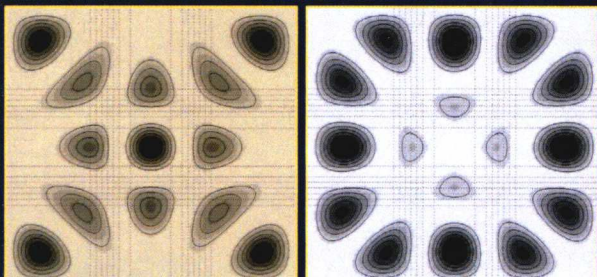
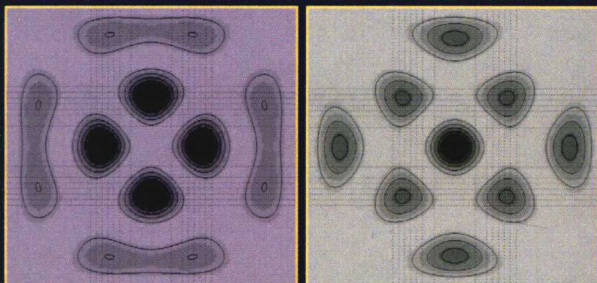
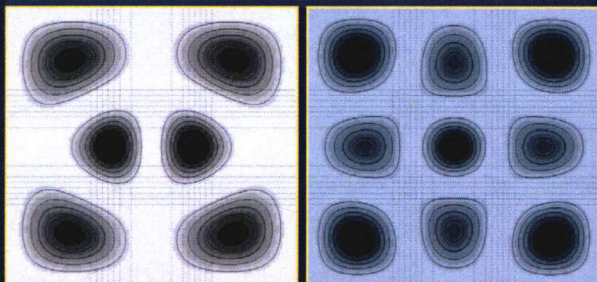


# Diffractive Optics and Nanophotonics



Edited by V. A. Soifer



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# Diffractive Optics and Nanophotonics

The phenomenon of diffraction, which was originally seen as a limiting factor in optics, is now a fundamental basis for the creation of a new component base and advanced information technology. **Diffractive Optics and Nanophotonics** is devoted to achievements in diffractive optics. It focuses on the development of new nanophotonic components and devices, as well as instrumentation and information technology based upon them.

Organized into five chapters authored by experts in the field, each chapter is exceptionally detailed and gives special attention to crucial topics and developments. Coverage includes rigorous mathematical methods for calculation of components of diffractive optics and nanophotonics, the focusing of laser radiation to overcome diffraction limitations, and rare coverage of resonant gratings, widely used in nanophotonics. This reference is a valuable tool for researchers and students in the field of chemistry, lasers, optics, and physics.



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**Soifer**



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# **Diffractive Optics and Nanophotonics**

# Introduction

The phenomenon of diffraction, which was originally seen as a limiting factor in optics, is now a fundamental basis for the creation of a new component base and advanced information technology.

The development of diffractive optics and nanophotonics devices is based on a computer solution of direct and inverse problems of diffraction theory, based on Maxwell's equations. Among the numerical methods for solving Maxwell's equations most widely used are: the finite-difference time-domain method, the finite element method and the Fourier modal method, and also approximate methods for calculating diffraction integrals.

The book is devoted to modern achievements of diffractive optics, focused on the development of new components and devices for nanophotonics, and devices and information technologies based on them.

The first chapter describes the Fourier modal method, designed for the numerical solution of Maxwell's equations, as well as some of its applications in problems of calculation of diffractive gratings with the resonance properties and plasmon optics components.

The Fourier modal method (or rigorous-coupled wave analysis) has a wide range of applications. In the standard formulation the method is used to solve the problems of diffraction of a monochromatic plane wave on diffraction gratings. Introduction of the light beam in a plane-wave basis allows to use the method for modelling the diffraction of optical pulses. Using the so-called perfectly matched layers in combination with artificial periodization enables the method to be used efficiently to solve the problems of the diffraction of light waves by non-periodic structures. In this chapter the Fourier modal method is considered for solving the diffraction of a plane wave in the two-dimensional and three-dimensional diffractive gratings, as well as in the case of non-periodic structures. The implementation of the method is based on the numerical-stable approach, known as the scattering matrix method.

The resonance features in the spectra of periodic diffraction structures are studied using the methods developed by the authors for calculating the scattering matrix poles. These methods take into account the form of the matrix in the vicinity of the resonances associated with the excitation of eigenmodes in the lattice and, compared with the known methods, have better convergence.

The Fourier modal method and scattering matrix formalism are applied to the calculation of diffractive gratings with the resonance properties for the conversion of optical pulses. The chapter proposes a theoretical model of resonant gratings performing operations of differentiation and integration of the optical pulse envelope and the results of the calculation and research of diffractive gratings for the differentiation and integration of picosecond pulses are presented.

The non-periodic variant of the Fourier modal method is used in the problem of calculating the diffractive optical elements for controlling the propagation of surface plasmon-polaritons. The principle of operation is based on the phase modulation of surface plasmon-polaritons by dielectric steps with changing height and length and located on the surface of the metal.

The Fourier modal method is also applied to the task of calculating the diffractive gratings forming, in the near-field, interference patterns of evanescent electromagnetic waves and, in particular plasmon modes. The chapter provides a theoretical description and a number of numerical examples of calculation of the gratings forming interference patterns of evanescent electromagnetic waves and plasmon modes with a substantially subwavelength period and demonstrates the ability to control the type and period of the interference patterns of damped waves due to changes in the parameters of the incident radiation.

The practical use of the results of the first chapter includes systems for optical computing and ultra-fast optical information processing, the creation of high-performance components plasmon optics, the contact lithography systems and the systems for optical trapping and manipulation of nanoscale objects.

The second chapter deals with the nanophotonics components based on photonic crystals: the gradient planar photonic crystal (PC) lens and photonic crystal fibers. The ultra-compact nanophotonic device is described for effectively connecting two-dimensional waveguides of different widths using the PC-lens. It is shown that the PC-lens focuses the light into a small focal spot directly behind the lens whose size is substantially smaller than the scalar diffraction



limit. The simulation was performed using a finite difference solution of Maxwell's equations.

The second chapter also describes the method of calculation of optical fibres with the PC cladding. In this waveguide the light propagates within the core not due to the effect of total internal reflection from the core-cladding interface, and by reflection from a multilayer Bragg mirror formed by the system of periodically spaced holes around the core. Calculation of spatial modes in PC-fibres is based on partitioning the inhomogeneous fibre cross-section into a set of rectangular cells, each with a set of known spatial sinusoidal modes. Further cross-linking of all modes is carried out at the interfaces of all cells. The PC waveguides differ from the step and gradient waveguides by that they allow the modes to be localized within the core, that is all the modes propagate inside the core and almost do not penetrate into the cladding thus increasing the diameter of the mode localized within the core. In addition, the PC waveguides with a hollow core help to avoid chromatic dispersion in the fibre and transmit light with higher power. A short pulse of light passing through in a PC waveguide of finite size is transformed at the output to white light due to non-linear dispersion. Sections of the PC waveguides are used as filters, white light sources, and non-linear optics for second harmonic generation.

The third chapter discusses the focusing of laser radiation. The concept of the diffraction limit was established in the 19th century:  $d_{\min} = \lambda/(2n)$ , where  $\lambda$  is the wavelength of light in vacuum,  $n$  is the refractive index of the medium. The third chapter shows that using diffractive micro-optics components, focusing the light near their surface it is possible to overcome the diffraction limit. Attention is given to the sharp focusing of laser light using micro-components such as the axicon, the zone plate, the binary and gradient planar microlens, microspheres. Focusing light near the surface of the micro-components allows to overcome the diffraction limit as a result of the presence of surface waves and the influence of the refractive index of the material of the focusing element. Simulation of focusing the laser beam is carried out by the approximate Richards-Wolf vector method and the finite difference solution of Maxwell's equations.

Reducing the size of the focal spot and overcoming the diffraction limit is an urgent task in the near-field microscopy, optical micromanipulation, contact photolithography, increasing the density of recording information on an optical disc, and coupling planar waveguides of different widths.

The fourth chapter describes the focusing of singular vortex laser beams. At the point of singularity the intensity of the light field is zero, and the phase is not defined. There are abrupt phase changes in the vicinity of this point. Singularities in light fields can appear as they pass through randomly inhomogeneous and non-linear media. It is also possible to excite vortex fields in laser resonators and multimode optical fibres. The most effective method of forming the vortex laser beams is to use spiral diffractive optical elements, including spiral phase plates and spiral axicons. The fourth chapter discusses the formation of vortex beams represented as a superposition of Bessel, Laguerre–Gauss, Hermite–Gauss, etc. modes. When focusing the vortex beams attention is paid to the combination of different types of polarization and phase singularities which lead to overcoming the diffraction limit of the far-field diffraction zone.

Main applications of vortex laser beams are sharp focusing of laser light, manipulation of microscopic objects and multiplexing the channels of information transmission.

In the fifth chapter we consider the problem of optical trapping, rotation, moving, positioning of micro-objects through the use of diffractive optical elements. Micro-objects are rotated by light beams with an orbital angular momentum. Considerable attention is paid to the methods of calculating the forces acting on the micro-objects in light fields. The problem of creating the torque in micromechanical systems using light beams has a fairly long history. In a number of studies the problem of rotation is considered in conjunction with other tasks: sorting, moving, positioning, etc. It should be noted that in all the above cases the focus is primarily on the manufacturing technology of micromechanics elements and no attempts are made to improve the light beams. At the same time, the calculation and application of diffractive optical elements, forming the vortex light beams for a specific form of the micromechanical component can improve the transmission efficiency of the torque in micromechanical systems.

This chapter discusses two methods of calculating the diffractive optical elements for forming light fields with a given amplitude-phase distribution. One of them is based on calculating a focusator forming a light field with a predetermined phase gradient along the contour. Another method uses the superposition of zero-order Bessel beams to form light traps in the form of hollow beams for opaque microscopic objects. The results of experiments on optical trapping and relocation

of micro-objects are presented. The chapter examines the possibility of using light beams to move the biological micro-objects.

The book has been written by experts of the Image Processing Systems Institute, Russian Academy of Sciences. In the first chapter, sections 1.1 and 1.2 were written by D.A. Bykov, E.A. Bezus and L.L. Doskolovich, section 1.3 by D.A. Bykov, L.L. Doskolovich and V.A. Soifer, sections 1.4 and 1.5 by E.A. Bezus and L.L. Doskolovich. The second chapter was written by V.V. Kotlyar, A.A. Kovalev, A.G. Nalimov and V.A. Soifer. The third chapter – by V.V. Kotlyar, A.A. Kovalev, A.G. Nalimov and S.S. Stafeev. The fourth chapter was written by S.N. Khonina and the fifth chapter by R.V. Skidanov, A.P. Porfir'ev and V.A. Soifer.

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