



# Wavelet Analysis in Civil Engineering

Pranesh Chatterjee



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Tata Steel, Netherlands



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# Preface

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When I started my doctoral research, my supervisor introduced me to the concept of wavelets. Initially, I was quite suspicious about the term *wavelets*. In time I came to learn that a wavelet is a powerful signal processing tool and can do some amazing things – extract the features hidden in a signal, for example. I became fond of wavelets and used the wavelet analytic technique for most of my research work. However, I realized that this particular topic was not easy to digest. More specifically, it was difficult to find suitable books outlining the application of wavelets in engineering. Since then I had in mind a latent desire to write such a book from my experience in this area that would be beneficial to interested postgraduate students and researchers. The opportunity came when I visited the stall of CRC Press at the 15th World Conference on Earthquake Engineering in 2012 in Lisbon, Portugal. I expressed my wish to write a book on applications of wavelets in civil engineering, and was approached soon thereafter when we all decided to go ahead.

I am greatly indebted to Professor Biswajit Basu and Professor Mira Mitra, who shared their valuable experiences with me and never hesitated to carry out in-depth discussions at times on the topic. I am grateful to my wife, Tanima, who has been a constant source of inspiration in my work, as well as to my two children, Sraman and Soham, who spent quality time with me to break up the monotony of writing. Last but not the least, sincere thanks are due to my friend Dr Debashish Bhattacharjee, who always encouraged me to remain strong in the face of daunting tasks.

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## About the Author

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**Pranesh Chatterjee, PhD**, earned undergraduate and postgraduate degrees in civil engineering and a doctorate in engineering from Jadavpur University, India. His main research focus during his doctoral study was in the field of structural dynamics. During this research work he extensively used the wavelet-based analytical technique to formulate various problems in soil–structure–fluid interaction analyses. In later stages, Dr Chatterjee took up a postdoctoral fellowship in the Structural Mechanics section at Katholieke Universiteit te Leuven in Belgium and then was selected as a prestigious Pierse Newman Scholar at the University College Dublin in Ireland. After spending a considerable amount of time in academics and participating in a number of interesting research works that resulted in several journal and conference publications, he decided to move to industry. Since then he has worked in different fields and currently works as manager of the plasticity and tribology group of Tata Steel Europe in the Netherlands. Dr Chatterjee has always been active in research and its publication.

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# Introduction

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The main objective of Chapter 1 is to introduce to readers the concept and utility of wavelet transform. It begins with a brief history of wavelets referring to earlier works completed by renowned researchers, followed by an explanation of the Fourier transform. The chapter also shows the advantages of the wavelet transform over the Fourier transform through simple examples, and establishes the efficiency of the wavelet transform in signal processing and related areas. Chapter 2 first describes the discretization of ground motions using wavelet coefficients. Later, it explains the formulation of equations of motion for a single-degree-of-freedom system in the wavelet domain, and subsequently the same is used to build the formulation for multi-degree-of-freedom systems. The systems are assumed to behave in a linear fashion in this chapter. The wavelet domain formulation of equilibrium conditions of the systems and their solutions in terms of the expected largest peak responses form the basis of the technique of wavelet-based formulation for later chapters. Chapter 3 focuses on two distinct problems. The first is to explain how to characterize nonstationary ground motion using statistical functionals of wavelet coefficients of seismic accelerations. The second is to develop the formulation of a linear single-degree-of-freedom system based on the technique as described in Chapter 2 to obtain the pseudospectral acceleration response of the system. The relevant results are also presented at the end. Chapter 4 shows stepwise development of the formulation of a structure idealized as a linear multi-degree-of-freedom system in terms of wavelet coefficients. The formulation considers dynamic soil–structure interaction effects and also dynamic soil–fluid–structure interaction effects for specific cases. A number of interesting results are also presented at the end of the chapter, including a comparison between wavelet-based analysis and time history simulation. Chapter 5 describes the wavelet domain formulation of a nonlinear single-degree-of-freedom system. In this case, the nonlinearity is introduced into the system using a Duffing oscillator, and the solution is obtained through the perturbation method. Chapter 6 introduces the concept of probability in the wavelet-based theoretical formulation of a nonlinear two-degree-of-freedom system. The nonlinearity is considered



through a bilinear hysteretic spring, and the probability conditions are introduced depending on the position of the spring with respect to its yield displacement condition. The analysis is supplemented with some numerical results. In the last chapter (Chapter 7), focus is on diverse applications to make readers aware of the use of wavelets in these areas. For this purpose, three different cases are discussed. The first one is related to the analysis of signals from bridge vibrations to identify axles of vehicles passing over the bridge. The second example explains the basic concept and formulation of stiffness degradation using a physical model. Thereafter, the chapter focuses on using a numerical technique to obtain the results of a degraded model (stiffness degradation through formation of cracks) and then compares the wavelet-based analysis of the results obtained from linear and nonlinear models. The third example is related to soil–structure–soil interaction. In this example, the wavelet analytic technique is used to obtain the results at the base of a structure considering dynamic soil–structure interaction. Subsequently, the forces, shears and moments thus obtained at the base of the model are applied at the supporting soil surface and a three-dimensional numerical model of this structure–soil interaction problem is used to obtain a nonstationary response within the soil domain.

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# Introduction to wavelets

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## 1.1 HISTORY OF WAVELETS

In 1807, Joseph Fourier developed a method that could represent a signal with a series of coefficients based on an analysis function. The mathematical basis of Fourier transform led to the development of wavelet transform in later stages. Alfred Haar, in his PhD thesis in 1910 [1], was the first person to mention wavelets. The superiority of Haar basis function (varying on scale/frequency) to Fourier basis functions was found by Paul Levy in 1930. The area of wavelets has been extensively studied and developed from the 1970s. Jean Morlet, who was working as a geophysical engineer in an oil company, wanted to analyse a signal that had a lot of information in time as well as frequency. With the intention of having a good frequency resolution at low-frequency components, he could have used narrow-band short-time Fourier transform. On the other hand, in order to obtain good time resolution corresponding to high-frequency components, he could also have opted for broad-band short-time Fourier transform. However, aiming for one meant losing the other, and Morlet did not want to lose any of this information. Morlet used a smooth Gaussian window (representing a cosine waveform) and chose to compress this window in time to get a higher-frequency component or spread it to capture a lower-frequency component. In fact, he shifted these functions in time to cover the whole time range of interest. Thus, his analysis consisted of two most important criteria – dilation (in frequency) and translation (in time) – which form the basis of wavelet transform. Morlet called his wavelets ‘wavelets of constant shape’, which later was changed by other researchers only to ‘wavelets’. J.O. Stromberg [2] and later Yves Meyer [3] constructed orthonormal wavelet basis functions. Alex Grossmann and Jean Morlet in 1981 [4] derived the transformation method to decompose a signal into wavelet coefficients and reconstruct the original signal again. In 1986, Stephen Mallat and Yves Meyer developed multiresolution analysis using wavelets [3, 5, 6], which later in 1998 was used by Daubechies to construct her own family of wavelets. In

1996, Daubechies [7] gave a nice, concise description of the development of wavelets starting from Morlet through Grossmann, Mallat, Meyer, Battle and Lemarié to Coifman, from the 1970s through mid-1990s. A pool of academicians, including pure mathematicians, engineers, theoretical and applied physicists, geophysical specialists and many others, have developed various kinds of wavelets to serve specific or general purposes as and when needed. Thus, though initiated mainly by the mathematicians, wavelets have gained immense popularity in all fields of applied sciences and engineering due to their unique time–frequency localization feature. It is due to this unique property that the wavelet transform has proved its ability (and reliability) in analysing nonstationary processes to reveal apparently hidden information that no other tool could provide. The application areas are wide, e.g. geophysics, astrophysics, image analysis, signal processing, telecommunication systems, speech processing, denoising, image compression and so forth. The wavelets have been applied analysing vibration signals. Some special techniques like discrete and fast wavelet transforms have been developed for this purpose. Before going into the discussion on wavelet analytic technique any further, it would be wise to review the basic theory on Fourier transform at this point.

## 1.2 FOURIER TRANSFORM

Most of the single-valued functions may be written as the summation of a series of harmonic functions within a desired range. This series is termed Fourier series. The concept of such a series has already been used by Daniel Bernoulli in connection to solving problems of string vibrations. However, it was Joseph Fourier, the French mathematician, who did a systematic study on Fourier series for the first time. Fourier series has found many applications in the fields of heat conduction, acoustics, vibration analysis, etc. The Fourier series for a function  $f(x)$  in the interval  $\alpha < x < \alpha + 2\pi$  is written as follows:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx) \quad (1.1)$$

where

$$a_0 = \frac{1}{\pi} \int_{\alpha}^{\alpha+2\pi} f(x) dx \quad (1.2)$$

$$a_n = \frac{1}{\pi} \int_{\alpha}^{\alpha+2\pi} f(x) \cos(nx) dx \quad (1.3)$$

$$b_n = \frac{1}{\pi} \int_{\alpha}^{\alpha+2\pi} f(x) \sin(nx) dx \quad (1.4)$$

The readers should note that once the value of  $\alpha$  is chosen as zero, the interval becomes  $0 < x < 2\pi$ , and on choosing  $\alpha = -\pi$ , the range becomes  $-\pi < x < \pi$ . Thus, Fourier series can actually represent any periodic (and also nonperiodic) function as the sum of simple sine and cosine waves. This idea forms the basis of Fourier transform (FT), which is an extension of the Fourier series. In Fourier transform, the period of the function may extend to infinity. The Fourier transform retrieves the frequency content of a signal. It decomposes a signal into orthogonal trigonometric basis functions. The Fourier transform  $\hat{X}(\omega)$  of a continuous function  $x(t)$  is defined in the following equation:

$$\hat{X}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \quad (1.5)$$

In the above equation, the term  $\hat{X}(\omega)$  gives the global frequency distribution of the time-dependent original signal  $x(t)$ . The original signal  $x(t)$  can be further reconstructed using the inverse Fourier transform as defined below:

$$x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{X}(\omega) e^{i\omega t} d\omega \quad (1.6)$$

The following Dirichlet conditions must be satisfied for Fourier transform and its reconstruction:

1. The time function  $x(t)$  and its Fourier transform  $\hat{X}(\omega)$  must be single-valued and piece-wise continuous.
2. The integral  $\int_{-\infty}^{\infty} |x(t)| dt$  must exist that insists that if  $|\omega| \rightarrow \infty$ ,  $\hat{X}(\omega) \rightarrow \infty$ .
3. The functions  $x(t)$  and  $\hat{X}(\omega)$  have upper and lower bounds (however, this is not a necessary condition).

In case of signals obtained from experiments, they are discrete in nature as they are sampled at  $N$  discrete time points with a sampling time of, say,  $\Delta t$ . These signals are analysed in the frequency domain using the concept of discrete Fourier transform (DFT),  $\hat{X}_{DFT}(\omega)$ , as defined below:

$$\hat{X}_{DFT}(f_n) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-i2\pi n \Delta t} \quad (1.7)$$

It may be seen from Equation (1.7) that the DFT may be evaluated at discrete frequencies  $f_n = \frac{n}{N\Delta t}$ , where  $n = 0, 1, 2, \dots, N-1$ . The inverse DFT, as shown below, may be used to get back the original discrete time signal.

$$x(n) = \frac{1}{\Delta t} \sum_{f_n=0}^{\frac{N-1}{N\Delta t}} \hat{X}_{DFT}(f_n) e^{i2\pi f_n n \Delta t} \quad (1.8)$$

It may be noted here that the  $N\Delta t$  in the equation above denotes the time length of the signal. The discrete Fourier transform computation requires evaluation of real and imaginary parts separately; thus,  $2N^2$  numbers of operations would be required. So, DFT works quite well when the signal length is short. If the signal becomes large with numerous discrete time points, DFT could become very tedious. The idea of fast Fourier transform (FFT) is developed, which is computationally more efficient in such cases because the FFT algorithm works on signals that must have as many samples as the power of 2 (i.e.  $2^m$  samples). The FFT is much faster because it uses the results from previous computations and thereby reduces the number of operations required. It utilizes the periodicity and symmetry of trigonometric functions to compute the transform with approximately  $N \log N$  numbers of operations.

If the time-dependent function  $x(t)$  in Equation (1.6) has only one frequency, the corresponding frequency spectrum,  $\hat{X}(\omega)$ , is a Dirac delta function. So, if the frequency spectrum has only one frequency, say,  $\hat{X}(\omega) = \delta(\omega - \omega_0)$ , then on substituting  $\hat{X}(\omega)$  in Equation (1.6) the following expression of  $x(t)$  is obtained:

$$x(t) = \frac{1}{\sqrt{2\pi}} e^{-i\omega_0 t} \quad (1.9)$$



On substituting  $x(t)$  on the right-hand side in Equation (1.5), the following equation is obtained, which is one of the definitions of the Dirac delta function.

$$\hat{X}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j(\omega - \omega_0)t} dt = \delta(\omega - \omega_0) \quad (1.10)$$

The readers should also know about Parseval's theorem and Parseval's identity at this stage, as these will be used in later chapters. Parseval's theorem is written as follows:

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |\hat{X}(\omega)|^2 d\omega \quad (1.11)$$

This implies that the total energy content of a function  $x(t)$  summed over all time  $t$  is equal to the total energy contained in its Fourier transform summed across all of its frequency components.

$$\frac{1}{\pi} \int_{-\pi}^{\pi} [f(t)]^2 dt = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2 + b_n^2 \quad (1.12)$$

The identity tells us that the sum of the squares of the Fourier coefficients of a certain function, say  $f(t)$ , is equal to the integral of the square of the function.

### 1.3 RANDOM VIBRATION

The random vibration is a nondeterministic motion that has a unique randomness in its characteristic. The vibrations induced in trains and road vehicles due to track and road surface roughness, wind excitations, ground motions and wave loading are common examples of random vibrations. Typically, a random vibration may be either a stationary or a nonstationary process. A common characteristic feature of these vibrations is that these are randomly varying in time, which obviously means that these are nondeterministic (and hence nonperiodic) in nature. This implies that in case of random vibrations, it is not possible to predict the amplitude of vibration accurately at any specific instant; however, one may predict the probability of occurrence of acceleration or displacement amplitude at an instant. Unlike a pure sinusoidal vibration, a random vibration contains a continuous spectrum of frequencies. It may be worth mentioning here that the histogram of a random datum or