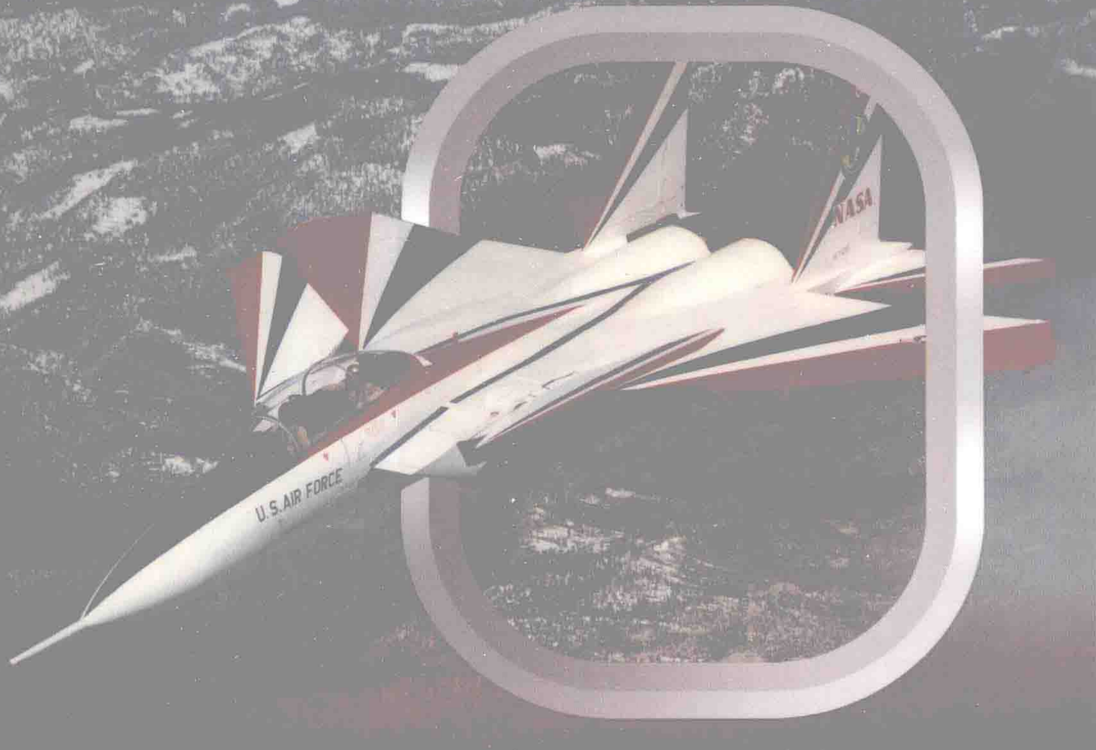

Wayne Durham
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Aircraft Control Allocation

Aerospace Series
Editors Peter Belobaba, Jonathan Cooper and Allan Seabridge

with website

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AIRCRAFT CONTROL ALLOCATION

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WILEY

This edition first published 2017

Registered office John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom

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Library of Congress Cataloging-in-Publication Data

Names: Durham, Wayne, 1941- author. | Bordignon, Kenneth A., author. | Beck, Roger, 1977- author.
Title: Aircraft control allocation / Wayne Durham, Kenneth A. Bordignon, Roger Beck.
Description: Chichester, West Sussex, United Kingdom : John Wiley & Sons, Inc., [2017] | Includes bibliographical references and index.
Identifiers: LCCN 2016024806| ISBN 9781118827796 (cloth) | ISBN 9781118827772 (epub) | ISBN 9781118827765 (Adobe PDF)
Subjects: LCSH: Airplanes–Control systems–Mathematics. | Airplanes–Control systems–Design and construction.
Classification: LCC TL678 .D87 2017 | DDC 629.135–dc23 LC record available at https://lccn.loc.gov/2016024806

A catalogue record for this book is available from the British Library.

Cover photo: NASA's NF-15B research aircraft is shown during an early yaw-vectoring mission in the ACTIVE flight research project. Credits: NASA Photo

Typeset in 10/12pt TimesLTStd by SPi Global, Chennai, India

Printed and bound in Malaysia by Vivar Printing Sdn Bhd

10 9 8 7 6 5 4 3 2 1

AIRCRAFT CONTROL ALLOCATION

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The aerospace sector draws upon many specialist areas of expertise; mechanical, electrical/electronic design, systems and computer engineering, planning, logistics, psychology (the human-machine interface), communications, risk analysis, and economics. Each title in the Aerospace Series explores a different aspect or cohesive topic that those working in or aspiring to work in the aerospace sector will find useful and informative. Books in the series range from high level student texts, explanation and dissemination of technology and good practice, through to interesting and important research that is immediately relevant to industrial development or practice.

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Dedication

For Craig Steidle, Bob Hanley, and John Foster. Thanks guys.

Series Preface

The field of aerospace is multi-disciplinary and wide ranging, covering a large variety of products, disciplines and domains, not merely in engineering but in many related supporting activities. These combine to enable the aerospace industry to produce innovative and technologically advanced vehicles. The wealth of knowledge and experience that has been gained by expert practitioners in the various aerospace fields needs to be passed onto others working in the industry and also researchers, teachers and the student body in universities.

The *Aerospace Series* aims to be a practical, topical and relevant series of books aimed at people working in the aerospace industry, including engineering professionals and operators, engineers in academia, and allied professions such commercial and legal executives. The range of topics is intended to be wide ranging, covering design and development, manufacture, operation and support of aircraft, as well as topics such as infrastructure operations and current advances in research and technology.

Modern aircraft are designed with multiple control surfaces, and possibly other control effectors e.g. thrust vectoring, and therefore problems can arise as to how to combine these control devices in an optimal manner, particularly in low-speed flight regimes where the aerodynamic surfaces lose their effectiveness.

This book, *Aircraft Control Allocation*, provides a detailed explanation of some selected topics relating to the aircraft control allocation problem. After providing some background material in aircraft control and control laws, a number of approaches that can be used to solve the control allocation problem are illustrated and the influence that they have on control law design discussed. Of particular note is the chapter describing some of the lessons learnt whilst designing the X-35 Flight Control System.

Peter Belobaba, Jonathan Cooper and Alan Seabridge

Glossary

$(\dot{\ })$ Dot over quantity: the derivative with respect to time of the contents of the parentheses ( ).

$(\hat{\ })$ Hat over quantity: the contents of the parentheses ( ) are approximate.

α Angle-of-attack: the aerodynamic angle between the projection of the relative wind onto the airplane's plane of symmetry and a suitably defined body fixed x-axis.

β Sideslip angle: The aerodynamic angle between the velocity vector and the airplane's plane of symmetry.

$\ell_1, \ell_2, \ell_\infty$ Vector norms: ℓ_2 is the square root of the sum of the squares of the entries in the vector. It appears everywhere. ℓ_1 is the sum of the absolute values of the entries and ℓ_∞ is the greatest absolute value. ℓ_1 and ℓ_∞ frequently appear in linear programming problems.

Ω Either:

1. Every combination of control effector deflections that are admissible; in other words, that are within the limits of travel or deflection.
2. A normally diagonal matrix used to specify the dynamics in a dynamic-inversion control law.

Φ The effects, usually body-axis moments, moment coefficients, or angular accelerations, of every combination of control effector deflections in Ω, q.v. (sense 1). Sometimes called the AMS, for 'attainable moment set' or subset.

ϕ Bank angle: one of three angles that define a 3-2-1 (z-y-x) rotation from inertial to body-fixed reference frames.

Π Either:

1. (Primarily) A subset of the attainable moments (Φ) consisting of all the moments that are generated by a particular control allocation method.
2. A plane surface that arises in Banks' method of allocation for the three-moment problem.

ψ Heading angle: one of three angles that define a 3-2-1 (z-y-x) rotation from inertial to body-fixed reference frames.

Θ A subset of Ω: all admissible controls that a particular control-allocation method can return as solutions to a control-allocation problem.

θ Pitch attitude: one of three angles that define a 3-2-1 (z-y-x) rotation from inertial to body-fixed reference frames.

B One of the matrices of the linearized equations of motion: *A* is the system matrix, *B* is control effectiveness matrix, and *C* is the output matrix.

C_{x_y} The non-dimensional stability or control derivative of *x* with respect to *y*: it is the non-dimensional form of X_y.

Comp Complementary: a superscript to certain dynamic responses.

Cont Controllable: a superscript to certain dynamic responses.

d, *des* Desired: a subscript to a dynamic response, or any other quantity.

F_B Body-fixed reference frames. The origin is at the airplane's center of mass. The axes x_B and z_B lie in the airplane's plane of symmetry. y_B completes the right-hand system. Once defined, a body-fixed reference system's orientation with respect to the body does not change. Two frequently used body-fixed reference frames are the principal axes and the stability-axis system.

F_H Local-horizontal reference frame. The axes x_H, y_H, and z_H are oriented north, east, and down, respectively. The earth is flat.

F_W Wind-axis system. The axis x_W lies in the direction of flight, opposite the relative wind. z_W is in the plane of symmetry, oriented downward.

g Either:

1. Acceleration of gravity, or
2. The non-dimensional units of load factor *n*, q.v.

I With subscripts; moment of inertia.

Kine Kinematic: a superscript to certain dynamic responses.

L, C, D Lift, side force, and drag: wind-axis forces in the x_W-, y_W- and z_W-directions, respectively.

L, M, N Body-axis moments about the x_B axis (rolling), y_B axis (pitching), and z_B axis (yawing), respectively.

L Either:

1. Lift, or
2. Rolling moment, depending on context.

LD Lateral-directional, meaning all motions, accelerations, forces, and so on, that are not longitudinal, q.v. Sometimes *lat-dir*.

Long Longitudinal, meaning all motions, accelerations, forces, and so on, that take place in the airplane's plane of symmetry. Pitching moments, velocities, and accelerations are about the airplane's y_B-axis but the motion is in the x_B–z_B plane.

m The mass of the airplane.

n Load factor, the ratio of lift to weight, $n = L/W$. Measured in *g*s.

p, q, r Body-axis roll rate, pitch rate, and yaw rate, respectively.

P A generalized inverse of a matrix *B*: $BPB = B$ and $PBP = P$, with appropriate dimensions.

Ref Subscript, 'evaluated in reference conditions'.

u Vector of control effector variables.

$\mathbf{u}_{min}$, $\mathbf{u}_{max}$ Vector of control effector limits, minimum or maximum.

$\mathbf{u}_l$, $\mathbf{u}_u$ Vector of control effector limits, lower or upper. This notation seems preferred by linear programmers over $\mathbf{u}_{min}$, $\mathbf{u}_{max}$, q.v.

x_B, y_B, z_B Names of body-axes.

W A weighting matrix, generally diagonal and positive.

x_W, y_W, z_W Names of wind axes.

X_y Where X is a force or moment and y is a state or control, a dimensional derivative, $\partial X/\partial y$. It is the dimensional form of C_{x_y}, q.v. The definition does *not* include division by mass or moment of inertia. If y is a control effector the result is called a control derivative, otherwise it is called a stability derivative.

X, Y, Z Body-axis forces in the x-, y- and z-directions, respectively.

x, y, z Names of axes. With no subscripts usually taken to be body-axes.

ACTIVE Advanced Control Technology for Integrated Vehicles. A research F-15 with differential canards, axisymmetric thrust vectoring, and other novel features.

ADMIRE Aero-Data Model In a Research Environment, simulation code. See Appendix B.

Admissible Of a control effector or suite of control effectors, those deflections that are within the physical limits of employment.

AMS Attainable moment subset or set, Φ.

Angular accelerations See Objectives.

ARI Aileron-rudder interconnect. Normally used to reduce adverse yaw due to aileron deflection.

Attainable Of moments or accelerations; that which can be generated by some admissible combination of control effectors. The term may be applied globally, meaning there is some theoretical combination, or locally, to a particular control allocation method, meaning those combinations of control effectors that the method will generate using its rules.

Basic feasible solution Of linear programs, a basic solution to the equality constraints in a linear program that also solves the inequality constraints.

Basic solution Of linear programs, a solution to the l linear equality constraints of a linear program in 'standard form' with $k - l$ of the decision variables at their bound.

CAS Control augmentation system.

Control effectiveness A measure of the effect of utilizing a control effector, either moment, moment coefficient, or angular acceleration.

Control authority The aggregate effect of the effectiveness of all the control effectors in whatever combination.

Control power Angular acceleration per unit of control deflection.

CHR Cooper–Harper rating; sometimes HQR.

Constraint Of a control effector, a limiting position, usually imposed by the hardware. It may also refer to a limit on the rate of travel. In linear programming, a constraint may refer to the position limits, but also of an equality that must be satisfied. Thus $\mathbf{u} \leq \mathbf{u}_{\max}$ is an inequality constraint, and $B\mathbf{u} = \mathbf{m}_{des}$ is an equality constraint.

Control effector The devices that directly effect control by changing forces or moments, such as ailerons or rudders. When we say 'the controls' with no qualification, we usually mean the control effectors. The sign convention for conventional flapping control effectors follows a right-hand rule, with the thumb along the axis about which the effector is designed to generate moments, and the curled fingers denoting the positive deflection of the trailing edge.

Control inceptor Cockpit devices that control, through direct linkage or a flight-control system or computer, the control effectors. Positive control inceptor deflections correspond to positive deflections of the effectors they are connected to, barring such things as aileron–rudder interconnects (ARI, q.v.).

Cycling Of a linear program, a condition in which a sequence of vertices is visited by a solver for which the objective function does not decrease, eventually returning to the starting point in the cycle. Cycling represents a failure to converge and must be addressed by choosing an exchange rule designed to prevent it.

Degenerate basic solution Of linear programs, a basic solution to a linear program in which one of the l decision variables in the basis is at its bound in addition to the non-basic variables.

Decision variables The set of unknown parameters being optimized in a linear program.

FBW Fly by wire. The pilot flies the computer, the computer flies the airplane.

FQ Flying qualities.

Ganged Said of mechanical devices that are linked so that they move in fixed relation to each other, such as ailerons.

HARV High angle-of-attack research vehicle.

HQ Handling qualities.

HQR Handling qualities rating.

Interior point method One of a family of numerical methods that seek to find the optimal solution to a linear program by moving through the interior of the feasible set.

Intersection Of two objects (q.v.), an object that is wholly contained in each of the two.

Lat-Dir Lateral-directional.

LEU, LED Leading-edge up, down. Terms used to describe the deflection of leading-edge control surfaces.

Linear programming A problem, or the method of solving that problem, of optimization of an objective subject to linear equality and inequality constraints. To the purpose of this book, a method of allocating controls subject to position constraints.

Moments See Objectives.

Moment coefficients See Objectives.

Object A generalization of any of the several polytopes that describe sets of admissible controls and attainable moments.

Objectives Those which control effectors are intended to generate. Originally control allocation sought to find the control effectors that generated specified moments, or moment coefficients. Subsequently researchers have tended toward using angular accelerations as the objectives. We will generally speak of the objectives as being moments.

Object notation A method of identifying objects (q.v.) using a 0 for a control at its lower limit, a 1 at its upper limit, and a 2 if it can be anywhere in between.

OBM On-board model. A set of aerodynamic data for an aircraft stored in the aircraft's flight control computer.

Over-actuated control system See Redundant controls.

Phase one/two program Phase one of a linear programming solver solves a modified problem in order to locate an initial feasible solution for the phase two solver that will optimize the original problem.

PIO Pilot-induced oscillation. There's a more politically correct term that removes the onus from the pilot.

PR Pilot rating; sometimes HQR, q.v.

Preferred Of a solution to the control allocation problem, a control effector configuration that is as close as possible to to one that is preferred. Minimum norm solutions are used as preferred solutions often.

Pseudo control A combination of control effectors intended to create a certain effect, such as the excitation of a particular dynamic response mode of the airplane.

Redundant controls Control effectors are seldom *redundant*, in the sense that the designer had no use for them in mind. The control effectors that are redundant in higher-speed flight may be critical in slow-speed flight. The term just means that there are more control effectors than objectives, q.v. As used in this book, it means there are more than three control effectors to generate the three moments or angular accelerations.

SAS Stability augmentation system.

Simplex An extension of a triangle (two-dimensional), or tetrahedron (three-dimensional), to an arbitrary number of dimensions. An n-dimensional simplex is defined by the convex hull of $n + 1$ vertices.

Simplex method Either:

1. (Dantzig) Algorithms based on Dantzig's original numerical algorithm for the solution of linear programs, introduced in 1947. The simplex method moves between neighboring vertices, basic solutions, of the feasible set, decreasing the cost function until the optimum is found.
2. (Nelder Mead) Also known as downhill simplex. Numerical solution algorithm that iterates an n-dimensional simplex to minimize n-dimensional, non-linear, unconstrained optimization problems. Heuristic rules at each step govern how to modify the simplex.

Slack variable Variables augmenting the decision variables in a linear program so that inequality constraints can be converted to equality constraints.

TEU, TED, TEL, TER Trailing-edge up, down, left, right. Terms used to describe the deflection of flapping control surfaces.

Union Of two objects (q.v.), the smallest object of which the two given objects are both members.

Warm start A heuristic method for initializing a linear program solver given a pre-existing optimal solution to a similar problem.

About the Companion Website

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