

Studies in Fuzziness and Soft Computing

Deng-Feng Li

Linear Programming Models and Methods of Matrix Games with Payoffs of Triangular Fuzzy Numbers

Deng-Feng Li

Linear Programming Models and Methods of Matrix Games with Payoffs of Triangular Fuzzy Numbers

Deng-Feng Li
School of Economics and Management
Fuzhou University
Fuzhou
China

ISSN 1434-9922 ISSN 1860-0808 (electronic)
Studies in Fuzziness and Soft Computing
ISBN 978-3-662-48474-6 ISBN 978-3-662-48476-0 (eBook)
DOI 10.1007/978-3-662-48476-0

Library of Congress Control Number: 2015951775

Springer Heidelberg New York Dordrecht London
© Springer-Verlag Berlin Heidelberg 2016

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

Springer-Verlag GmbH Berlin Heidelberg is part of Springer Science+Business Media
(www.springer.com)

Studies in Fuzziness and Soft Computing

Volume 328

Series editor

Janusz Kacprzyk, Polish Academy of Sciences, Warsaw, Poland
e-mail: kacprzyk@ibspan.waw.pl

About this Series

The series “Studies in Fuzziness and Soft Computing” contains publications on various topics in the area of soft computing, which include fuzzy sets, rough sets, neural networks, evolutionary computation, probabilistic and evidential reasoning, multi-valued logic, and related fields. The publications within “Studies in Fuzziness and Soft Computing” are primarily monographs and edited volumes. They cover significant recent developments in the field, both of a foundational and applicable character. An important feature of the series is its short publication time and world-wide distribution. This permits a rapid and broad dissemination of research results.

More information about this series at <http://www.springer.com/series/2941>

*To my wife, Wei Fei and To my son,
Wei-Long Li*

Preface

Two-person zero-sum finite games, which often are called matrix games for short, are an important part of noncooperative games. Matrix games have been extensively studied and successfully applied to many fields such as management science, decision science, operational research, economics, finance, business, social science, and biology as well as engineering. However, the assumption that all payoffs are precise common knowledge to the players is not realistic in many competitive or antagonistic decision occasions. In fact, more often than not, in real competitive or antagonistic situations, the players cannot exactly estimate payoffs in the game due to lack of adequate information and/or imprecision of the available information on the environment. This lack of precision and certainty may be appropriately modeled by using the fuzzy set. Intervals and triangular fuzzy numbers, which are special and simple cases of the fuzzy sets, seem to be suitable and convenient for dealing with fuzziness or imprecision of payoffs in matrix games. On the other hand, in some real-life game problems, choice of strategies for the players is constrained due to some practical reason why this should be, i.e., not all mixed (or pure) strategies in a game are permitted for each player. As a result, there appear four important types of matrix games, which are interval-valued matrix games, matrix games with payoffs of triangular fuzzy numbers, interval-valued constrained matrix games, and constrained matrix games with payoffs of triangular fuzzy numbers. As far as I know, however, there is less investigation on them. Therefore, this book focuses on studying the concepts, properties, models, and methods of the aforementioned four types of matrix games.

This book is divided into two parts. Each part includes two chapters. Chapter 1 discusses interval-valued matrix games, mainly including interval-valued mathematical programming models of interval-valued matrix games, acceptability-degree-based linear programming models and method of interval-valued matrix games, the lexicographic method of interval-valued matrix games, and primal-dual linear programming models and method of interval-valued matrix games. Chapter 2 studies matrix games with payoffs of triangular fuzzy numbers, mainly including fuzzy multi-objective programming models and fuzzy linear programming method of matrix games with

payoffs of triangular fuzzy numbers, two-level linear programming models and method of matrix games with payoffs of triangular fuzzy numbers, the lexicographic method of matrix games with payoffs of triangular fuzzy numbers, and Alfa-cut-based primal-dual linear programming models and method of matrix games with payoffs of triangular fuzzy numbers. Chapter 3 expatiates interval-valued constrained matrix games, including the concepts of solutions of interval-valued constrained matrix games and properties, and primal-dual linear programming models and method of interval-valued constrained matrix games. Chapter 4 expounds constrained matrix games with payoffs of triangular fuzzy numbers, mainly including fuzzy multi-objective programming models and method of constrained matrix games with payoffs of triangular fuzzy numbers, and Alfa-cut-based primal-dual linear programming models and method of constrained matrix games with payoffs of triangular fuzzy numbers. The aim of this book was to develop and establish simple, efficient, and effective linear programming models and methods for solving interval-valued matrix games, interval-valued constrained matrix games, matrix games with payoffs of triangular fuzzy numbers, and constrained matrix games with payoffs of triangular fuzzy numbers. I tried my best to ensure that the models and methods developed in this book are of practicability, maneuverability, and universality.

This book is addressed to people in theoretical researches and practical applications from different fields and disciplines such as decision science, game theory, management science, fuzzy sets or fuzzy mathematics, applied mathematics, optimizing design of engineering and industrial system, expert system, and social economy as well as artificial intelligence. Moreover, it is also addressed to teachers, postgraduates, and doctors in colleges and universities in different disciplines or majors: decision analysis, management, operation research, fuzzy mathematics, fuzzy system analysis, systems engineering, project management, industrial engineering, hydrology, and water resources.

First of all, special thanks are due to my coauthor Chun-Tian Cheng and my doctoral graduates Jiang-Xia Nan, Fang-Xuan Hong for completing and publishing several articles. This book was supported by the Key Program of the National Natural Science Foundation of China (No. 71231003), the National Natural Science Foundation of China (Nos. 71171055, 71101033, and 71001015), the “Chang-Jiang Scholars” Program (the Ministry of Education of China), the Program for New Century Excellent Talents in University (the Ministry of Education of China, NCET-10-0020), and the Specialized Research Fund for the Doctoral Program of Higher Education of China (No. 20113514110009) as well as “Science and Technology Innovation Team Cultivation Plan of Colleges and Universities in Fujian Province.” I would like to acknowledge the encouragement and support of my wife as well as the understanding of my son.

Last but not least, I would like to acknowledge the encouragement and support of all my friends and colleagues.

Ultimately, I should claim that I am fully responsible for all errors and omissions in this book.

About the Author



Deng-Feng Li was born in 1965. He received the B.Sc. and M.Sc. degrees in applied mathematics from the National University of Defense Technology, Changsha, China, in 1987 and 1990, respectively, and the Ph.D. degree in system science and optimization from the Dalian University of Technology, Dalian, China, in 1995.

From 2003 to 2004, he was a Visiting Scholar with the School of Management, University of Manchester Institute of Science and Technology, Manchester, UK. He is currently a Distinguished Professor of “Chang-Jiang Scholars” Program, Ministry of Education of China and “Min-Jiang Scholarship” Distinguished Professor with the School of Economics and Management, Fuzhou University, Fuzhou, China. He has been conferred the Outstanding Contribution Experts of the National Middle-Aged and Young of China and was approved as an expert of the Enjoyment of the State Council Special Allowance of China. He has authored or coauthored more than 300 journal papers and seven monographs. He has coedited one proceeding of the international conference and two special issues of journals and won 23 academic achievements and awards such as Chinese State Natural Science Award and 2013 IEEE Computational Intelligence Society IEEE Transactions on Fuzzy Systems Outstanding paper award. His current research interests include classical and fuzzy game theory, fuzzy decision analysis, group decision making, supply chain, fuzzy sets and system analysis, fuzzy optimization, and differential game. He is the Editor-in-chief of *International Journal of Fuzzy System Applications* and Associate Editors and/or Editors of several international journals.

Abstract

This book is an academic monograph based on the papers published in international famous journals by the author. The focus of this book is on theoretical models and methods of interval-valued constrained and unconstrained matrix games, and constrained and unconstrained matrix games with payoffs of triangular fuzzy numbers. This book includes four chapters. Chapter 1 mainly discusses interval-valued mathematical programming models of interval-valued matrix games, acceptability-degree-based linear programming models and method of interval-valued matrix games, the lexicographic method of interval-valued matrix games, and primal-dual linear programming models and method of interval-valued matrix games. Chapter 2 mainly studies fuzzy multi-objective programming models and fuzzy linear programming method of matrix games with payoffs of triangular fuzzy numbers, two-level linear programming models and method of matrix games with payoffs of triangular fuzzy numbers, the lexicographic method of matrix games with payoffs of triangular fuzzy numbers, and Alfa-cut-based primal-dual linear programming models and method of matrix games with payoffs of triangular fuzzy numbers. Chapter 3 expatiates the concepts of solutions of interval-valued constrained matrix games and properties, and primal-dual linear programming models and method of interval-valued constrained matrix games. Chapter 4 mainly expounds fuzzy multi-objective programming models and method of constrained matrix games with payoffs of triangular fuzzy numbers, and Alfa-cut-based primal-dual linear programming models and method of constrained matrix games with payoffs of triangular fuzzy numbers. The aim of this book was to develop and establish simple, efficient, and effective linear programming models and methods for solving interval-valued matrix games, interval-valued constrained matrix games, matrix games with payoffs of triangular fuzzy numbers, and constrained matrix games with payoffs of triangular fuzzy numbers.

This book is addressed to people in theoretical researches and practical applications from different fields and disciplines such as decision science, game theory, management science, fuzzy sets or fuzzy mathematics, applied mathematics, optimizing design of engineering and industrial system, expert system, and social economy as well as artificial intelligence. Moreover, it is also addressed to teachers,

postgraduates, and doctors in colleges and universities in different disciplines or majors: decision analysis, management, operation research, fuzzy mathematics, fuzzy system analysis, systems engineering, project management, industrial engineering, applied mathematics, hydrology, and water resources.

Contents

**Part I Models and Methods of Matrix Games with Payoffs
of Triangular Fuzzy Numbers**

1	Interval-Valued Matrix Games	3
1.1	Introduction	3
1.2	Matrix Games and Auxiliary Linear Programming Models	5
1.3	Interval-Valued Mathematical Programming Models of Interval-Valued Matrix Games	8
1.3.1	Arithmetic Operations Over Intervals	8
1.3.2	Concepts of Solutions of Interval-Valued Matrix Games and Properties	10
1.3.3	Auxiliary Interval-Valued Mathematical Programming Models	13
1.3.4	Solving Methods of 2×2 Interval-Valued Matrix Games	18
1.4	Acceptability-Degree-Based Linear Programming Models of Interval-Valued Matrix Games	23
1.4.1	Concepts of Acceptability Degrees of Interval Comparison and Properties	23
1.4.2	Interval-Valued Mathematical Programming Models and Satisfactory Equivalent Forms	25
1.4.3	Auxiliary Linear Programming Models of Interval-Valued Matrix Games	26
1.4.4	Real Example Analysis of Market Share Problems	34
1.5	The Lexicographic Method of Interval-Valued Matrix Games	36

1.6	Primal-Dual Linear Programming Models of Interval-Valued Matrix Games	41
1.6.1	The Monotonicity of Values of Interval-Valued Matrix Games	41
1.6.2	Auxiliary Linear Programming Models of Interval-Valued Matrix Games	42
1.6.3	Real Example Analysis of Investment Decision Problems	48
	References	61
2	Matrix Games with Payoffs of Triangular Fuzzy Numbers	65
2.1	Introduction	65
2.2	Triangular Fuzzy Numbers and Alfa-Cut Sets	67
2.3	Fuzzy Multi-Objective Programming Models of Matrix Games with Payoffs of Triangular Fuzzy Numbers	69
2.3.1	Order Relations of Triangular Fuzzy Numbers	69
2.3.2	Concepts of Solutions of Matrix Games with Payoffs of Triangular Fuzzy Numbers	71
2.3.3	Fuzzy Linear Programming Method of Matrix Games with Payoffs of Triangular Fuzzy Numbers	73
2.4	Two-Level Linear Programming Models of Matrix Games with Payoffs of Triangular Fuzzy Numbers	82
2.5	The Lexicographic Method of Matrix Games with Payoffs of Triangular Fuzzy Numbers	89
2.6	Alfa-Cut-Based Primal-Dual Linear Programming Models of Matrix Games with Payoffs of Triangular Fuzzy Numbers	96
2.6.1	Interval-Valued Matrix Games Based on Alfa-Cut Sets of Triangular Fuzzy Numbers	97
2.6.2	Linear Programming Method of Matrix Games with Payoffs of Triangular Fuzzy Numbers	107
2.6.3	Computational Analysis of a Real Example	110
	References	119
 Part II Models and Methods of Constrained Matrix Games with Payoffs of Triangular Fuzzy Numbers		
3	Interval-Valued Constrained Matrix Games	123
3.1	Introduction	123
3.2	Constrained Matrix Games and Auxiliary Linear Programming Models	124

3.3	Primal-Dual Linear Programming Models of Interval-Valued Constrained Matrix Games	126
3.3.1	Monotonicity of Values of Constrained Matrix Games . . .	127
3.3.2	Linear Programming Methods of Interval-Valued Constrained Matrix Games	128
3.3.3	Real Example Analysis of Market Share Problems	130
	References	134
4	Constrained Matrix Games with Payoffs of Triangular Fuzzy Numbers	135
4.1	Introduction.	135
4.2	Fuzzy Multi-Objective Programming Models of Constrained Matrix Games with Payoffs of Triangular Fuzzy Numbers.	136
4.2.1	Constrained Matrix Games with Payoffs of Triangular Fuzzy Numbers	136
4.2.2	Fuzzy Multi-Objective Programming Method of Constrained Matrix Games with Payoffs of Triangular Fuzzy Numbers	139
4.3	Alfa-Cut-Based Primal-Dual Linear Programming Models of Constrained Matrix Games with Payoffs of Triangular Fuzzy Numbers	148
4.3.1	Concepts of Alfa-Constrained Matrix Games with Payoffs of Triangular Fuzzy Numbers	148
4.3.2	Linear Programming Models of Constrained Matrix Games with Payoffs of Triangular Fuzzy Numbers.	149
4.3.3	Algorithm of Linear Programming Method of Constrained Matrix Games with Payoffs of Triangular Fuzzy Numbers	157
4.3.4	Real Example Analysis of Market Share Problems with Payoffs of Triangular Fuzzy Numbers	158
	References	165

Part I
Models and Methods of
Matrix Games with Payoffs
of Triangular Fuzzy Numbers

Chapter 1

Interval-Valued Matrix Games

1.1 Introduction

Game theory is engaged in competing and strategic interaction among players in management science, operational research, economics, finance, business, social science, biology, engineering, and others. It began in the 1920s and has achieved a great success [1, 2]. The simplest game is the zero-sum game involving only two players with finite pure strategies (i.e., options), which is often called the matrix game for short. A matrix game is usually expressed by a payoff matrix $A = (a_{ij})_{m \times n}$, where a_{ij} is the amount of reward/loss which the player I wins (and hereby the player II loses) when the players I and II choose their pure strategies δ_i ($i = 1, 2, \dots, m$) and β_j ($j = 1, 2, \dots, n$), respectively. Here, m and n are two arbitrary positive integers.

Traditionally, the payoffs a_{ij} are represented by crisp values, which indicate that they are precisely known. This assumption is reasonable for clearly defined games, which have many useful applications, especially in finance, management and decision making systems [3, 4]. In the real world, however, there are some cases in which the payoffs are not fixed numbers known and have to be estimated even though two players do not change their strategies. An example is one in which different advertising strategies of two competing companies lead to different market shares and the market shares must be estimated. Hence, fuzzy games have been extensively studied. Dubois and Prade [5] gave a brief overview and discussion on the fuzzy games with crisp sets of strategies and fuzzy payoffs due to the lack of precision on the knowledge of the associated payoffs. Nishizaki and Sakawa [2] and Bector and Chandra [4] made good overviews on update research of this topic. Bector et al. [6, 7] studied the matrix games with fuzzy goals and fuzzy payoffs by using the defined fuzzy linear programming duality, respectively. Campos [8], Campos and Gonzalez [9] and Campos et al. [10] proposed ranking function based