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*Seven Tales of the  
Pendulum*

Gregory L. Baker



The pendulum is a unique physical system which exhibits remarkably varied and complex behavior under many different conditions. It is also a system which, in its many manifestations, has left a significant imprint on human thought and culture. Using graphs, figures, and narrative to explain scientific ideas and models, Gregory Baker gives a lucid account of the physics of the pendulum, showing the reader how the context of the pendulum progresses over four centuries from that of a simple system of classical physics, to that of a chaotic system, and eventually to that of a modern quantum system. He also describes its fascinating presence in cultural history, from its role in timekeeping and measurements of the earth to its importance as a literary symbol of doom.

Seven “tales”, detailing different important facets of the pendulum, show the exciting diversity of the science of the pendulum, and its untold significance in the history of human intellectual development.

This book will appeal to anyone with an interest in science and its impact on human thought and culture.

**“Gregory Baker spins a series of tales about how the pendulum became central to a wide range of scientific inquiry and technological development. This book is highly recommended both for the professional scientist and the curious layman, each of whom will find much of interest and will surely learn a thing or two.”**

*Julien Clinton Sprott, University of Wisconsin-Madison*

**“Reader friendly and educative, this book would compel one to have a second look at the pendulum the next time one crosses it.”**

*R. Balashankar, The Organiser Weekly*

**“a useful and interesting book for physics students who want to find out more about pendulums.”**

*Brian Clegg, Popular Science*

**“... a captivating book ...”**

*European Journal of Physics*

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Baker

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*Seven Weeks*

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# SEVEN TALES OF THE PENDULUM

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OXFORD  
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## SEVEN TALES OF THE PENDULUM



To Margaret





## Preface

In 2005 Oxford University Press published a unique physics textbook, *The Pendulum: A Case Study In Physics*, by Gregory Baker (this author) and James Blackburn. This new book, *Seven Tales of the Pendulum*, grew from that work. The subject of the original work was a single physical system – the pendulum – which exhibits remarkably varied and complex behavior under many different conditions. We used this single system to build a comprehensive description of the pendulum’s classical, chaotic, and quantum behavior into a single book. In the course of our research it became apparent to us that, beyond the physics, the pendulum has importance for the historical development of applied science, and as such, has left a significant imprint on human thought and culture. Many of these nonscientific, but culturally enriching, connections found their way into the text. The combination of these factors resulted in that publication being devoted to the science, history, and culture of the pendulum.

*The Pendulum* was written as a text for students and researchers of mathematics, physics, and other quantitative sciences, and therefore utilized advanced mathematics. Yet, the diversity of the science of the pendulum and its importance in the history of human intellectual development are exciting stories that needed to be presented to a wider audience. Therefore, one of us (GLB) decided to write an almost non-technical version, the book you have in hand.

*Seven Tales of the Pendulum* is intended for the general reader with an interest in science and its relation to other human endeavors. It is an accessible description of the science of the pendulum, liberally sprinkled with related historical and cultural matters. What about the mathematical prerequisite? Almost all of the mathematical formulas from the old work have disappeared. Instead, I describe quantitative relationships with words, pictures, and graphs. Occasionally, I use “scientific” notation for numbers that are very big or very small. For example, the approximate size of an atom is about 0.0000000002 meters or, in scientific notation,  $2 \times 10^{-10}$  meters. The United States’ national debt is, at this writing, about 13 trillion dollars or, in scientific notation,  $13 \times 10^{12}$  dollars. Occasionally, we will use a letter or symbol to represent a quantity. For example, we use the Greek letter theta  $\theta$  to signify the

angle of a pendulum away from the downward vertical position. And very occasionally, a simple algebraic formula will be used to reinforce the meaning of the graphs and words. (I do sneak in the sine curve from trigonometry, but pictures should make its meaning clear.) Readers may rest easy knowing that I am mindful of the warning made famous by Stephen Hawking, that every formula reduces the readership by a factor of two.

While this book relies heavily on *The Pendulum: A Case Study in Physics*, it is structured somewhat differently, with a clearer distinction between small-amplitude and large-amplitude motion. The material has been rearranged, completely edited, condensed in many places, and expanded in others. Furthermore, certain sections of the 2005 production that may be either too esoteric or too mathematical for the general reader are not included. For example, beyond passing mention, I do not describe the mathematical analogy of the damped driven pendulum with a superconducting Josephson junction, a topic that previously occupied a whole chapter. Similarly, the chapter on synchronization is now heavily edited to omit difficult technicalities. Yet many surprising features are still described in sufficient detail to give the reader a sense of that growing field of study. As behooves a popular work, detailed references are not included. Instead, there is a short section of “further reading” that points to generally accessible additional material. Those wishing references must consult the over two hundred sources in the 2005 book. Finally, there is a short glossary of terms.

The passage of several years has only strengthened the conviction that the story of the pendulum is both interesting and important. It is part of our cultural and scientific history, providing roots of that history as well as pointing toward a future for the pendulum that is yet to be charted. It is most gratifying to be offering this story to a wider audience.

Gregory L. Baker  
Huntingdon Valley  
Pennsylvania  
2010

## Acknowledgments

Because this book rests heavily on *The Pendulum: A Case Study in Physics*, much credit must go to those who aided in that production. However, their contributions are recognized in that book and I refer the reader to the acknowledgments there. In this book, I would like to recognize individuals who contributed specifically to this new work. These include some who helped with the earlier work as well as new people.

I am grateful to Jonathan Betts, senior curator of horology at the National Maritime Museum, Greenwich, for pointing me to certain documents on how early clocks were calibrated. Further information on clocks was kindly provided by Bob Holmstrom. I would like to thank many people who freely allowed me to again use diagrams or figures from the earlier work or that are new to this book. These include Dan Allen, John Byrd, Beryl Clotfelter, Richard Crane, Jens Gundlach, J. Carter Harris, John Lindner, Juan Sanmartin, Bill Underwood, Margaret Walker, and James Yorke. I would like to thank Steve Fazzio for allowing me to choose from among his lovely photographs of O Botafumeiro. Jason Gallas produced beautiful diagrams using his scientific expertise and many hours of computer time, for which I am very grateful. I also appreciate the help of organizations that freely allowed me to use new diagrams or those from the previous work. This group includes the American Association of Physics Teachers, the American Physical Society, the American Chemical Society, Cambridge University Press, the Long Now Foundation, the National Watch and Clock Museum (USA), Oxford University Press, the Rare Books and Special Collections section of Princeton University Library, and the Society of Exploration Geophysicists. I also acknowledge permissions for figure use from the National Maritime Museum, l'Observatoire de Paris, and the Science Museum of London.

The nature of this book provides a strong incentive for the insertion of many figures, taken from a variety of sources. I have made every effort to determine original sources and obtain permissions for the use of these illustrations. Credit is found in the figure captions. Figures that were generated either by James Blackburn or myself for *The Pendulum: A Case Study in Physics* are typically not given specific credit. New figures generated by me for this book are again not given specific credit.

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I appreciate help from David Perry of the Swedenborg Library at Bryn Athyn College in acquiring important materials. Various people read preliminary drafts of certain chapters and provided valuable comments. These include Thane Glenn, Robert Hilborn, Robert Johns, and Julian Clint Sprott.

I feel a special sense of gratitude to Sonke Adlung, the senior physical science editor at Oxford University Press. His patience and encouragement fortified my resolve to search for, and hopefully find, a style and format that would be suitable for a popular audience. I also appreciate the help of April Warman as I worked through the publication process. I valued her clear and timely responses to my many questions.

My co-author for the previous book, colleague with other projects, and friend, James Blackburn, deserves special thanks. Many of the diagrams that he created for *The Pendulum: A Case Study in Physics* populate this new work. He also very kindly created new or improved artwork for this book. His aid, encouragement, and wise counsel have been a strong support for me as I took on this new challenge. I value his professional acumen and his abiding friendship.

Last, and most deserving of my gratitude, is my wife, Margaret. She read every submission and every draft of the book, catching my grammatical slips, but more importantly, acting as my archetypical reader. Her persistence in asking that I explain things more clearly or more fully has been immeasurably important to creating an understandable and readable book. Without her gentle and determined insistence on continual improvement, this book would not have happened. And during the process, she has continued to be the wonderful partner that she has been for more than four decades. I am profoundly grateful for our life together.



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