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Quantum Confined Laser Devices

Optical gain and recombination in semiconductors

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QUANTUM CONFINED LASER DEVICES

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Preface

My initial interest in quantum-confined lasers was not driven by scientific curiosity but by necessity: at Philips Research Laboratories in the 1980s I was asked to work on quantum well lasers. No kind of laser had featured in my physics degree course 20 years earlier: at that time the first lasers had only just been made. I pleaded ignorance.

The learning experience that then followed has shaped this book: it is a book for the newcomer, of any age. At the risk of making the text too long I have tried to avoid the implicit in favour of the explicit; I have stated the obvious at the risk of boring some readers.

The aim is to provide an account of the physics of quantum confined semiconductor gain media that exploit conduction to valence band transitions—wells and dots—and to relate this to evaluation of the properties of practical devices. It has been written for final-year MPhys students, PhD students, and researchers involved in the practical study of diode lasers and assumes background knowledge at about the level of the second year of an undergraduate physics course. My guiding principle, with a few exceptions at the behest of the publisher, has been to write about topics of which I have personal experience, where I feel I can contribute to the reader's understanding. The book has been designed to exploit and complement others in this Oxford Master Series, particularly the comprehensive account of Laser Physics by Hooker and Webb and Mark Fox's books on Quantum Optics and Optical Properties of Solids.

Historically, quantum wells came on the scene about 20 years before practical quantum dots emerged, and early research on dots used some concepts and language from the world of confinement in one dimension that were not strictly appropriate for a fully localised system. For this book quantum dots are regarded as "dots", with more in common with atoms and molecules than with solids. The pedagogical approach therefore is to begin with a two-level atom-like system, which is developed, through Bloch's theorem, to a hybrid system that has no confinement in two of its dimensions: the quantum well.

In wells and dots quantum confinement takes place on a scale much smaller than the wavelength of the light and over the years I have come to regard the modal gain of the structure as a more useful concept than material gain of the gain medium. This view pervades the text.

Beyond the assumed background knowledge the book has been written to be as self-contained as possible. Pointers to further reading are given at the end of each chapter. References to published work serve to acknowledge the contributions of others and to enable the reader to study the primary, first-hand accounts for themselves. To provide realistic illustrations I have produced many of the figures using simple models for typical structures rather than "artist's impressions"; nevertheless, their purpose is to illustrate and they should not be read as definitive calculations. The section "This Book" provides an outline of the structure, the nature of the exercises, and the conventions with regard to units.

It is a pleasure to acknowledge contributions to my learning from many colleagues over many years, in some cases unknowingly, sometimes in response to pleas for help. I would not be writing the book at all without that first push from John Walling and the collaboration with colleagues at Philips: John Orton, Phil Dawson, Geoff Duggan, Dennis Fletcher, the molecular beam epitaxy team, and friends in Eindhoven. For conversations at various times I am indebted to Alf Adams, David Bour, Larry Coldren, Jim Coleman, Martin Dawson, Dennis Deppe, John McInnerny, Stephan Koch, Luke Lester, and Luke Mawst. I am pleased to acknowledge the education I've received through working with former students and postdocs, particularly Huw Summers, Paul Rees, Mat Hutchings, Sam Shutts, Lewis Kalstein, Rob Thomas, and Ian O'Driscoll; Helen Pask originated the computer code that I used to generate many of the figures on quantum dot gain and emission. The IT team in the Physics Department kept my remote access system working through various disturbances, enabling me to write in proximity to home comforts.

I am grateful for specific advice on aspects of the text from Weng Chow, Shun Lien Chuang (sadly deceased), Mark Fox, Ian Galbraith, Cun Zheng Ning, Eoin O'Reilly, Peter Smowton, Stephen Sweeney, and Peter Zory. Special thanks go to Gareth Parry for his detailed reading of the entire manuscript; the infelicities that remain are all mine. Thanks go also to the staff at OUP for their interest in the book in the first instance, to Sonke Adlung, who encouraged me to use LaTeX as the means to produce the text (I was soon converted), and to the production team who turned my text into a book. Thanks also go to my family who have put up with a life dominated by "The Book" for too long.

I hope that this book will help others to begin their journey with quantum confined laser diodes and, one day, make their own contribution to the collective wisdom of the subject.

Pwll Bach and Cardiff
November 2014

About this book

Structure

The book begins with a short historical account of the emergence of lasers and the diode laser in particular. Thereafter it is organised into five major sections, set out as follows:

- Part I The diode laser
  Chapters 2, 3, 4, and 5 give an introduction to the concept of optical gain, the formation of heterostructures using semiconductor alloys, the optical waveguide, round-trip amplification, laser threshold, and definitions of quantum efficiency. They provide the essential framework for the whole book.
- Part II Fundamental processes
  Chapters 6, 7, and 8 establish the classical and quantum mechanical treatments of the interaction of light with atoms and the principles of quantum confinement by potential wells.
- Part III Device physics
  Based on the previous two parts, Chapters 9, 11, and 12 provide expressions for the modal gain and spontaneous emission in dots and wells, which is the basis of device operation. Two further chapters, 10* and 13*, use rate equations to describe the exchange of carriers between dots and the wetting layer, and to calculate the light–current relation above threshold. These two chapters could be omitted at an undergraduate level.
- Part IV Device operation
  Chapter 14 describes common Fabry–Perot and grating feedback lasers, including vertical cavity lasers. Drawing on Part III, Chapter 15 gives an account of the threshold characteristics of these devices and Chapter 16 describes the temperature dependence of these characteristics.
- Part V Studies of gain and recombination
  The final two chapters give an account of methods of measuring gain and radiative recombination and are written primarily with postgraduate readers and researchers in mind.

Each part is preceded by a short summary of its content and the knowledge assumed or required from previous chapters. A final-year undergraduate course could be based on material in Parts I and II, unstarred chapters of Part III, and selected material from Part IV. Seven appendices provide material that is relevant but not central to the main text.

Exercises

Exercises are an integral part of the text: they include some derivations and proofs and provide experience of putting numbers into equations. The exhortation to *estimate* something means just that. The aim of such an exercise is to give the reader a feel for the magnitude of certain quantities by looking up typical values of unknowns, or making sensible estimates of them. Often the relevant information is within the chapter or related chapters. The solution manual gives the values used by the author and indicates tutorial points to be drawn from the exercises. Some of the more difficult exercises and those that are part of the text could be used as worked examples in class and are indicated by §.

Units, conventions, and useful relations

Exercises often require numerical work, which raises the question of units. *Equations* in the text are given in the SI system unless otherwise explicitly stated. However, in everyday work non-SI units are more convenient for some quantities: for example the electron volt rather than the joule. When evaluating equations the reader is advised to convert all values into SI units before substituting them, and then convert the final result to a non-SI unit if required.

The symbol e denotes the magnitude of the electronic charge; the charge on an electron is $-e$. Energy level diagrams and band diagrams are drawn with the electron energy as the increasing positive quantity.

The symbol $f(E)$ is the probability of occupation of a state at energy E by *an electron*, irrespective of the band.

The symbol ν is used to denote frequency (in Hz) and ω to denote angular frequency (in rad s^{-1}), so photon energy is $h\nu$ or $\hbar\omega$. A referenced list of symbols is given. Some useful physical constants are listed at the end of the book.

Rapid, mental conversion can be made between photon energy in eV and wavelength in μm by

$$h\nu(\mathrm{eV}) = \frac{1.24}{\lambda(\mu\mathrm{m})}$$

This is used in many of the exercises.

List of symbols

In some cases it is inevitable that two different quantities are represented by the same symbol; the meaning should be clear from the context. The equation or section where a symbol is defined is given in the list below where relevant.

A	Area
A, $\boldsymbol{A}$	Vector potential, eqn 7.67
A_{21}	Einstein spontaneous emission coefficient, eqn 7.1
a	Lattice parameter
a_0	Unstrained lattice parameter
a_{H}	Bohr radius of hydrogen atom, eqn C.10
a_{ex}	Excitonic Bohr radius, eqn C.9
B	Radiative recombination coefficient, eqn 15.8, Section B.5
B_0	Rate constant for phonon-induced exchange of carriers between dot states and wetting layer, eqn 10.2
$B_{12}^{h\nu}$	Einstein coefficient for absorption (with respect to photon energy), eqn 7.4
$B_{21}^{h\nu}$	Einstein coefficient for stimulated emission (with respect to photon energy), eqn 7.5
C	Auger coefficient, eqn 15.9
c	Velocity of light
c_1, c_2	Probability amplitudes, eqn 7.22
$\boldsymbol{D}$	Electric displacement vector
d	Thickness of waveguide core
d	Effective length of dipole: dipole moment = $(-e)d$
E	Energy
$\mathcal{E}$	Electric field
E_{F}	Fermi energy
E_{Fc}	Conduction band quasi-Fermi energy
E_{Fv}	Valence band quasi-Fermi energy
E_{c}	Conduction band edge energy
E_{cb}	Energy of a state in the conduction band
E_{g}	Band gap energy
E_{v}	Valence band edge energy
E_{vb}	Energy of a state in the valence band
$\boldsymbol{e}$	Unit vector specifying direction of vector potential, eqn 7.69

F	Envelope function, eqn 8.4
F_{ext}	Light extraction factor, eqn 5.24
F_{w}, F_{b}	Envelope functions in well and barrier regions, Section 8.4.1
$f(E)$	Electron occupation probability of a state at energy E
G	Modal gain (coefficient), eqn 2.4
G_{pk}	Peak gain on a modal gain spectrum, Section 5.4
g	Material gain, Section 4.3.1
g_2	Number of final states per unit energy, eqn 7.77
g_{amp}	Amplitude material gain, eqn 6.19
g_{d}	Degeneracy factor, Section 9.2.2
g_{dl}	Material gain of a layer of dots, Section 9.4
$\hat{H}$	Hamiltonian operator, eqn 7.66
H_{21}	Perturbation, eqn 7.70
I	Current
I_{ov}	Wavefunction overlap integral, eqn 11.21
J	Current density (per unit area)
J_{spon}	Recombination current density due to spontaneous emission, eqn 5.19
$J_{\mathrm{th}}^{\mathrm{spon}}$	Spontaneous recombination current density at threshold, Section 5.3.3
J_{th}	Threshold current density (per unit area), Section 5.3
J_{trans}	Transparency current density, Section 5.4
J_{w}	Current density entering the quantum well (or quantum dot) system, eqn 5.20
K	Force constant, Section 6.4.1
$\boldsymbol{K}$	Wavevector of exciton centre of mass, eqn C.4
k, $\boldsymbol{k}$	Wavevector of an electron or hole, eqn 8.1
k	Propagation constant of light in a dielectric, eqn 4.1
k_0	Propagation constant of light in free space, $k_0 = 2\pi/\lambda_0$
k_{B}	Boltzmann's constant
k_{w}, k_{b}	Wavevectors in well and barrier regions, Section 8.4.1
L	Length, distance
$L(\hbar\omega)$	Lorentzian homogeneous broadening function in energy, eqn 6.54
$L(\omega)$	Lorentzian homogeneous broadening function, in angular frequency, eqn 6.49
L_{c}	Laser cavity length
L_z	Width of potential well
M	Momentum matrix element, eqn 7.71
M	Basis function momentum matrix element, Section 11.3.1
M_{cv}	Band-to-band momentum matrix element, eqn 11.8, Section 11.3.1
M_{dip}	Dipole moment of a dot, eqn 9.4

M_T	Transition momentum matrix element, eqn 11.23
m	Mass
m_0	Free-electron rest mass
m^*	Effective mass
m_e^*	Effective mass of an electron (in the conduction band)
m_h^*	Effective mass of a hole (in the valence band)
N_d	Number of dots per unit area, usually per layer
N_p	Number of photons
N_{ph}	Photon density (per unit area), Section 13.1
n	Carrier density, electron density, per unit area unless stated
n	Refractive index
n_d	Electrons per unit area occupying a layer of dots
n_{eff}	Effective index of a guided mode, eqn 4.9
n_g	Group index
n_{ph}	Bose–Einstein photon mode occupation number, eqn 7.10
n_{th}	Bose–Einstein phonon occupation number, eqn 10.1
n_{wl}	Electron density in wetting layer (per unit area)
P_{Lout}	Power output of laser, eqn 13.9
$\boldsymbol{P}$	Electrical polarisation vector, Section 6.3.1
$\tilde{P}$	Complex electrical polarisation, eqn 6.8
$P(E_i)$	Gaussian probability distribution in energy, eqn 9.9
p	Hole density, usually per unit area
$\boldsymbol{p}$, $\hat{\boldsymbol{p}}$	Momentum, momentum operator, eqn 7.66
p_d	Wavefunction dephasing probability rate, Section 7.3.4
p_d	Holes per unit area occupying a layer of dots
p_{wl}	Hole density in wetting layer (per unit area)
R	Power reflectivity, $= r_1 r_2, r^2$
$\boldsymbol{R}$	Position vector of exciton centre of mass, eqn C.3
R_A	Auger recombination rate, eqn 15.9, Section E.5
R_H	Hydrogen atom Rydberg (=13.6 eV), eqn C.7
R_{ex}	Excitonic Rydberg, eqn C.5
R_{nr}	Non-radiative recombination rate per unit area, SRH recombination rate, eqn 15.5, Section E.4
R_{nr}^{hi}	High-injection SRH recombination rate per unit area, eqn 15.6, Section E.4
R_{spon}	Spectrally integrated spontaneous emission rate per unit area, eqn 5.19, $[L]^{-2}\,[T]^{-1}$
$R_{spon}(h\nu)$	Spectral spontaneous emission rate, $[L]^{-2}\,[T]^{-2}\,[E]^{-1}$
R_{stim}	Stimulated downward transition rate
R_{stim}^{net}	Net downward stimulated rate, $[L]^{-2}\,[T]^{-2}$, eqn 13.2
$R_{up}(\omega)$	Upward absorption rate per atom, $[T]^{-1}$, eqn 7.53
r	Amplitude reflectivity, eqns 5.5, 14.1
$\boldsymbol{r}_i$	Position vector of ith lattice site, Section 8.2.2
r_{stim}^{net}	Net stimulated emission rate per photon, Section 4.3

$\boldsymbol{S}$	Poynting vector, energy flux [E] [L]$^{-2}$ [T]$^{-1}$, eqn 4.4; in a waveguide, Section 4.1.4
T	Absolute temperature, K
T_1	Population decay time, Section 7.3.7
T_2	Pure dephasing time, Section 7.3.7
U_e, U_m	Electrical and magnetic energy densities, [E] [L]$^{-3}$, Section 4.1
$u(\boldsymbol{r})$	Atomic-like, periodic part of Bloch function, eqn 8.1
u_c, u_v	Conduction and valence band functions, Section 11.3.1
u_i	Basis functions ($i = x, y, z$), Section 11.3.1
V	Volume in real space
V	Electric potential
$\hat{V}$	Perturbation, potential energy operator, eqn 7.28
V_0	Height of barrier of potential well
V_{fwd}	Forward voltage (on a diode)
v_E	Energy velocity, eqn 4.5
v_g	Group velocity
v_{ph}	Phase velocity, Section 4.1.1
W	Energy of a harmonic oscillator, eqn 6.26
W_{12}	Transition rate from a state to a continuum of states, [T]$^{-1}$, Fermi's Golden Rule, eqn 7.77
W_x	Lateral width (x direction) of guided mode
w_{mode}	Effective transverse mode width, eqn 4.47
x	Position coordinate, direction
x	Alloy composition, Section 3.3
$\hat{x}$	Position coordinate operator
y	Position coordinate, direction
y	Alloy composition, Section 3.3
z	Position coordinate, direction
α	Optical absorption coefficient, [L]$^{-1}$, eqn 2.8
α_{cav}	Optical cavity loss coefficient, [L]$^{-1}$, eqn 5.12
α_i	Internal optical mode loss coefficient, [L]$^{-1}$, Section 4.5
α_m	Distributed mirror loss coefficient, [L]$^{-1}$, eqn 5.12
β	Propagation constant of a guided mode, eqns 4.7, 4.9
$\tilde{\beta}$	Complex propagation constant of a guided mode, eqn 5.2
β_{spon}	Fraction of spontaneous emission entering the mode, Section 5.1.1
Γ	Optical confinement factor, Section 4.4.3, eqn 4.38
γ	Energy decay rate, eqn 6.26
γ_{cv}	Momentum matrix element polarisation factor, eqn 11.30
γ_{trans}	Number of allowed transitions between a pair of energy levels in a dot, Section 9.2.2
γ_{well}	Fractional absorption of a quantum well at normal incidence, eqn 11.34

ΔE_c	Conduction band offset energy, Section 3.4.1
ΔE_g	Band gap difference at heterobarrier, Section 3.4.1
ΔE_v	Valence band offset energy, Section 3.4.1
ϵ_0	Permittivity of free space
ϵ_r	Relative permittivity, sometimes called the dielectric constant, Section 4.1.1
η_0	Overall internal quantum efficiency, eqn 5.22
η_0^d	Overall internal differential efficiency above threshold, eqn 15.29
η_{ext}^d	External differential quantum efficiency, eqn 5.26
η_{inj}^d	Differential injection efficiency, eqn 15.26
η_{int}^d	Internal differential radiative quantum efficiency, eqn 5.25
η_s^d	Differential current spreading efficiency, eqn 15.25
η_{ext}	External quantum efficiency, 5.23
η_{inj}	Injection efficiency, eqn 5.20
η_{int}^{spon}	Internal spontaneous quantum efficiency, eqn 5.21
η_{pow}	Power conversion efficiency, eqn 5.28
κ	Transverse propagation constant, Section 4.2
Λ	Full-width of Lorentzian broadening function in energy, eqn 6.54
λ	Wavelength of light in medium of index n
λ_0	Wavelength of light in free space.
λ_B	Bragg wavelength of grating, eqn 14.4
μ	Microscopic dipole moment
μ_0	Permeability of free space
μ_{12}	Dipole matrix element of an atom, eqn 7.40 and preceding text
μ_{cv}	Dipole matrix element between states in conduction and valence bands, eqn 9.4
μ_{ex}	Exciton reduced mass, Section C.1.1
μ_{ij}	Dipole matrix element, eqn 7.39
μ_r	Relative permeability
ν	Frequency, Hz.
ρ	Charge density, $[L]^{-3}$, eqn 7.24
ρ	Density of electronic continuum states for a quantum well per unit area, eqn 8.44
$\rho(h\nu)$	Spectral photon density $[E]^{-1}\,[L]^{-3}$, eqn 7.11
$\rho_{mode}(h\nu)$	Density of modes in a large optical cavity, $[L]^{-3}\,[E]^{-1}$, eqn 7.9
σ	Optical cross section of atom or dot, Section 6.6.1
σ_0	Spectrally integrated optical cross section Section 6.6.3
σ_E	Standard deviation of Gaussian probability distribution, eqn 9.9
σ_n, σ_p	Electrical conductivity, n-, p-type

τ	Decay time of macroscopic polarisation, eqn 6.39; FWHM of Lorentzian in frequency is $2/\tau$, eqn 6.49
τ_{cap}	Lifetime for phonon-induced capture from the wetting layer, per empty dot state, eqn 10.5
τ_{em}	Lifetime for phonon-induced emission from an occupied dot state to the wetting layer, eqn 10.3
τ_{ph}^{0}	Cold cavity photon lifetime, eqn 13.4
$\tau_{\mathrm{ph}}^{\mathrm{stim}}$	Average photon lifetime due to net stimulated recombination, eqn 13.2
τ_{spon}	Lifetime for spontaneous emission (between two levels), eqn 7.3
Φ	Photon flux, $[\mathrm{L}]^{-2}\,[\mathrm{T}]^{-1}$
ϕ	Optical phase, wavefunction phase
χ	Electric susceptibility, eqn 6.2
$\tilde{\chi}$	Complex electric susceptibility. eqn 6.9
Ψ	Wavefunction
ψ	Wavefunction (usually for a single state)
Ω_{R}	Rabi frequency, Section 7.3.6
ω	Angular frequency, $\mathrm{rad\,s^{-1}}$
ω_0	Resonant frequency of classical oscillator, eqn 6.25
ω_0	Frequency corresponding to energy separation of two states, eqn 7.35

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