 \delta(\sum p_{\mathrm{i}} - \sum p_{\mathrm{f}})$ which enforces overall energy–momentum conservation for the process.

(iii) an integral $\int \mathrm{d}^4 k$ for each independent loop momentum in the diagram.

(iv) a symmetry factor S^{-1} for the whole diagram, where S is the number of different ways in which the internal lines can be arranged with the external lines fixed. For example, S is 1 for the diagrams of fig. 1.2.1 but 2 for fig. 1.2.2 because of the possibility of interchanging the two internal lines.

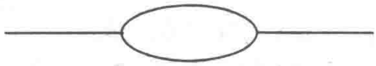

FIGURE 1.2.2 A diagram with $S \neq 1$.

(In some books (e.g. Bjorken & Drell, 1965) the rules are given in a form which distributes the factors of 2π differently. The form given above is quite general and will hold for theories with trilinear or quadrilinear vertices. The other prescriptions are specific to the case of trilinear vertices.)

Symmetric integration

A diagram with l independent loops and n internal lines will have associated with it an integral which is proportional to

$$\lim_{\varepsilon \to 0} \int \mathrm{d}^4 k_1 \ldots \mathrm{d}^4 k_l / \prod_{r=1}^{n} (q_r^2 - m^2 + \mathrm{i}\varepsilon). \tag{1.2.5}$$

For convenience we have omitted writing the numerical factors which can be read off from the rules given above. The next task is to perform the integrals over the loop momenta.

An indispensable preliminary is the combination of denominators in (1.2.5) by using Feynman's identity

$$\frac{1}{d_1 d_2 \ldots d_n} = (n-1)! \int_0^1 \frac{\mathrm{d}\alpha_1 \ldots \mathrm{d}\alpha_n \, \delta(\sum \alpha - 1)}{\left[\sum_{i=1}^{n} \alpha_i d_i\right]^n}. \tag{1.2.6}$$

In passing we may note the useful generalisation of (1.2.6)

$$\frac{1}{d_1^{1+r_1} d_2^{1+r_2} \dots d_n^{1+r_n}} = \frac{(n+\sum r_i - 1)!}{\prod_{i=1}^{n} r_i!} \int_0^1 \frac{\prod \alpha_i^{r_i} \mathrm{d}\alpha_i \, \delta(\sum \alpha - 1)}{[\sum \alpha_i d_i]^{n+\Sigma r_i}}. \quad (1.2.7)$$

For integral r_i this can be proved from (1.2.6) by differentiating with respect to the d_i. It is also true for general r_i provided the factorials are interpreted via the gamma function: $n! \equiv \Gamma(n+1)$.

Although applying (1.2.6) to (1.2.5) appears to introduce a further complication in the form of an auxiliary Feynman parameter α_i associated with each internal line, we shall find that it enables the loop momenta k_j to be integrated out.

The expression (1.2.5) becomes

$$\Gamma(n) \frac{\int_0^1 \prod \mathrm{d}\alpha_i \, \delta(\sum \alpha - 1) \prod \int \mathrm{d}^4 k_j}{[\sum \alpha_i (q_i^2 - m^2) + \mathrm{i}\varepsilon]^n}, \quad (1.2.8)$$

where there is now a single denominator. The qs are linear combinations of the loop momenta k and the external momenta p so that the denominator in (1.2.8) can be written as

$$\begin{aligned} \psi(p,k,\alpha) &\equiv \sum \alpha_i (q_i^2 - m^2) + \mathrm{i}\varepsilon \\ &= \boldsymbol{k}^{\mathrm{T}} \cdot \boldsymbol{A} \cdot \boldsymbol{k} - 2\boldsymbol{k}^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \boldsymbol{p} + (\boldsymbol{p}^{\mathrm{T}} \cdot \boldsymbol{\Gamma} \cdot \boldsymbol{p} - \sigma) + \mathrm{i}\varepsilon, \end{aligned} \quad (1.2.9)$$

where

$$\sigma = \sum \alpha_i m^2. \quad (1.2.10)$$

If there are l independent loop momenta and e external lines (and thus, by overall momentum conservation, $e-1$ independent external momenta) $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{\Gamma}$ are respectively $l \times l$, $l \times (e-1)$, and $(e-1) \times (e-1)$ matrices. Their elements are linear in the αs and the matrices act on row and column vectors made up of the l independent loop momenta ($\boldsymbol{k}$) and the $e-1$ independent external momenta ($\boldsymbol{p}$). The superscript T represents the transpose in matrix space. In these row and column vectors each entry is a momentum and is thus itself a Lorentz vector. Thus the first term of (1.2.9) written out fully takes the form of a multiple sum over matrix and Lorentz indices

$$\boldsymbol{k}^{\mathrm{T}} \cdot \boldsymbol{A} \cdot \boldsymbol{k} = \sum_{i,j=1}^{l} \sum_{\mu=0}^{3} k_i^{\mu} \cdot A_{ij} \cdot k_{j\mu}, \quad (1.2.11)$$

and similarly for the other terms.