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Computational Methods in Power System Analysis

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Preface

There are many excellent books on power systems that treat power system analysis, and its most important computational problem: the power flow problem. Some of these books also discuss the traditional computational methods for solving the power flow problem, i.e., Newton power flow and Fast Decoupled Load Flow. However, information on newer solution methods is hard to find outside research papers.

This book aims to fill that gap, by offering a self-contained volume that treats both traditional and newer methods. It is meant both for researchers who want to get into the subject of power flow and related problems, and for software developers that work on power system analysis tools.

Part I of the book treats the mathematics and computational methods needed to understand modern power flow methods. Depending on the knowledge and interest of the reader, it can be read in its entirety or used as a reference when reading Part II. Part II treats the application of these computational methods to the power flow problem and related power system analysis problems, and should be considered the meat of this publication.

This book is based on research conducted by the authors at the Delft University of Technology, in collaboration between the Numerical Analysis group of the Delft Institute of Applied Mathematics and the Electrical Power Systems group, both in the faculty Electrical Engineering, Mathematics and Computer Science.

The authors would like to acknowledge Kees Vuik, the Numerical Analysis chair, and Lou van der Sluis, the Electrical Power Systems chair, for the fruitful collaboration, as well as all colleagues of both groups that had a part in our research. Special thanks are extended to Robert van Amerongen, who was vital in bridging the gap between applied mathematics and electrical engineering.

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Delft, October 2013

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Chapter 1
Introduction

Electricity is a vital part of modern society. We plug our electronic devices into wall sockets and expect them to get power. Power generation is a subject that is in the news regularly. The issue of the depletion of natural resources and the risks of nuclear power plants are often discussed, and developments in wind and solar power generation, as well as other renewables, are hot topics. Much less discussed is the transmission and distribution of electrical power, an incredibly complex task that needs to be executed reliably and securely, and highly efficiently. To achieve this, both operation and planning require many complex computational simulations of the power system network.

Traditionally, power generation is centralised in large plants that are connected directly to the transmission system. The high voltage transmission system transports the generated power to the lower voltage local distribution systems. In recent years, decentralised power generation has been emerging, for example in the form of solar panels on the roofs of residential houses, or small wind farms that are connected to the distribution network. It is expected that the future will bring a much more decentralised power system. This leads to many new computational challenges in power system operation and planning.

Meanwhile, national power systems are being interconnected more and more, and with it the associated energy markets. The resulting continent-wide power systems lead to much larger power system simulations.

The base computational problem in steady-state power system simulations is the power flow (or load flow) problem. The power flow problem is a nonlinear system of equations that relates the bus voltages to the power generation and consumption. For given generation and consumption, the power flow problem can be solved to reveal the associated bus voltages. The solution can be used to assess whether the power system will function properly. Power flow studies are the main ingredient of many computations in power system analysis.

Contingency analysis simulates equipment outages in the power system, and solves the associated power flow problems to assess the impact on the power system. Contingency analysis is vital to identify possible problems, and solve them before

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they have a chance to occur. Many countries require their power system to operate in such a way that no single equipment outage causes interruption of service.

Monte Carlo simulations, with power flow calculations for many varying generation and consumption inputs, can be used to analyse the stochastic behaviour of a power system. This type of simulation is becoming especially important due to the uncontrollable nature of wind and solar power.

Operation and planning of power systems further lead to many kinds of optimisation problems. What power plants should be generating how much power at any given time? Where to best build a new power plant? Which buses to connect with a new line or cable? All these questions require the solution of some optimisation problem, where the set of feasible solutions is determined by power flow problems, or even contingency analysis and Monte Carlo simulations.

Traditionally, the power flow problem is solved using Newton power flow or the Fast Decoupled Load Flow (FDLF) method. Newton power flow has the quadratic convergence behaviour of the Newton-Raphson method, but needs a lot of computational work per iteration, especially for large power flow problems. FDLF needs relatively little computational work per iteration, but the convergence is only linear. In practice, Newton power flow is generally preferred because it is more robust, i.e., for some power flow problems FDLF fails to converge, while Newton power flow can still solve the problem. However, neither method is viable for very large power flow problems. Therefore, the development of fast and scalable power flow solvers is very important for the continuous operation of future power systems.

In this book, Newton-Krylov power flow solvers are treated that are as fast as traditional solvers for small power flow problems, and many times faster for large problems. Further, contingency analysis is used to demonstrate how these solvers can be used to speed up the computation of many slightly varying power flow problems, as found not only in contingency analysis, but also in Monte Carlo simulations and some optimisation problems.

In Part I the relevant computational methods are treated. The theory behind solvers for linear and nonlinear systems of equations is treated to provide a solid understanding of Newton-Krylov methods, and convergence theory is discussed, as it is needed to be able to make the right choices for the Krylov method, preconditioning, and forcing terms, and to correctly interpret the convergence behaviour of numerical experiments.

In Part II power system analysis is treated. The relevant power system theory is described, traditional solvers are explained in detail, and Newton-Krylov power flow solvers are discussed and tested, using many combinations of choices for the Krylov method, preconditioning, and forcing terms.

It is explained that Newton power flow and FDLF can be seen as elementary Newton-Krylov methods, indicating that the developed Newton-Krylov power flow solvers are a direct theoretical improvement on these traditional solvers. It is shown, both theoretically and experimentally, that well-designed Newton-Krylov power flow solvers have no drawbacks in terms of speed and robustness, while scaling much better in the problem size, and offering even more computational advantage when solving many slightly varying power flow problems.

Part I
Computational Methods

Chapter 2
Fundamental Mathematics

This chapter gives a short introduction to fundamental mathematical concepts that are used in the computational methods treated in this book. These concepts are complex numbers, vectors, matrices, and graphs. Vectors and matrices belong to the field of linear algebra. For more information on linear algebra, see for example [1], which includes an appendix on complex numbers. For more information on spectral graph theory, see for example [2].

2.1 Complex Numbers

A complex number $\alpha \in \mathbb{C}$, is a number

$$\alpha = \mu + \iota\nu, \tag{2.1}$$

with $\mu, \nu \in \mathbb{R}$, and ι the imaginary unit[1] defined by $\iota^2 = -1$. The quantity $\operatorname{Re}\alpha = \mu$ is called the real part of α, whereas $\operatorname{Im}\alpha = \nu$ is called the imaginary part of the complex number. Note that any real number can be interpreted as a complex number with the imaginary part equal to 0.

Negation, addition, and multiplication are defined as

$$-(\mu + \iota\nu) = -\mu - \iota\nu, \tag{2.2}$$

$$\mu_1 + \iota\nu_1 + \mu_2 + \iota\nu_2 = (\mu_1 + \mu_2) + \iota(\nu_1 + \nu_2), \tag{2.3}$$

$$(\mu_1 + \iota\nu_1)(\mu_2 + \iota\nu_2) = (\mu_1\mu_2 - \nu_1\nu_2) + \iota(\mu_1\nu_2 + \mu_2\nu_1). \tag{2.4}$$

[1] The imaginary unit is usually denoted by i in mathematics, and by j in electrical engineering because i is reserved for the current. In this book, the imaginary unit is sometimes part of a matrix or vector equation where i and j are used as indices. To avoid ambiguity, the imaginary unit is therefore denoted by ι (iota).

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The complex conjugate is an operation that negates the imaginary part:

$$\overline{\mu + \iota\nu} = \mu - \iota\nu. \tag{2.5}$$

Complex numbers are often interpreted as points in complex plane, i.e., 2-dimensional space with a real and imaginary axis. The real and imaginary part are then the Cartesian coordinates of the complex point. That same point in the complex plane can also be described by an angle and a length. The angle of a complex number is called the argument, while the length is called the modulus:

$$\arg(\mu + \iota\nu) = \tan^{-1}\frac{\nu}{\mu}, \tag{2.6}$$

$$|\mu + \iota\nu| = \sqrt{\mu^2 + \nu^2}. \tag{2.7}$$

Using these definitions, any complex number $\alpha \in \mathbb{C}$ can be written as

$$\alpha = |\alpha|\, e^{\iota\varphi}, \tag{2.8}$$

where $\varphi = \arg\alpha$, and the complex exponential function is defined by

$$e^{\mu+\iota\nu} = e^{\mu}\left(\cos\nu + \iota\sin\nu\right). \tag{2.9}$$

2.2 Vectors

A vector $\mathbf{v} \in K^n$ is an element of the n-dimensional space of either real numbers ($K = \mathbb{R}$) or complex numbers ($K = \mathbb{C}$), generally denoted as

$$\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}, \tag{2.10}$$

where $v_1, \ldots, v_n \in K$.

Scalar multiplication and vector addition are basic operations that are performed elementwise. That is, for $\alpha \in K$ and $\mathbf{v}, \mathbf{w} \in K^n$,

$$\alpha\mathbf{v} = \begin{bmatrix} \alpha v_1 \\ \vdots \\ \alpha v_n \end{bmatrix}, \quad \mathbf{v} + \mathbf{w} = \begin{bmatrix} v_1 + w_1 \\ \vdots \\ v_n + w_n \end{bmatrix}. \tag{2.11}$$

The combined operation of the form $\mathbf{v} := \alpha\mathbf{v} + \beta\mathbf{w}$ is known as a vector update. Vector updates are of $O(n)$ complexity, and are naturally parallelisable.