Mathematics for Physicists

物理学中的数学方法

Wang Huaiyu

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Introduction

This book is mainly for graduate students in physics and engineering, and also suitable for undergraduate students at senior level and those who intend to enter the field of theoretical physics. The contents cover the mathematical knowledges of the following fields: the theories of variational method, of Hilbert space and operators, of ordinary linear differential equations, of Bessel functions, of Dirac delta function, of the Green's function in mathematical physics, of norm, of integral equations, the application of number theory in physics, the basic equations in multidimensional spaces and non-Euclid spaces. The book explains the concepts and deduces the formulas in great details. It is very learner-friendly with its content level gradually from being easy to being difficult. A great amount of exercises are beneficial to readers.

Preface

This book grew from a graduate course that I have taught at Tsinghua University. It was originally written in Chinese and published by Science Press in Beijing. World Scientific Publishing and Science Press have been very kind to jointly publish this English version. This English version has been compressed in space compared to the original Chinese version.

The aim of this book is to integrate the necessary aspects of mathematics for graduate students in physics and engineering. The undergraduate students at senior level and researchers who intend to enter the field of theoretical physics were supposed possible readers. Readers are assumed to have knowledges of linear algebra and complex analysis. Some familiarity with elementary physical knowledges could be a prerequisite for deriving full benefit from reading this book.

This book consists of ten chapters. Chapter 1 introduces the variational method. Chapter 2 introduces the theories of Hilbert space and operators. In chapter 3, the theory of ordinary linear differential equations of second order is systematically presented. The polynomial solutions are given. The method of drawing series solutions based on the complex analysis is given. The problems concerning the adjoint equations are discussed. Chapter 4 comprehensively introduces Bessel functions and their various deforms. Chapter 5 introduces Dirac delta function. Chapter 6 presents the theory of Green's function in mathematical physics. By the way, the author has written a graduate textbook systematically introducing the Green's function in condensed matter physics. Chapter 7 introduces the theory with respect to norm. Chapter 8 introduces the theory of integral equations. In these chapters, the author payed attention to the link up with the contents that readers may have learnt in undergraduate courses.

The last two chapters are an attempt to introduce some recent achievements of scientific research into the textbook while presenting mathematical basic knowledges.

Chapter 9 introduces the basis of number theory and its application in physics, material science and other scientific fields. This kind of application was initiated by a Chinese scientist, Prof. Chen Nanxian. The author thought that this ingenious method was worthy of being introduced to readers. The mathematical basis in this chapter is relatively simple, but it leads to useful and wide results. This achievement has never been introduced in textbooks except a literary work by Chen himself.

Chapter 10 introduces the fundamental equations in spaces with arbitrary dimensions. This is because in modern physics, research has been not limited to three-dimensional space and one-dimensional time, and not limited to Euclid space. The author tries hard to introduce some basic knowledge in multidimensional spaces starting from the ordinary differential equations of second order. The associated Gegenbauer equation and its solutions enable us to realize the values of the angular momentum and their projections in Euclid spaces. The pseudo spherical coordinates were introduced. Although they were employed for the discussion on Euclid spaces, they apparently were also useful to investigations on non-Euclid spaces. Plain terminologies were utilized to present the concept of metric, without resorting to symmetry or group theory. The author believed that this is a way easily grasped by readers. The work on the Klein-Gorden equation and Maxwell equation were new and interesting.

When presenting the mathematical basic theory, the logic rigor was assured without loss of understandability. The author tried hard to clearly narrate the basic concepts and the relations between them. The derivations of the formulas were given in detail as far as possible. For proofs of the theorems, when they were too long or needed knowledges that were beyond the scope of this paper, we had to omit them. The explanations of the questions asked by students in the course of my teaching have been covered.

This textbook does not include the content of group theory, for there have been textbooks specialized for group theory.

The author thought that a great amount of exercises are beneficial to readers. Most of the exercises in this book were collected from materials. A few were prepared by the author.

The author thanks Prof. Chen Nanxian for introducing his smart work to the author. The contents in Chapter 9 in this book are all from Prof. Chen's work.

The investigations of the Klein-Gorden equation and Maxwell equations in de Sitter spacetime are from the work of Prof. Zhou Bin. The author thanks him for providing his achievements and for his helpful discussions.

I acknowledge and express my deep sense of gratitude to the valuable discussions and helps from Professors Wang Chongyu, Zhou Yunsong, Xun Kun, Han Rushan, Tong Dianmin, Zheng Yujun and Yu Yabin.

I wish to express my thanks to my wife Miao Qing and my family members Miao Hui, Miao JiChun and Wang Nianci for their constant help in my work and life.

A special thank goes to Prof. Lu Xiukun, who taught me mathematics when I was a student in secondary school.

Finally, I thank editor Qian Jun for his help in publishing the original Chinese version and present English version.

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This English version has been translated from Chinese by myself, with slight updating over the original Chinese version. I shall be most grateful to those readers who are kind enough to bring to my notice any remaining mistakes, typographical or otherwise for remedial action. Please feel free to contact me.

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