



GINÉS LIFANTE PEDROLA

BEAM PROPAGATION
METHOD FOR
DESIGN OF OPTICAL
WAVEGUIDE DEVICES



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Ginés Lifante Pedrola

Universidad Autónoma de Madrid, Spain

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*To my beloved sisters:
Belín, María Cinta, María José, Lidia, Pilar and Zoila.*

Preface

The aim of this book is to provide the fundamentals and the applications of the beam-propagation method (BPM) implemented by finite-difference (FD) techniques, which is a widely used mathematical tool to simulate light wave propagation along axially varying optical waveguide structures. The content covers from the background, variations of the method, numerical implementations and applications of the methodology to many practical examples. Thus, the book gives systematic and comprehensive reviews and tutorials on the analysis and design of integrated photonics devices based on optical waveguides using FD-BPM. It treats almost all aspects of BPM analysis, from fundamentals through the advancements developed by extension and modifications, to the most recent applications to specific integrated optical devices.

The book can be a text for postgraduate courses devoted to numerical simulation of integrated photonic devices. Also, it is suitable for supplementary or background reading in modern curricula graduate courses such as 'Optoelectronics', 'Optical engineering', 'Optical-wave electronics', 'Photonics' or 'Integrated optics'. This book is also of interest for professional researchers and engineers in the area of integrated optics, optoelectronics and optical communications. Although BPM codes are commercially available, or even free, many engineers must develop their own software to suit their particular requirements. This book can serve both the building of home-made codes, as well for use of existing software by understanding the underlying approaches inherent in the BPM and its range of applicability.

Integrated photonics devices are based on optical waveguides with transversal dimensions of the order of microns. This means that the light propagation along these structures cannot be analysed in terms of ray optics; instead the light must be treated as electromagnetic waves. Hence, Chapter 1 presents the basics of the electromagnetic theory of light, starting from Maxwell's equations in inhomogeneous media. Wave equations in terms of the transverse field components in inhomogeneous media are obtained, including the treatment of anisotropic media and second-order non-linear media. Using the slowly varying approximation, full vectorial equations for the electric and magnetic fields are obtained in Chapter 2, which are the basic differential equations for developing BPM algorithms. Finite-difference approximations

of the wave equations are then derived for the simplest case of scalar propagation in two-dimensional structures that allows us to study the stability and numerical dissipation of FD-BPM schemes. Chapter 3 develops full vectorial FD-BPM algorithms for the simulation of light propagation in 2D and 3D structures where the numerical implementations of the FD-BPM are detailed.

Extensions and modifications of the BPM approaches based on finite-difference techniques are presented in Chapter 4. These include wide-angle BPM, which relaxes the restriction of the application of BPM to paraxial waves and allows the simulation of light beams with large propagation angles respect of the longitudinal direction. BPM algorithms, which can handle multiple reflections known as bidirectional-BPM, are then discussed. Simulation of light propagation in active media, second-order non-linear media and anisotropic media are also topics covered in Chapter 4. The last sections include the description of time-domain (TD) simulation techniques based on finite differences, which can simulate the propagation of optical pulses and can manage backward waves due to reflections at waveguide discontinuities. Both the time-domain BPM and finite-difference time-domain (FDTD) are explained in detail. The different BPMs supply almost universal numerical tools for describing the performance of a great variety of integrated optical devices. Although particular devices have specific routes to be modelled with their own constraints, the great advantage of the BPM lies in the fact that, as few approximations have been made for its derivation, its applicability is quite wide and almost any integrated photonic device can be modelled by using it. The last chapter, Chapter 5, presents selected examples of integrated optical elements commonly used in practical integrated photonic devices, where their performance and relevant characteristics are analysed by the appropriate BPM approach.

Some appendices have been added at the end of the book. They include material related to BPM algorithms or BPM simulations of some integrated photonics devices, but which is not indispensable to the understanding of the different topics developed along the book chapters. The appendices include mathematical derivations of some formulae, physical phenomena descriptions and even relevant program listing.

Commonly accepted notation and symbols have been utilized throughout this book. However, some of the symbols have multiple meanings and therefore a list of symbols and their meanings is provided at the beginning of the book to clarify symbol usage. Also, a list of acronyms is given to help the reader.

A selection of BPM programs are made available free of charge for the readers at the website of the author (www.uam.es/personal_pdi/ciencias/glifante). Among others, this selection includes the programs ‘Vectorial mode solver for planar waveguides’, ‘Vectorial light propagation in 2D-structures’, ‘Vectorial light propagation in 3D-structures’ and ‘2D-light propagation in the time domain’.

*Ginés Lifante Pedrola
Madrid, February 2015*

List of Acronyms

ABC	absorbing boundary conditions
ADI	alternating direction implicit
ASE	amplified spontaneous emission
AWG	arrayed waveguide grating
BC	boundary condition
Bi-BPM	bidirectional beam propagation method
BPM	beam propagation method
CCS	complementary coplanar strip
CFL	Courant–Friedrichs–Levy stability criterion
CN	Crank–Nicolson scheme
CS	coplanar strip
CW	continuous wave
DFR	distributed feedback reflector
EIM	effective index method
EM	electromagnetic
EO	electro-optic
ESW	equivalent straight waveguide
FD	finite difference
FD-BPM	finite-difference beam-propagation method
FDTD	finite-difference time domain
FE	finite element
FE-BPM	finite-element beam-propagation method
FFT	fast Fourier transform
FFT-BPM	fast Fourier transform beam-propagation method
FT	Fourier transform
FPR	free propagation region
FSR	free spectral range
FV-BPM	full vectorial beam propagation method
IFD-NL-BPM	iterative finite difference non-linear beam-propagation method

IL	insertion loss
Im-Dis-BPM	imaginary distance beam-propagation method
IR	infrared
MMI	multimode interference
MPA	modal propagation analysis
MZ	Mach–Zehnder
MZI	Mach–Zehnder interferometer
NL	non-linear
NL-BPM	non-linear beam propagation method
OI	overlap integral
PHASAR	phase-array
PML	perfectly matched layer
QPM	quasi-phase matching
RE	rare earth
SH	second harmonic
SHG	second harmonic generation
SV-BPM	semi-vectorial beam-propagation method
SVE	slowly varying envelope
SVEA	slowly varying envelope approximation
TBC	transparent boundary conditions
TD-BPM	time-domain beam-propagation method
TE	transverse electric field
TF/SF	total field/scattered field
TM	transverse magnetic field
UV	ultraviolet
WDM	wavelength division multiplexing

List of Symbols

Roman Symbols

a_j	tridiagonal system coefficient
a_ν	modal weight
A	attenuation; also, waveguide cross section
A_x	x dependent part of the operator P_{xx} (or Q_{xx})
A_y	y dependent part of the operator P_{xx} (or Q_{xx})
A_{ij}	spontaneous emission probability
b	normalized propagation constant
b_j	tridiagonal system coefficient
B_x	x dependent part of the P_{yy} (or Q_{yy}) operator
B_y	y dependent part of the P_{yy} (or Q_{yy}) operator
$\mathcal{B}(\mathbf{r}, t)$	magnetic flux density vector
c	speed of light in free space
c_j	tridiagonal system coefficient
C	P_{xy} (or Q_{xy}) operator (cross-coupling term)
$C_{M,N}(i,j)$	coefficient for the FDTD algorithm
d	thickness, depth
d_{ijk}	second-order non-linear tensor
d_{PML}	PML thickness
D	P_{yx} (or Q_{yx}) operator (cross-coupling term); also, distance, separation
$D_{M,N}(i,j)$	coefficient for the FDTD algorithm
\mathbf{D}	(complex amplitude of) displacement vector
\mathcal{D}	differential operator (for wide-angle BPM)
$\mathcal{D}(\mathbf{r}, t)$	electric displacement vector
E_i	i Cartesian component of E
E_{yx}, E_{yz}	splitting (sub-components) of the magnetic field component E_y in FDTD
$E(\mathbf{r})$	complex amplitude of $\mathcal{E}(\mathbf{r}, t)$ for monochromatic waves
$E_t(\mathbf{r})$	transverse component of E
$\mathcal{E}(\mathbf{r}, t)$	electric field
$f(x)$	mode transversal profile

$f_v(x, y)$	eigenmode (eigenvector, transversal field distribution)
F_j	energy flux leaving the j -boundary
\mathcal{F}	operator for NL-BPM
G	differential operator (in 3D-scalar wave equation)
G_x, G_y	split of the differential operator G
h	Planck's constant; also, height
\mathcal{H}	differential operator for TD-BPM
H_j	i Cartesian component of H
$\mathcal{H}_x, \mathcal{H}_y$	split of the differential operator for TD-BPM
H_{yx}, H_{yz}	splitting (sub-components) of the magnetic field component H_y in FDTD
$\mathbf{H}(\mathbf{r})$	complex amplitude of $\mathcal{H}(\mathbf{r}, t)$ for monochromatic waves
$H_t(\mathbf{r})$	transverse component of \mathbf{H}
$\mathcal{H}(\mathbf{r}, t)$	magnetic field
$\mathcal{H}_0(\mathbf{r})$	magnetic field amplitude for monochromatic waves
$\hat{\mathbf{H}}$	matrix differential operator for Ψ_t (or Φ_t)
i	imaginary unity; also, integer
I	intensity (or irradiance)
I_0	intensity of a monochromatic plane wave
I_p	pump intensity
j	integer
$\mathcal{J}(\mathbf{r}, t)$	electric current density
k	wavenumber in the medium; also, integer
k_0	wavenumber in free space
K	reference wavenumber
\tilde{K}	complex-valued reference wavenumber
L	window size; also, length
L_x, L_y	transversal grid dimensions
L_z	longitudinal length
L_c	coupling length
$\mathcal{L}_A, \mathcal{L}_B$	pseudo-differential operators
\mathcal{L}_j	operators for NL-BPM
m	integer
M_m	polynomial of degree m
\mathcal{M}	overall transfer matrix (for Bi-BPM)
$\mathcal{M}(\mathbf{r}, t)$	magnetic current density
n	refractive index; also, integer
n_0	reference refractive index
n_c	complex refractive index
n_p	refractive index of the PML medium
N	concentration of active ions
N_e	effective index of the symmetric mode
N_{eff}	effective index of the mode
N_i	population density in the i -th level
N_n	polynomial of degree n
N_o	effective indices of the anti-symmetric mode
N_x, N_y	number of transversal grid points

$\mathcal{O}[\cdot]$	approximation order in FD schemes
p	power exponent of the PML profile
$P(z)$	complex field amplitude correlation function
$P(\xi)$	Fourier transform of $P(z)$
\mathcal{P}_j	propagation matrix (for bidirectional BPM)
P_{ij}	differential operator for the transverse SVE field ψ_i
\mathcal{P}	differential operator for wide angle BPM; also, for wide-band BPM
$\mathcal{P}(\mathbf{r}, t)$	polarization vector
\mathcal{P}_{NL}	non-linear polarization
Q_{ij}	differential operators for the transverse SVE field Φ_i
Q_j^m	Von Neumann analysis parameter
Q	operator for wide band BPM
r	reflection coefficient
r_j	tridiagonal system coefficient
r_0	maximum reflection coefficient at the PML region
\mathbf{r}	position
R	reflectivity
R_j, R_{ij}	coefficient for FD schemes of BPM
R_{ij}	pump rate (or stimulated emission rate)
$S1_{ij}-S4_{ij}$	coefficients for FD schemes of BPM
\mathcal{S}	Poynting vector
S	complex Poynting vector
t	time; also, transmission coefficient
T	period
T_j	transmission coefficient
T_{AB}	interface matrix (for bidirectional BPM)
u	SVE-field component Ψ_x (or Φ_x)
$u(x, y, z)$	SVE scalar optical field
u_f	SVE for the fundamental wave
u_s	SVE for the SH wave
u_j^m	discretized SVE optical field
u_j^+	discretized incident field ψ_A^+
u_j^-	discretized reflected field ψ_A^-
$u(\mathbf{r}, t)$	temporal envelop of the electric field
u_t	SVE of the transverse electric field
$\mathbf{u}_x, \mathbf{u}_y, \mathbf{u}_z$	unitary vectors along the x -, y - and z -axis
v	SVE-field component Ψ_y (or Φ_y); also, propagation speed of an EM wave
w	width
$w(k, t)$	spatial frequencies
w_j^+	discretized forward field ψ_B^+
W	width
W_{ij}	stimulated emission rate
W_{ij}^{NR}	non-radiative probability
W_ν	relative power carried by the ν -th mode
x	Cartesian coordinate

\mathcal{X}_j	dimensionless operator
y	Cartesian coordinate
z	Cartesian coordinate
Z	total propagation length
$Z1_{ij}$ – $Z4_{ij}$	coefficients for FD schemes of BPM

Greek Symbols

α	Crank–Nicolson scheme parameter; also, absorption coefficient
α_{eff}	effective attenuation coefficient (of PML)
$\tilde{\alpha}_s$	intrinsic propagation losses
β	propagation constant
β_ν	propagation constant of the ν th order eigenmode
χ_i	polynomial coefficient
χ_L	linear susceptibility
$\chi^{(2)}$	coefficient of second-order non-linear susceptibility
χ_{ijk}	element of the second-order non-linearity susceptibility tensor
δ	delta Kronecker function; also, ABC region thickness (or PML region)
$\Delta x, \Delta y$	grid size
Δz	longitudinal step size
Δk	mismatch parameter
Δt	time step
ϵ	scalar dielectric permittivity
ϵ_r	dielectric constant (relative dielectric permittivity)
ϵ_0	dielectric permittivity of free space
ϵ_{ij}	element of the permittivity matrix
$\boldsymbol{\epsilon}$	permittivity tensor
$\phi(i)$	transversal field distribution of a waveguide mode for FDTD
γ	amplification factor (for Von Neumann analysis); also, damping factor (for bi-BPM)
Γ	correlation between optical fields; also, overlap integral
$\boldsymbol{\eta}$	impermeability tensor
η_0	free space impedance
$\varphi(\mathbf{r})$	initial phase
φ	incident angle
Φ_x	x -component of the SVE transversal magnetic field
Φ_y	y -component of the SVE transversal magnetic field
$\boldsymbol{\Phi}_t$	SVE field of $\mathbf{H}_t(\mathbf{r})$
κ	absorption index; also, coupling coefficient
κ_{max}	maximum value of $\kappa(x)$
$\kappa(x)$	absorption index profile (in ABC)
λ	wavelength
Λ	grating period
μ	magnetic permeability
μ_0	magnetic permeability of free space
ν	frequency
θ_i	angle of reflection (or transmission)

ρ	parameters for ABC region
ρ_i	magnetic conductivity of the PML
$\rho(\mathbf{r}, t)$	charge density
$\sigma(\mathbf{r})$	electrical conductivity
$\sigma(\rho), \sigma_i$	electrical conductivity profile of the Bérenger layer
σ_{ij}	absorption (or emission) cross-section
σ_{\max}	maximum conductivity of the PML
τ	pulse temporal width; also, lifetime
ω	angular frequency
$\omega(k)$	relation dispersion
ω_s	angular frequency for the SH wave
ω_f	angular frequency for the fundamental wave
ω_0	carrier frequency
ξ_i	polynomial coefficient
ξ_ν	eigenvalue (relative propagation constant)
$\psi(x, y, z)$	generic scalar field
ψ^+	forward field
ψ^-	backward field
ψ_A^+	incident field in region A
ψ_A^-	reflected field in region A
ψ_B^+	transmitted field in region B
Ψ	slowly varying electric field
Ψ_t	SVE field of $E_t(\mathbf{r})$
Ψ_x	x component of the SVE transverse electric field Ψ_t
Ψ_y	y component of the SVE transverse electric field Ψ_t

Mathematical Symbols

∂	partial differential
∇	gradient operator
∇	divergence operator
$\nabla \times$	curl operator

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