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## A Unified Approach to Vibrations and Controls

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Mechanical Systems

*for Janet*

# Preface

This text presents a unified treatment of vibrations and control. Both of these topics fit under the broad title of mechanical systems, hence the main title of the book. I have taught a one-semester course dealing with both of these topics for several years. I have used two excellent books, but the notation is not consistent between the two, and asking students to buy two books is an imposition. This book provides one book with one notation for both topics.

One thing my teaching experience has taught me is that one semester is not enough, and I have included enough material for a 1-year course. Chapters 1–5 cover the topics in a standard vibrations course with some added material on electric motors and vibration measurement. Chapter 6 introduces state space and forms a bridge from the first half of the book to the second. If one were to use the book only for vibrations or only for controls, I would suggest including Chap. 6 in either course. Chapters 7–10 cover linear controls starting with PID control and moving quickly to state space control, where I address controllability, observability, and tracking. Chapter 11 introduces feedback linearization as an easily understandable venture in nonlinear control. I have taken advantage of the fact that kinematic chains with only revolute joints can be controlled using a variant of feedback linearization to introduce nonlinear control of simple robots with only revolute joints. This chapter is a bonus, not necessary for a first course in controls, but there for those who find it useful.

The text is aimed at juniors and seniors in mechanical and related engineering programs, such as applied mechanics, aeronautical engineering, and biomedical engineering, but I hope that it will be useful to working engineers as well. I expect the students to have had exposure to ordinary differential equations, linear algebra, and an elementary course in engineering dynamics. I use free body diagrams early in the text and expect that idea to be familiar to them. I use Euler-Lagrange methods from Chap. 3 onwards. These methods are introduced assuming no previous familiarity with them.

I have found that students don't much like abstract mathematics, so I have tried hard to include examples with a real-world flavor. Some of these examples (a servomotor, suspension of a steel ball by an electromagnet, and the overhead crane) appear early in the text and then reappear to illustrate the new material introduced in each chapter. I have also tried to refresh student memories and/or include basic material they may not have seen. In particular, I spend some time in Chap. 1, which provides an overview of the whole text, on complex notation and the connection between the complex exponential and trigonometric functions. As I work through the first few chapters, I use trigonometric functions and the complex exponential interchangeably to get students used to the connection. The complex exponential is essential once the book moves to state space.

Chapter 2 covers one degree of freedom problems using free body diagrams to develop the equations of motion. I also introduce the concepts of kinetic and potential energy here. Chapter 3 deals with systems with more than one degree of freedom. It includes a careful discussion of degrees of freedom and introduces the whole Euler-Lagrange process for deriving equations of motion. I also introduce a simple DC motor model in Chap. 3 so that I can use voltage as an input for the rest of the text instead of just saying "Here's a force" or "Here's a torque." Chapter 4 covers modal analysis. Chapter 5 discusses vibration measurement, and Fourier analysis, Fourier series, and Fourier transforms, primarily as tools for vibration analysis. I discuss the Gibbs phenomenon and Nyquist phenomena, including aliasing. I prove no theorems.

Most of the world cannot be well modeled by linear systems, so I have included the simulation of nonlinear systems using commercial packages for the integrations. Linearization is a skill that is not generally taught formally. It is very important, even in the age of computers, and I attack it twice, once in Chap. 3 and again in Chap. 6, more formally. The control designs in the text, except for Chap. 11, are all linear, based on choosing gains to drive errors to zero, but I assess their efficacy using numerical simulation of the nonlinear equations.

I cover control in the frequency domain in Chap. 7, introducing the Laplace transform (without theory) and transfer functions. I go through the classical PID control of a second-order (one degree of freedom) system and apply the idea of integral control to automotive cruise control.

I think that control in state space is much more useful. I introduce state space in Chap. 6 and state space control in the time domain in Chap. 8. The rest of the text is cast in state space and in the time domain. Chapter 8 includes the controllability theorem (the algebraic theorem without proof), the reduction of a controllable dynamical system to companion form, the placement of poles by choosing gains in the companion form, and the mapping of the gains back to the original space. Chapter 8 is limited to using full state feedback driving a dynamical system to a (vector) zero. I deal with both linear and nonlinear systems. I design controls for nonlinear systems by linearizing and applying a state space control algorithm, but then I test the controls by solving the full nonlinear systems numerically.

Chapter 9 introduces observers. I go through the examples in Chap. 8 using an observer. I also note that the observer doubles the size of the system and that one needs to take account of that in assessing the control that one designs. In particular, I note that the original system and the observed system may be independently stable, but that does not necessarily extend to the combined system, which can actually be unstable.

Chapter 10 explains how to track a time-dependent reference state, both assuming full state feedback and using an observer. Again I use linear control on nonlinear systems and verify the controls using numerical integration.

The basic control paradigm, executed in state space, is to linearize the system (if necessary), design a linear control, and then assess the linear control on the nonlinear system in simulation.

Chapter 11 addresses nonlinear control by feedback linearization. This is applicable to some simple robots, and I discuss three-dimensional motion briefly in order to deal with realistic robots.

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# Acknowledgments

I owe thanks to several sets of students taking ME 213 at the University of Rochester. They showed what did and did not work. Even this year's class, whom I have only met five times at the time of writing, helped me clarify several points in Chap. 2. Thanks also to the TAs in this course, who gave me a separate feedback on what did and did not work. I also wish to thank several anonymous referees whose comments led me to further clarification.

# Contents

# Overview with Some Definitions and Mathematics

# 1

In which we introduce a number of concepts and techniques that will be important for the whole text.

## 1.1 General Introduction

Vibrations and controls are both key engineering topics. They have a great deal of mathematics and physics in common, which is why it makes sense to tackle them in the same text. Both deal with the response of mechanical systems to external forces. (I will include electromechanical systems in the term *mechanical systems*. This lets me use motors to control mechanical systems from time to time.) Mechanical systems can be as simple as a pendulum or as complicated as a jet aircraft. One of the important things an engineer has to do is to model complicated systems as simpler systems containing the important aspects of the real system. Our models will be mathematical—sets of differential equations. Real systems are generally nonlinear in a mathematical sense. Nonlinear differential equations are generally solvable only numerically, but it can be possible to *linearize* the differential equations, by which we mean: replace the nonlinear equations with linear equations that provide a good approximation to the nonlinear system over a limited range. Even the simple pendulum is a nonlinear system, one that can be approximated reasonably well by a linear system if the amplitude of the oscillation of the pendulum is not too large. (I'll discuss what I mean by too large later.) Linearization is an important topic that we will address in some detail later in the text.

We will restrict our mathematical analysis to linear systems, but we will not entirely neglect nonlinear systems, which we will deal with numerically. I will call a numerical model of a mechanical system a *simulation*, and I will use simulations to assess the utility of linear models. For example, we might design a control system based on a linear model of an actual system and test it on the nonlinear simulation. We can use simulations in general to assess the range of validity of a linearized model, the "too large" issue mentioned in the previous



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paragraph. The simulations will be sets of quasilinear first-order ordinary differential equations (*quasilinear* means that the derivatives enter linearly), which we will integrate using commercial software. These software packages typically implement something called the *Runge-Kutta method.* For more information, see Press et al. (1992).

Mechanical systems can be continuous, like a bending beam, or discrete, like a collection of masses connected by springs. Of course, at some level all systems are continuous (and at a still finer level, all systems are again discrete). Continuous systems can be modeled by discrete systems. This is the principle behind finite element analysis. This text addresses primarily discrete systems. I will look at three simple continuous systems in Chap. 4. Discrete vibrating systems can be modeled by finite collections of masses and springs and dampers. Mechanisms can also be so modeled, so the study of mechanisms and the study of vibrations are closely related. We want to be able to control mechanisms, so control also falls under this same broad umbrella. We will study mechanisms as mechanisms, as models for vibration, and as systems to be controlled.

A mechanism has some number of *degrees of freedom*. Each degree of freedom corresponds to an independent motion the mechanism can execute. A pendulum has but one degree of freedom; all it can do is swing back and forth in a fixed arc. A simplified model of a vehicle (a block lying on a plane) has three degrees of freedom: the location of its center of mass and the direction in which it is pointing. An object in space has six degrees of freedom, three translational and three rotational. An object confined to a plane has at most three degrees of freedom—two to define its location and one its orientation. This text deals almost exclusively with planar mechanisms (I will address some three-dimensional [robotic] control problems in Chap. 11), and I choose the Cartesian $x = 0$ plane as the plane in which they lie, so that the Cartesian translational variables will be $y$ and $z$. These are coordinates in the world. I let $z$ increase upward and $y$ to the right. This gives me a right-handed coordinate system with unit vectors **j** and **k,** respectively. I will denote the orientation angle by $\theta$. It will always increase in the counterclockwise direction, but I will choose different origins for different problems. Figure 1.1 shows a three-dimensional picture of a cylinder in the $y$, $z$ ($x = 0$ plane). The arrow indicates the center of mass and the angle is the angle between the horizontal and the cylinder.

In general the more degrees of freedom a mechanism has, the more complicated it is. However, a mechanism can be quite complicated mechanically and still have a small number of degrees of freedom. For example, a twelve-cylinder internal combustion engine fixed to the ground has dozens of parts but only one degree of freedom. The location and orientation of every part in the engine is determined by the crank angle. (If the engine is installed in a vehicle, it has seven degrees of freedom, because it can rotate and translate as a whole. The rotation and translation will be limited by the motor mounts, but those degrees of freedom still exist.) I will discuss degrees of freedom in more detail in Chap. 3.

Mechanical systems have *inputs* and *outputs*. The inputs are whatever forces and torques that the world applies to the system. Sometimes these are fairly abstract. The inputs to a robot are the voltages applied to the motors driving the arms.

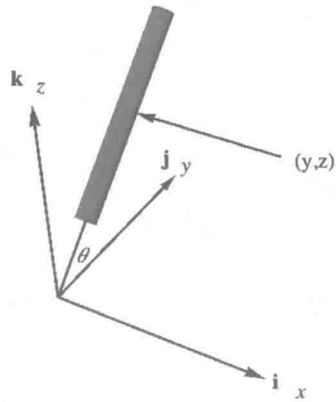


**Fig. 1.1** Inertial space showing the $x = 0$ plane, to which our planar mechanisms will be confined

The outputs are whatever variables of the system that the world needs to know and perhaps control. The input to a cruise control is fuel flow and the output is speed. I will introduce a simple electric motor model in Chap. 3. Its input is a voltage, and its outputs are torque and rotation. For most of the models in this text the inputs will be forces or torques and the outputs positions and speeds. The control of linear systems with one input (single-input, SI systems) is a well-understood and well-studied topic, and it will be the focus of control in this text.

The mathematics involved is that of ordinary differential equations and linear algebra. The independent variable is the time, $t$. In order to address the mathematics, we need to be able to derive the equations, which we do by looking at models. Building models is actually the most difficult task before us. We need to be able to take a problem presented to us in words, possibly poorly, and devise a mechanical model, a mechanism made up of parts connected in some way. Mechanical systems have inertia, perhaps (usually) some way to store and release energy and ways of dissipating energy. The simpler the model, the simpler the mathematics. Our goal should be to find the simplest model that represents the important parts of the physics of the mechanism. Once we can identify the important parts of a mechanism and how they are attached, derivation of the relevant equations becomes simple (in principle). There are (at least) two ways to build equations: using free body diagrams and Newton's laws or considering the energies of the system and forming the *Euler-Lagrange equations*. We will start with the former, but most of our analysis will be based on the latter, which I will introduce in Chap. 3.

Once we have differential equations we need to solve them. This requires some skills and techniques in mathematics, most of which I will develop during the course of the text. All the sets of differential equations that we will tackle analytically will consist of linear, ordinary differential equations with constant coefficients. (As I noted above, we will have some systems of quasilinear differential equations, but we will tackle these numerically.) I expect the reader to have some familiarity with linear ordinary differential equations with constant coefficients. The independent variable will always be time. We are interested in the evolution of systems and their control as time passes. I will consider sets of first-order equations written in matrix-vector form, so some familiarity with linear algebra will be very helpful. The equations for mechanical systems occur "naturally" as sets of second-order equations, so we will need a method of converting these to pairs of first-order equations. This will lead us to the concept of *state space*. The solution of linear ordinary differential equations with constant coefficients is based on exponential and trigonometric functions (which are actually equivalent, as we will see when we learn a little about *complex numbers* at the end of this chapter). Trigonometric functions suggest the use of *Fourier series*, and we will look at those in Chap. 5. Exponentials and the *Laplace transform* are closely related, and we will learn how to use Laplace transforms. (The general theory of Laplace transforms is beyond the scope of this text.)

I will be using vectors and matrices throughout. The vectors are not physical vectors such as displacement, velocity, force, etc. They are abstract vectors in the sense of linear algebra. I will generally denote vectors by bold face lowercase roman letters, such as $\mathbf{u}$. I will denote the general component of a vector by a subscript, such as $u_i$. (I will generally denote scalars by roman italic, sometimes uppercase and sometimes lowercase.) I will denote matrices by bold face uppercase roman letters, such as $\mathbf{A}$. I will denote the general component of a matrix by a double subscript, the first referring to the row and the second to the column, such as $A_{ij}$. I will denote the transpose of a matrix by a superscript T, as $\mathbf{A}^{\mathrm{T}}$. I will denote its inverse by a superscript $-1$, as $\mathbf{A}^{-1}$. I will denote the identity matrix by $\mathbf{1}$ and the null matrix (and null vector, which one will be clear from context) by $\mathbf{0}$. Vectors are column vectors ($N \times 1$ matrices). I will write row vectors ($1 \times N$ matrices) as transposes, as $\mathbf{u}^{\mathrm{T}}$.

I will use a dot to indicate the total derivative with respect to time, as

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt}$$

This means to differentiate each component of $\mathbf{x}$ with respect to time to form a new vector. It is frequently the case that $\mathbf{x}$ is not an explicit function of time, but that the time derivative exists because $\mathbf{x}$ depends on other variables that are themselves functions of time. For example

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}(q_1(t), q_2(t))}{dt} = \frac{\partial \mathbf{x}}{\partial q_1}\dot{q}_1 + \frac{\partial \mathbf{x}}{\partial q_2}\dot{q}_2$$

This is an example of the chain rule, and it will occur throughout the text.

I denote matrix multiplication by writing the matrix and vector next to each other with no symbol. For example, the vector differential equations governing the behavior of some dynamical system that has been written in state space form are

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{1.1a}$$