International Series in
Operations Research & Management Science

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Extension of Data Envelopment Analysis with Preference Information

Value Efficiency

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ISSN 0884-8289 ISSN 2214-7934 (electronic)
ISBN 978-1-4899-7527-0 ISBN 978-1-4899-7528-7 (eBook)
DOI 10.1007/978-1-4899-7528-7
Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2014950059

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

International Series in Operations Research & Management Science

Volume 218

More information about this series at http://www.springer.com/series/6161

To Paul, Elizabeth, and William
and
To Kaiju, Minna, Minni, Hannu, Janne,
Sasu, Merri, and Sissi

Preface

To improve performance is one of the key issues for managers in organizations. Demands to make operations more profitable and compatible as well as to cut public expenditure have been repeatedly present in headlines. The "goodness" of operations or performance is often simply measured in money; the firm making most money or the public sector unit having the least expenditure is considered to be the best one. However, the monetary measures do not always capture all aspects of the performance. Especially in the public sector it may be practically—or politically—impossible to attach prices to some goods or services produced: what is the price of a university degree, or of a medical operation saving a human life? We may be able to figure out the short-term costs of some operations, but what is—for instance—the price of the lost opportunity? How expensive it is not to educate, or to lose a life? Thus performance is clearly multidimensional in its nature, and several indicators (outputs) are required to characterize all essential aspects of performance. The factors (inputs) affecting performance are multidimensional as well. In practice, the relationships between outputs and inputs are often complex or unknown making direct performance evaluation a complicated task.

An alternative way to approach the performance evaluation problem is data envelopment analysis (DEA) developed by Charnes, Cooper, and Rhodes (1978 and 1979). Performance evaluation is carried out relatively by comparing decision-making units (DMUs) essentially performing the same task. The purpose is to study whether it is possible to find another comparable unit that produces more outputs with similar usage of inputs or achieves the same level of output production with less inputs. If such unit exists, it is quite clear that—other things being equal—the evaluated unit is not operating as well as it could be. In DEA, there is no need to explicitly know relationships between inputs and outputs. The values of inputs and outputs of the units are the only requisite information for the analysis.

DEA reveals the units which are supposed to be able to improve their performance and the units which cannot be recognized as poor performers. Because we use multidimensional factors to measure performance, "goodness" is not fully defined. For instance, we cannot name the best performer without preference information of somebody. DEA identifies technically efficient units, but it is value-free in the sense that it does not take into account importance of various factors. Whereas there are numerous books about DEA, none of them concentrates on incorporating preference or value information into the analysis. In many practical applications, the use of such information is a necessity.

The aim of this book is to provide an introduction to the methods currently available in the field of DEA to incorporate preference information. The book serves as a reference volume for the readers interested in those methods. In addition to theoretical considerations, numerous illustrative examples are included. Hence, the book can be used as a teaching text as well. Only a modest mathematical background is needed to understand the main principles. The only prerequisites are (a) familiarity with linear algebra, especially matrix calculus, (b) knowledge of the simplex method, and (c) familiarity with the use of computer software.

This book is organized as follows. Chapter 1 provides motivation and introduces the basic concepts. Chapter 2 provides the basic ideas and models of the DEA. The efficient frontier and production possibility set concepts play an important role in all considerations. That's why these concepts are considered closer in Chap. 3. Since the approaches introduced in this study are inspired by multiple objective linear programming, the basic concepts of this field are reviewed in Chap. 4. Chapter 5 also compares and contrasts DEA and multiple objective linear programming providing some cornerstones for approaches presented later in this book. Chapter 6 discusses the traditional approaches to take into account preference information in DEA. In Chap. 7 value efficiency is introduced, and Chap. 8 discusses practical aspects. Some extensions are presented in Chap. 9 and in Chap. 10 value efficiency is extended to cover the case, when a production possibility set is not convex. Three implemented applications are reviewed in Chap. 11.

The readers familiar with DEA may skip Chaps. 1 through 3, and the readers familiar with MOLP may skip Chap. 4. The readers interested in practical aspects may start to read Chap. 10 first and then "dig" necessary theory from the previous chapters into the extent needed.

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Chapter 1
Introduction

Basic Concepts

1.1 Motivation

Scarcity is one of the key concepts in economics. The fact that there are not enough resources to produce everything needed and wanted emphasizes the importance to utilize and allocate the existing ones in the best possible way. The demand for efficiency of operations both in private and public sectors has also been currently emphasized due to severe economic conditions and increased competition.

Most people inevitably associate efficiency with layoffs in the private sector and budget cuts in the public sector. The concept of efficiency itself is rather innocent: it just points out whether there is possibility to develop the firm or public organization—decision-making unit (DMU)—such that it performs better with the current resources or to keep the current performance with less resources. The methods which are used to evaluate efficiency aim at looking deeper than mere monetary figures: they examine the production process itself, how resources—inputs—are turned into products and services—outputs. This is a key factor contributing to their success: especially in the public sector it is often practically or politically difficult to put a price tag on some outputs, for example, when evaluating the performance of, say, the health care system.

With the absence of price information, we turn our attention to the production process itself and analyze *technical efficiency*. The DMUs are basically converting inputs into outputs. Schools use instructors, educational material, computer, and other facilities to provide education. Production uses plants, machinery, and man hours to produce physical products. In this framework it is possible to evaluate the technical efficiency of operations. In evaluating technical efficiency, we are primarily interested that the resources are efficiently used to produce outputs. Are we producing right things is a secondary question.

Because it is very hard to evaluate the absolute performance of the DMUs without any benchmarking unit, a more popular way is to evaluate relative

T. Joro, P.J. Korhonen, *Extension of Data Envelopment Analysis with Preference Information*, International Series in Operations Research & Management Science 218, DOI 10.1007/978-1-4899-7528-7_1

performance. As a performance measure, a *relative technical efficiency* concept is used. The problem is: Is it possible to find another comparable unit that can produce more outputs with similar usage of inputs, or achieves the same level of output production with less inputs? If such a unit exists, it is quite clear that the unit under evaluation is not operating as well as it could be. Hence forward, we use the term technical efficiency to refer to relative technical efficiency.

To measure technical efficiency of homogeneous units operating in similar conditions, Charnes et al. (1978, 1979) developed a method called data envelopment analysis (DEA). Currently, DEA has become one of the most widely used methods in operations research/management science (OR/MS) (see Bragge et al. 2012). It has gained popularity in performance evaluation both in public and private sectors (see Seiford 1996 for a bibliography on DEA). See also a Fortune magazine article by Norton (1994) and an OR Newsletter article by Simons (1996). Being a linear programming-based performance evaluation tool, DEA has strong connections to both OR/MS and production economics fields.

Based on information about the performance of those units, the purpose of DEA is to empirically characterize the *efficient frontier*. If a DMU lies on that frontier, it is referred to as an *efficient unit*, otherwise *inefficient*. When a unit is inefficient, a *target* (*reference*) *unit* is sought for each inefficient unit by projecting it *radially* onto the efficient frontier. In *radial projection* the values of controllable (input or output) variables are proportionally improved until the boundary of the efficient frontier is achieved. The input/output values of the target unit are considered *target values* for the inefficient unit. DEA calculates for the inefficient units a measure—called *efficiency score*—that illustrates its degree of efficiency. Occasionally, we measure inefficiency, then we call the score *inefficiency score*. The appealing feature in DEA is inevitably its capability to compress information on the usage of several inputs and the production of several outputs into a single figure: the (in)efficiency score.

The underlying assumption in the radial projection technique is that the "most suitable" target values for each inefficient unit are found without any additional information merely by proportionally improving controllable variables. Radial projection is a value-free technique in the sense that it does not require the intervention of a decision maker (DM). It also enables a straightforward technique to specify an efficiency score. Radial projection does not allow any flexibility for a DM to choose a target unit for an inefficient unit. This can undermine the significance of a target unit in practice. Consequently, the standard use of radial projection has encountered occasional critique and suggestions of other methods. For example, see Thanassoulis and Dyson (1992) and Färe and Grosskopf (2000). Nevertheless, the actual behavior of DM searching for a target unit in practice has rarely been studied. An exception is the paper by Korhonen et al. (2003).

Figuratively speaking DEA makes it possible to add apples and oranges together without pricing them. However, as promising as it sounds, also this is a double-edged sword: the caveat lies very much in the recognition of the relevant inputs and outputs. This may sound rather straightforward, but this is not usually the case in practice. In most of the cases the importance of different outputs, and also inputs, is very different. A university can list Ph.D. and master's degrees and journal articles as outputs, but also, e.g., working papers, conference presentations, and so forth. Consulting done by faculty members may be considered as one of the outputs of a university department. The way technical efficiency is defined makes it possible for a particular department to become efficient solely by concentrating on consulting at the expense of academic research and teaching. If indeed it would be acceptable to specialize in any input or output, the analysis of technical efficiency would work well and the results would be relevant.

Rarely all the outputs produced are of equal value to the DM. Resulting from the way efficiency is defined, units have different possible strategies to become efficient: they can specialize in producing different outputs. In efficiency analysis this may lead sometimes to results that are not plausible: for example, it is likely that we—as customers—strongly prefer a hospital specialized on excellence in medical treatments to one that is specialized in excellence in administrations. The latter is an important factor as well. In the literature there exist some approaches to overcome these difficulties, but they require some partial price information to be included into the analysis—which, as we concluded, may be difficult.

Several DEA extensions have been introduced to deal with the situations where value-free specialization is not acceptable. The models are often inspired by real-life applications where there has emerged a need to avoid unrealistic specialization. They aim at introducing some *preference information* into the analysis of technical efficiency. The term "preference information" refers to the additional information based on market prices, expert opinion, preferences, values, or judgment of a DM having the control over the units whose performance is under evaluation.

Technically preference information is in most of the existing approaches incorporated into the analysis via restrictions placed on weighting parameters in a mathematical optimization problem. These weights have the economic interpretation of prices for inputs and outputs. Although in these approaches no exact price information is needed, they still require the DM to think in terms of prices for example, how many master's degrees are equivalent to one Ph.D. degree? What is the price of a medical treatment x with respect to treatment y? Thus these approaches share some of the problems related to the definition of the prices themselves. Although there is not necessarily a need to figure out the price level, we need to have some idea of relative prices or marginal rates of substitution. They may be practically or politically very difficult to determine.

In the book, we will also introduce another approach which is based on the idea to combine multiple objective linear programming (MOLP) and DEA. In MOLP, the purpose is to help a DM find the most preferred solution (MPS) on the efficient frontier. The MPS is the solution on the efficient frontier which pleases the DM most. It can be a real unit or a hypothetical unit (=a point on the efficient frontier). Sometimes, we use the term most preferred unit (MPU) as a synonym to MPS, when we would like to emphasize that the solution is a real DMU, especially in the context of non-convex models. No price or weight information is needed in advance to determine the MPS. The system may help the DM to search the efficient frontier until the MPS is found. By making general assumptions about the *value function* of the DM, we may introduce a new concept: a *value efficiency score* or *value* *inefficiency score* depending on which one is more convenient to use. The value efficiency score behaves like efficiency score, but it may be very low to the technically efficient unit very "far" from the MPS. The analysis in which preference information is incorporated into the DEA in the way described above is called value efficiency analysis (VEA).

In this chapter we first briefly discuss the key concepts of efficiency and production analysis relating DEA to this framework. Then we reproduce the basic DEA models as well as introduce some generalizations. Coelli et al. (2005) is a good source for a more comprehensive introduction to efficiency and productivity analysis. Charnes et al. (1994) and Cooper et al. (2007) provide a thorough presentation on DEA.

1.2 Preliminary Considerations

In efficiency and productivity analysis the aim is to evaluate the performance of firms, public organizations, or more generally DMUs that convert *inputs* into *outputs*.

1.2.1 Decision-Making Units, Inputs, and Outputs

The term DMU refers to the units whose performance is evaluated. They may be firms or parts of firms such as branches or public sector entities. Possible examples range from production facilities, supermarkets, and banks to schools, hospitals, and government agencies. What is essential is that the DMUs have control over their operations and that they are comparable: they perform essentially the same task using similar inputs to produce similar outputs and operate in similar environmental conditions.

Inputs are the resources consumed by the DMUs. The inputs can be, e.g., working hours, number of physicians or teachers, or sales space. Respectively, outputs are the goods and services produced by the DMUs. Number of products produced, number of customers served, sales volume, and number of students graduating are typical examples.

1.2.2 Productivity and Efficiency

The *productivity* of a DMU is defined as the ratio of the output(s) that it produces to the input(s) that it uses:

Definition 1.1 Productivity $= \frac{\text{Output(s)}}{\text{Input(s)}}$

When there is only a single input and a single output, the ratio is trivial. With the presence of multiple inputs and/or outputs, both the inputs and the outputs must be aggregated into a single index. Typically, the definition of the productivity is expanded to a ratio of the weighted sum of outputs over the weighted sum of inputs:

Definition 1.2 Productivity $= \dfrac{\sum_{r=1}^{s} \mu_r * \text{Output}_r}{\sum_{i=1}^{m} v_i * \text{Input}_i}$,

where s is the number of outputs and m the number of inputs.

Sometimes the weighted sums are referred to as a *virtual input* and a *virtual output*. How to select the weights? Traditionally evaluating DMUs with multiple inputs and/or outputs has required information on the prices and the price interpretation of the weights. DEA offers one possibility to come up with a single aggregated index without the need of having a priori price information. We discuss this later in this section when relating DEA into the framework of production economics. In our terminology, productivity refers to *total factor productivity*, which is a productivity measure involving all factors of production. The measures like labor productivity in a factory are known as *partial* measures of productivity. These partial productivity measures can provide a misleading indication of overall productivity when considered in isolation (Coelli et al. 2005).

It is important to make a clear distinction between productivity and *efficiency*. Productivity is an absolute measure of performance. Based on productivity measure, it is possible to construct the *production frontier* enveloping the production possibility set (PPS). The production frontier represents the maximal output attainable from each input level. Efficiency[1] on the other hand tells whether DMUs are operating on the production frontier or beneath it. The units operating on the frontier are *technically efficient* and those operating beneath it are *technically inefficient*. To illustrate the difference between the terms, consider a single input and a single output. The concept of efficiency goes back to Pareto (1906) who first defined the concept of efficiency. Since a point on the efficient frontier is also called a *Pareto optimal* point. Actually, Koopmans (1951) defined the technical efficiency in the context of the production analysis.

Figure 1.1 illustrates the situation, where one input and one output are assumed. DMUs 1 and 2 are technically efficient whereas DMU_0 is inefficient. The curve going through DMUs 1 and 2 represents the production frontier and the shaded is the PPS. The productivity of DMU_2 is less than DMU_1, but there is no other DMU at the same input level with higher productivity. DMU_0 is inefficient because it uses the same amount of input as DMU_2, but its output level is lower.

[1] We formally define efficiency later on.

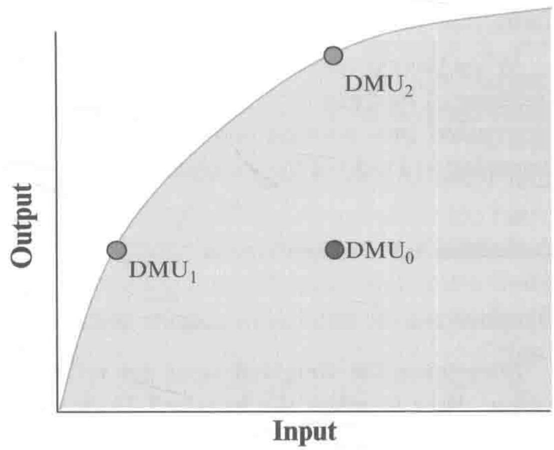

Fig. 1.1 Efficiency and productivity

1.2.3 *Technical, Overall, Allocative Efficiency*

Assume we are evaluating DMUs consuming two inputs to produce the same amount of one output (or consuming the same amount of one input to produce two outputs). Figure 1.2 illustrates the classical concepts of efficiency with the above assumptions in two pictures. In each picture, the curve going through DMU_1 illustrates the production frontier. DMU_0 is technically inefficient in each picture: it does not operate on the production frontier. The tangent line for the production frontier at DMU_1 is the isocost (isorevenue) line containing the information on input (output) prices. In the left side picture the approach is called *input oriented* and in the right side picture the approach is called *output oriented*. As we can see DMU_1 is the unit on the production frontier having minimum costs (maximum profits). Thus among all technically efficient units, DMU_1 is the only one that is *overall efficient* (currently often called *economic efficiency*).

For DMU_0 the ratio[2] $TE = \frac{O\text{-}DMU_0^T}{O\text{-}DMU_0}$ reflects technical efficiency, and ratio $OE = \frac{O\text{-}DMU_0^O}{O\text{-}DMU_0}$ overall efficiency. (The letter "O" refers to the origin.) *Allocative efficiency* is defined as the ratio $AE = \frac{O\text{-}DMU_0^O}{O\text{-}DMU_0^T}$. Thus the overall efficiency can be decomposed into technical and allocative efficiency: $OE = TE \times AE$, where technical efficiency addresses DMU distance from the production frontier, and allocative efficiency its distance from the optimal input (output) allocation. The decomposition was first proposed by Farrell (1957).

[2] Notation of type $O - DMU_0^T$ refers to the length of the line between the origin and point DMU_0^T.