M. Hanif Chaudhry

# Applied Hydraulic Transients

Third Edition



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# Preface

Applied Hydraulic Transients covers transient flow in closed conduits and in open channels in a systematic and comprehensive manner from introduction to advanced level, with an emphasis on the presentation of efficient and robust computational procedures for analysis and simulation. These procedures, based on modern numerical methods, are suitable for machine computations, provide more accurate results as compared to the available traditional methods and allow the analysis of large and complex systems. The field of application is very broad and diverse and covers systems, such as hydroelectric power plants, pumped storage schemes, water-supply systems, oil pipelines, cooling-water and industrial piping systems. The book is suitable as a reference for practicing engineers and researchers and as a text for senior-level undergraduate and graduates students.

Because of diverse nature, the material in each chapter is presented more or less as stand-alone. Practical applications are emphasized throughout by including case studies of real-life projects, problems of applied nature and photographs and design criteria. Design charts and empirical formulas are presented in the appendix for approximate analyses and for comparing different alternatives for feasibility studies and preliminary design and for the selection of parameters for detailed analyses. Solved examples and sample computer programs are included to facilitate learning. SI units are used throughout. However, equivalent values of empirical constant in the Customary English units are provided which should allow the use of these units without much difficulty.

The general sequence of presentation in this third edition is similar to that in the earlier editions. However, revisions are made throughout for clarity and the references are updated. A new chapter on leak and partial blockage detection is added. In each chapter, a chapter-opener photograph is included as an illustrative introduction to the chapter. In Chapter 1, the historical background is updated and a section on wave reflection and transmission is added. A new section introduces the

inclusion of unsteady friction in the governing equations in Chapter 2 and the simulation of unsteady friction and the application of higher-order numerical methods are presented in Chapter 3. Coverage of the modeling of pump turbines is expanded in Chapter 5. A new section in Chapter 8 outlines the determination of the functional significance of stenosis in cardiovascular systems. The material in Chapters 10, 11, and 13 is revised and a new Chapter 12 discusses the detection of leaks and partial blockages in pipelines. Design charts and other data are presented in Appendix A and sample computer programs in FORTRAN along with sample input and output data are included in Appendices B through E.

I have used Chapters 1 through 5 and 10 as a textbook for a three-credit graduate course on hydraulic transients at Old Dominion University, Washington State University and University of South Carolina. Different chapters or parts thereof may be used for instructional material for advanced level, specialized courses and workshops.

Thanks are extended to British Columbia Hydro and Power Authority and California Department of Water Resources for data and prototype test results on their projects and to my former colleagues, R. E. Johnson for the instrumentation for the prototype tests and R. M. Rockwell and J. Gurney for Figs. 5-11 and 5-29, and G. Vandenburg for Fig. 10-4. Dr. Sam Martin generously provided technical advice, photographs, and figures. Assistance provided by my former graduate student, Dr. Elkholy, for the preparation of the manuscript, by Dr. Mohapatra for proofreading the final draft, and by Rebecca Wessinger for the preparation of the figures for inclusion in the manuscript is thankfully acknowledged. Several individuals from all over the globe were very kind in providing photographs and other material; these are acknowledged throughout the book. I am thankful to my family, especially our grandchildren, Aryaan, Amira, and Rohan, for many hours that should have been spent with them but were required by the preparation of this edition.

Columbia, SC, USA

M. Hanif Chaudhry

# Contents

| 1 | BAS                      | IC CONCEPTS                        | 1    |  |
|---|--------------------------|------------------------------------|------|--|
|   | 1-1                      | Introduction                       | 2    |  |
|   | 1-2                      | Terminology                        | 2    |  |
|   | 1-3                      | Historical Background              | 3    |  |
|   | 1-4                      | Basic Waterhammer Equations        | 8    |  |
|   | 1-5                      | Wave Propagation                   | 11   |  |
|   | 1-6                      | Wave Reflection and Transmission   | 15   |  |
|   | 1-7                      | Transient Flow Analysis            | 19   |  |
|   | 1-8                      | Causes of Transients               | 19   |  |
|   | 1-9                      | System Design and Operation        | 20   |  |
|   | 1-10                     | Accidents and Incidents            | 20   |  |
|   | 1-11                     | Summary                            | 26   |  |
|   | Prob                     | lems                               | 27   |  |
|   | Refer                    | ences                              | 29   |  |
| 2 | TRANSIENT-FLOW EQUATIONS |                                    |      |  |
|   | 2-1                      | Introduction                       | 36   |  |
|   | 2-2                      | Reynolds Transport Theorem         | 36   |  |
|   | 2-3                      | Continuity Equation                | 39   |  |
|   | 2-4                      | Momentum Equation                  | 43   |  |
|   | 2-5                      | General Remarks                    | 45   |  |
|   | 2-6                      | Wave Velocity                      | 49   |  |
|   | 2-7                      | Solution of Governing Equations    | 54   |  |
|   | 2-8                      | Unsteady Friction                  | 55   |  |
|   | 2-9                      | Summary                            | 59   |  |
|   | Probl                    | ems                                | 59   |  |
|   | Refer                    | chces                              | ((() |  |
| 3 | СНА                      | RACTERISTICS AND FINITE-DIFFERENCE |      |  |
|   |                          | THODS                              | 65   |  |
|   | 3-1                      | Introduction                       |      |  |
|   |                          |                                    |      |  |

| Con |  |
|-----|--|

|   | 3-2   | Characteristic Equations          |
|---|-------|-----------------------------------|
|   | 3-3   | Boundary Conditions               |
|   | 3-4   | Convergence and Stability         |
|   | 3-5   | Method of Specified Intervals     |
|   | 3-6   | Computational Time Interval 88    |
|   | 3-7   | Unsteady Friction                 |
|   | 3-8   | Explicit Finite-Difference Method |
|   | 3-9   | General Remarks                   |
|   | 3-10  | Implicit Finite-Difference Method |
|   | 3-11  | Comparison of Numerical Methods   |
|   | 3-12  | Analysis Procedure                |
|   | 3-13  | Case Study                        |
|   | 3-14  | Summary                           |
|   | Prob  | lems                              |
|   |       | rences                            |
|   |       |                                   |
| 4 | TRA   | ANSIENTS IN PUMPING SYSTEMS       |
|   | 4-1   | Introduction                      |
|   | 4-2   | Pump Operations                   |
|   | 4-3   | Pump Characteristics              |
|   | 4-4   | Power Failure                     |
|   | 4-5   | Complex Systems                   |
|   | 4-6   | Pump Start-Up                     |
|   | 4-7   | Design Criteria                   |
|   | 4-8   | Case Study                        |
|   | 4-9   | Summary148                        |
|   | Prob  | lems                              |
|   | Refer | ences                             |
|   |       |                                   |
| 5 |       | ANSIENTS IN HYDROELECTRIC         |
|   |       | VER PLANTS                        |
|   | 5-1   | Introduction                      |
|   | 5-2   | Hydroelectric Power Plant         |
|   | 5-3   | Conduit System                    |
|   | 5-4   | Hydraulic Turbine                 |
|   | 5-5   | Hydraulic Turbine Governor        |
|   | 5-6   | Mathematical Model                |
|   | 5-7   | Design Criteria                   |
|   | 5-8   | Governing Stability               |
|   | 5-9   | Stability Analysis                |
|   | 5-10  | Pumped-Storage Projects           |
|   | 5-11  | Pump Turbine                      |
|   | 5-12  | Case Study                        |
|   | 5-13  | Summary                           |
|   |       | ems                               |
|   | Refer | ences 916                         |

| 6 | TRA   | ANSIENTS IN COOLING-WATER SYSTEMS .         | 221 |
|---|-------|---|-----|
|   | 6-1   | Introduction                                | 222 |
|   | 6-2   | Cooling-Water Systems                       | 222 |
|   | 6-3   | Causes of Transients                        |     |
|   | 6-4   | Analysis Procedures                         |     |
|   | 6-5   | Boundary Conditions                         |     |
|   | 6-6   | Summary                                     |     |
|   | Prob  | olems                                       |     |
|   |       | rences                                      |     |
| 7 | TR.   | ANSIENTS IN LONG OIL PIPELINES              | 237 |
|   | 7-1   | Introduction                                |     |
|   | 7-2   | Definitions                                 |     |
|   | 7-3   | Causes of Transients                        |     |
|   | 7-4   | Methods of Analysis                         |     |
|   | 7-5   | Design Considerations                       |     |
|   | 7-6   | Summary                                     |     |
|   | Prob  | lems  |     |
|   |       | rences                                      |     |
| 8 | PER   | RIODIC FLOWS AND RESONANCE                  | 249 |
|   | 8-1   | Introduction                                |     |
|   | 8-2   | Terminology                                 |     |
|   | 8-3   | Development of Periodic Flow                |     |
|   | 8-4   | Methods of Analysis                         |     |
|   | 8-5   | Block Diagram                               |     |
|   | 8-6   | Transfer Matrices                           |     |
|   | 8-7   | Frequency Response                          | 285 |
|   | 8-8   | Spatial Variation of Pressure and Discharge |     |
|   | 8-9   | Pressure Nodes and Antinodes                |     |
|   | 8-10  | Resonant Frequencies                        |     |
|   | 8-11  | Verification of Transfer-Matrix Method      |     |
|   | 8-12  | Variable-Characteristics Pipeline           |     |
|   | 8-13  | Stenosis in Human Cardiovascular System     |     |
|   | 8-14  | Case Study                                  |     |
|   | 8-15  | Summary                                     |     |
|   | Probl | lems  |     |
|   |       | rences                                      |     |
| 9 | CAV   | TTATION AND COLUMN SEPARATION               | 327 |
|   | 9-1   | Introduction                                |     |
|   | 9-2   | General Remarks                             |     |
|   | 9-3   | Causes of Column Separation                 |     |
|   | 9-4   | Energy Dissipation                          | 330 |
|   | 9-5   | Wave Velocity in a Gas-Liquid Mixture       |     |

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|---------|---------|---------|---|
| XII     | Con     | Exvisit | 0 |
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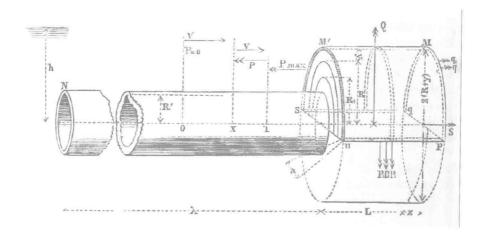
|    | 9-6       Analysis of Cavitation       335         9-7       Design Considerations       336         9-8       Case Study       337         9-9       Summary       341         Problems       341         References       343  |
|----|--|
|    |  |
| 10 | TRANSIENT CONTROL       347         10-1 Introduction       348         10-2 Surge Tank       349         10-3 Air Chamber       351         10-4 Valves       354         10-5 Optimal Transient Control       366         10-6 Case Study       367         10-7 Summary       370         Problems       371         References       373   |
| 11 | SURGE TANKS  |
|    | 11-1       Introduction       378         11-2       Types of Surge Tanks       378         11-3       Governing Equations       379         11-4       Solution of Governing Equations       387         11-5       Surge Oscillations in Frictionless System       388         11-6       Terminology       389         11-7       Normalization of Equations       393         11-8       Phase-Plane Method       393         11-9       Stability of Simple Tank       395         11-10       Stability of Closed Surge Tank       401         11-11       Multiple Surge Tanks       405         11-12       Design Considerations       407         11-13       Case Study       410         11-14       Summary       414         Problems       414         References       418 |
| 12 | LEAK AND PARTIAL BLOCKAGE DETECTION $\dots\dots421$  |
|    | 12-1 Introduction       422         12-2 Terminology       422         12-3 Partial Blockage Detection       424         Frequency Analysis       424         System Parameters       425         PPFR for Partial Blockage       426         12-4 Leak Detection       428         Engagement Responses       420   |
|    |  |

|    |  | Contents | XIII  |
|----|--|----------|---|
|    | Leak Detection Procedure  12-5 Real-life Application  12-6 Summary  Problems  References   |          | . 433<br>. 433<br>. 433   |
| 13 | TRANSIENT OPEN-CHANNEL FLOWS  13-1 Introduction 13-2 Terminology 13-3 Examples of Transient Flows 13-4 Surge Height and Celerity 13-5 Governing Equations 13-6 Methods of Solution 13-7 Method of Characteristics 13-8 Explicit Finite-Difference Methods 13-9 Initial Conditions 13-10 Implicit Finite-Difference Methods 13-11 Second-Order Explicit Schemes 13-12 Comparison of Finite-Difference Methods 13-13 Special Topics 13-14 Case Study 13-15 Summary Problems References |          | $\begin{array}{c} .438 \\ .438 \\ .440 \\ .440 \\ .445 \\ .450 \\ .451 \\ .454 \\ .462 \\ .465 \\ .470 \\ .473 \\ .474 \\ .484 \\ .487 \\ .487 \end{array}$ |
| A  | DESIGN CHARTS  A-1 Equivalent Pipe A-2 Valve Closure A-3 Valve Opening A-4 Power Failure to Centrifugal Pump A-5 Air Chamber A-6 Simple Surge Tank A-7 Surges in Open Channels A-8 Data for Pumping Systems References   |          | . 503<br>. 504<br>. 504<br>. 504<br>. 506<br>. 511<br>. 514<br>. 522  |
| В  | TRANSIENTS CAUSED BY OPENING OR CL<br>A VALVE  B-1 Program Listing  B-2 Input Data  B-3 Program Output   |          | . 527<br>. 531  |

### XIV Contents

| $\mathbf{C}$ | TRANSIENTS CAUSED BY POWER FAILURE TO      |             |
|--------------|--|-------------|
|              | PUMPS                                      | . 53        |
|              | C-1 Program Listing                        | . 535       |
|              | C-2 Input Data                             |             |
|              | C-3 Program Output                         |             |
|              |  |             |
| D            | FREQUENCY RESPONSE OF A SERIES PIPING      |             |
|              | SYSTEM                                     | . 545       |
|              | D-1 Program Listing                        | . 545       |
|              | D-2 Input Data                             | . 547       |
|              | D-3 Program Output                         |             |
| 12           | WATER LEVEL OCCUL ATIONS IN A CIMPLE CUROR |             |
| E            | WATER LEVEL OSCILLATIONS IN A SIMPLE SURGE | No. Chicago |
|              | TANK                                       |             |
|              | E-1 Program Listing                        | . 549       |
|              | E-2 Input Data                             | . 552       |
|              | E-3 Program Output                         | . 552       |
| F            | SI AND ENGLISH UNITS AND CONVERSION        |             |
| L            |  |             |
|              | FACTORS                                    |             |
|              | F-1 SI Units                               |             |
|              | F-2 Conversion Factors                     |             |
|              | Author Index                               | . 557       |
|              | Subject Index                              | 565         |

# BASIC CONCEPTS



Rigid metal pipe and elastic rubber section for Isebree Moens' experiments [1877].

(Courtesy, Arris Tijsseling.)

### 1-1 Introduction

In this chapter, a number of common terms related to hydraulic transients are defined, and a brief history of the developments in hydraulic transients is presented. The basic waterhammer equations relating the change in pressure due to an instantaneous change in flow velocity and an expression for the velocity of pressure waves in a pipe are derived. The propagation and reflection of waves in a pipeline are discussed, followed by different approaches for the analysis of hydraulic transients. The chapter concludes with brief information on a number of accidents and incidents caused by hydraulic transients.

# 1-2 Terminology

A number of common terms are defined in this section.

Steady and Unsteady Flow. Flow is called steady if the flow conditions, such as pressure and velocity, at a point are constant with time. If the conditions change with time, the flow is termed unsteady. Strictly speaking, turbulent flows are always unsteady since the conditions at a point in these flows are changing continuously. However, these flows are considered steady if the temporal mean conditions over a short period do not change with time. When referring to the steady or unsteady turbulent flows herein, we will consider the temporal mean conditions for this designation.

**Transient Flow.** The intermediate-stage flow, when the flow conditions change from one steady-state to another steady state, is called transient flow.

Uniform and Nonuniform Flow. If the flow velocity is constant with respect to distance at any given time, the flow is called uniform flow, whereas if the velocity varies with distance, the flow is called nonuniform.

Steady-Oscillatory or Periodic Flow. If the flow conditions are varying with time and if they repeat after a fixed time interval, the flow is called periodic or steady-oscillatory flow. The time interval at which conditions are repeating is termed as the period. If T is the period in seconds, then the frequency of oscillations, f, in cycles/s and in rad/s is 1/T and  $2\pi/T$ , respectively. Frequency expressed in rad/s is called circular frequency and is usually designated by  $\omega$ .

Column Separation. If the pressure in the flow drops to the vapor pressure of the liquid, then cavities are formed in the liquid and many times the liquid column may separate over the entire cross section.

Waterhammer. In the past, terms such as waterhammer, oilhammer, and steamhammer referred to the pressure fluctuations caused by a flow change depending upon the fluid involved. However, hydraulic transients has become common since the 1960s.

**Pressure Surge.** Transients involving slowly varying pressure oscillations are referred to as *pressure surges* or *surges* in North America. In Europe especially in the United Kingdom, however, the term *pressure surge* includes both rapid (i.e., waterhammer) and slow transients. In this book, we shall follow the North American practice.

To clarify the preceding definitions, let us discuss the flow conditions in a pipeline following instantaneous closure of the downstream valve (Fig. 1-1). Initially, the downstream valve is fully open, and the flow velocity throughout the pipeline is  $V_o$ . At time  $t=t_o$ , the valve is suddenly closed, reducing the flow through the valve instantly to zero. Because of the conversion of kinetic energy, pressure rises at the valve, and a pressure wave travels in the upstream direction. This wave is reflected from the reservoir and then travels back and forth between the closed valve and the reservoir. Due to losses in the system, this wave is dissipated as it travels in the pipeline. Finally, let us say at time  $t_1$ , the flow is completely stopped and the pressure in the entire pipeline is the same as the reservoir head.

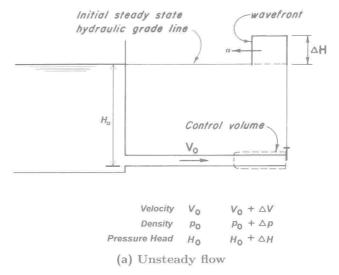
Based on the preceding definitions, the flow for  $t < t_o$  and  $t > t_1$  is steady when the conditions are constant with respect to time. However, the intermediate flow (i.e.,  $t_o \le t \le t_1$ ) when the flow conditions are changing from the initial steady state to the final steady state is called transient flow.

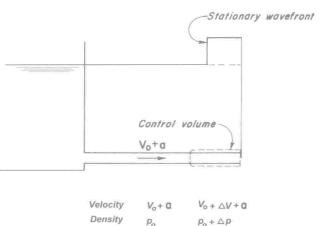
Now let us consider another flow situation in which the downstream valve is opened and closed periodically at frequency,  $\omega_f$ . After a number of cycles, the transient flow in the pipeline becomes periodic with the same period as that of the opening and closing valve. This flow is called *steady-oscillatory flow* or *periodic flow*.

# 1-3 Historical Background

A brief history of the developments in hydraulic transients is presented in this section (most of the material is based on Wood [1970]). Interested reader should see papers by Tijsseling and Anderson [2004, 2007, 2008, 2012].

The study of fluid transients began with the investigation of the propagation of sound waves in air, the propagation of waves in shallow water, and the flow of blood in arteries. Newton [1687] studied the propagation of sound waves in air and the propagation of water waves in canals. Both Newton and Lagrange obtained theoretically the velocity of sound in air as 298.4 m/s as compared to their experimental value of 348 m/s. Lagrange erroneously attributed this difference to experimental error, whereas Newton indicated that





(b) Unsteady flow converted to steady flow

 $H_{o}$ 

Hn+ AH

Pressure Head

Fig. 1-1. Propagation of pressure wave.

the theoretical velocity was incorrect and that this discrepancy was due to spacing of the solid particles of air and the presence of vapors in air. By comparing the oscillations of a liquid in a U-tube to that of a pendulum, Newton derived an incorrect expression for the celerity of water waves in a canal as  $\pi\sqrt{L/g}$ , where L= the wavelength and g= acceleration due to gravity. Euler [1759] derived the following partial differential equation for wave propagation:

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \tag{1-1}$$

in which a = wave speed. He developed a general solution of this equation as

$$y = F(x+at) + f(x-at)$$
(1-2)

in which F and f are the traveling waves. Euler [1775] tried, but failed, to obtain a solution for the flow of blood through arteries.

Lagrange [1788] analyzed the flow of compressible and incompressible fluids utilizing the concept of velocity potential. He also derived a correct expression for the celerity of waves in a canal as  $c=\sqrt{gd}$ , in which d= flow depth. Monge developed a graphical method for integrating the partial differential equations [1789] and introduced the term method of characteristics. Around 1808, Laplace explained the difference between the theoretical and measured values of the velocity of sound in air as follows: The relationships derived by Newton and Lagrange were based on Boyle's law. This law was not valid under varying pressures since the air temperature did not remain constant. He reasoned that the theoretical velocity would increase by about 20 percent if the adiabatic conditions were assumed instead of the isothermal conditions.

Young [1808] investigated the propagation of pressure waves in pipes. Helmholtz appears to be the first to point out that the velocity of pressure waves in water contained in a pipe was less than that in unconfined water. He correctly attributed this difference to the elasticity of pipe walls. In 1869, Riemann [1869] developed a three-dimensional equation of motion and applied its simplified one-dimensional form to analyze vibrating rods and sound waves. Weber [1866] studied the flow of an incompressible fluid in an elastic pipe and conducted experiments to determine the velocity of pressure waves. He also developed the dynamic and continuity equations. Marey [1875] conducted extensive tests to determine the velocity of pressure waves in water and in mercury and concluded that the wave velocity was independent of the amplitude of the pressure waves, was three times greater in mercury than in water, and was proportional to the elasticity of the tube. Resal [1876] developed the continuity and dynamic equations and a second-order wave equation and compared his analytical studies with Marey's experimental results. Lord Rayleigh published his book "Theory of Sound" in 1877.

Korteweg [1878] was the first to determine the wave velocity considering the elasticity of both the pipe wall and the fluid; earlier investigators had considered only one of the two at a time. Tijesseling and Anderson [2012] indicated that Korteweg gave Moens' smei-empirical expression for wave speed a mathematical basis and credited Moen [1877] for extensive experiments on metal pipes equipped with air pockets.

Although Wood lists Michaud [1878] as the first to study waterhammer, recent investigations by Anderson [1976] have shown that actually Menabrea [1858] was the first to do so. Michaud [1878] presented the design and use of

air chambers and safety valves. Gromeka [1883] included the friction losses in the waterhammer analysis for the first time, assuming the liquid to be incompressible and the friction losses to be directly proportional to the flow velocity.

Weston [1885] and Carpenter [1893-1894] conducted experiments to develop a theoretical relationship between the reduction of flow velocity in a pipe and the corresponding pressure rise. However, neither one succeeded because their pipelines were short. Frizell [1898] analyzed waterhammer in the Ogden hydroelectric development in Utah with a 9.45 km long penstock, developed expressions for the velocity of waterhammer waves and for the pressure rise due to instantaneous reduction of the flow. He stated that the wave velocity would be the same as that of sound in unconfined water if the modulus of elasticity of the pipe walls were infinite. He also discussed the effects of branch lines, wave reflections, and successive waves on speed regulation. Unfortunately, for some unknown reason, Frizell's work has not been recognized as much as that of his contemporaries, such as Joukowski [1898] and Allievi [1903, 1913 and 1937].

In 1897, Joukowski conducted extensive experiments in Moscow on 7.62 km long and 50 mm in diameter, 305 m long and 101.5 mm in diameter, and 305 m long and 152.5 mm in diameter pipelines. Based on his experimental and theoretical studies, he published his classic report [1898, 1900] on the basic theory of waterhammer. He developed a formula for the wave velocity, taking into consideration the elasticity of both the water and the pipe walls. He also developed the relationship between the reduction of flow velocity and the resulting pressure rise by using the conservation of energy and the continuity condition. He discussed the propagation of a pressure waves in a pipe and the wave reflection from the open end of a branch. He studied the utilization of air chambers, surge tanks, and safety valves to control waterhammer pressures. He found that the pressure rise was a maximum for closing times  $\leq 2L/a$ , in which L = length of the pipeline and a = wave speed.

Allievi published the general theory of waterhammer in 1903. His dynamic equation was more accurate than that of Korteweg. He showed that the term  $V\left(\partial V/\partial x\right)$  in the dynamic equation was not important as compared to the other terms and could be dropped. He introduced two dimensionless parameters:

$$\rho = \frac{aV_o}{2gH_o} 
\theta = \frac{aT_c}{2L}$$
(1-3)

in which a = wave velocity;  $V_o =$  steady-state velocity; L = length of the pipeline;  $T_c =$  valve-closure time;  $\rho =$  one half of the ratio of the kinetic energy of the fluid to the potential energy stored in the fluid and the pipe walls at pressure head  $H_o$  and  $\theta =$  the valve-closure characteristics. For the valve-closure time,  $T_c$ , Allievi obtained an expression for the pressure rise at the valve and presented charts for the pressure rise and drop caused by a