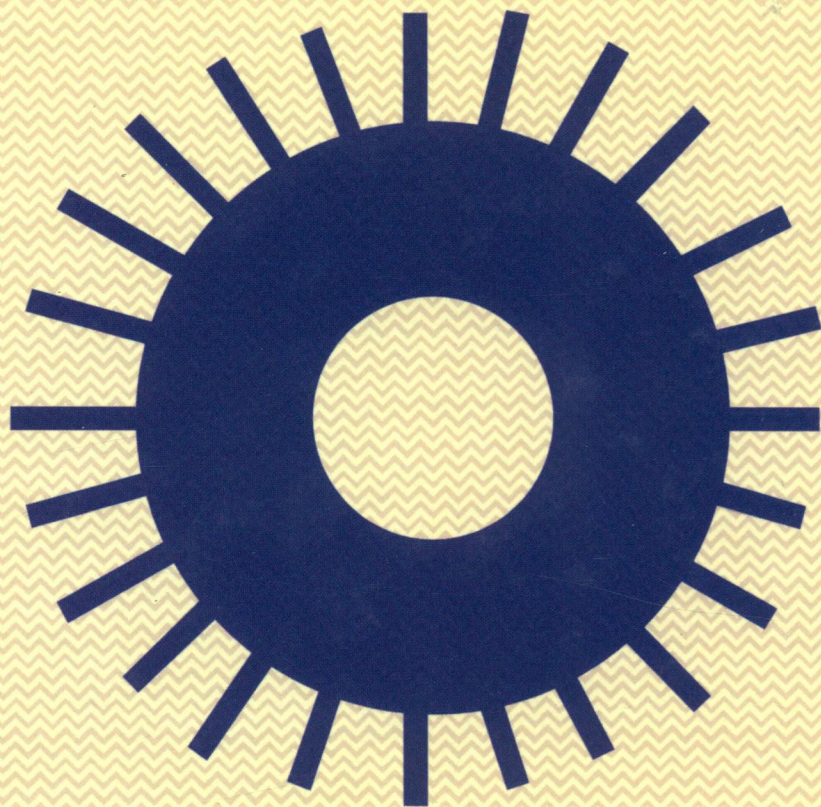




Vibration of **Nearly**
Periodic Structures and
Mistuned Bladed Rotors

ALOK SINHA

This is the first comprehensive volume on nearly periodic structures and mistuned blade vibration. Alok Sinha presents fundamental concepts and state-of-the-art techniques in the analysis of free and forced response of a nearly periodic structure, weaving together his own work (covering 35 years of research in this field) with works by other researchers. He also discusses similarities between tools used in bladed rotor analysis and condensed matter physics.

Specific subjects covered include:

- Reasons behind mode localization
- Reasons behind amplitude amplification of steady-state response
- State-of-the-art computational techniques for mistuned bladed rotors including multistage rotors
- Identification of mistuning from measured response
- Vibration localization in linear atomic chains
- Analysis of two-dimensional periodic structures

ALOK SINHA is Professor of Mechanical Engineering at Pennsylvania State University. He also has served as visiting professor of aeronautics and astronautics at Massachusetts Institute of Technology and Stanford University. His areas of teaching and research include vibration, control systems, jet engines, robotics, neural networks, and nanotechnology. He is a Fellow of the American Society of Mechanical Engineers and the American Association for the Advancement of Science, and an Associate Fellow of the American Institute of Aeronautics and Astronautics. He has received a NASA Certificate of Recognition for Significant Contribution to the Space Shuttle Microgravity Mission. He is the author of *Linear Systems: Optimal and Robust Control* (2007), and *Vibration of Mechanical Systems* (2010). He has also served as an Associate Editor of the *ASME Journal of Dynamic Systems, Measurement and Control*, the *ASME Journal of Turbomachinery*, and the *AIAA Journal*.

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To my

Granddaughter: Meera

Daughter: Swarna

Daughter and Son-in-Law: Divya and Vishal

Wife: Hansa

Acronyms

CMM:	Coordinate Measurement Machine
CMS:	Component Mode Synthesis
FE:	Finite Element
FEM:	Finite Element Model
FMM:	Fundamental Model of Mistuning
FOD:	Foreign Object Damage
IBR:	Integrally Bladed Rotor
KL:	Karhunen-Loeve
LFT:	Linear Fractional Transformation
LMI:	Linear Matrix Inequality
MAC:	Modal Assurance Criterion
MDA:	Modal Domain Analysis
MMDA:	Modified Modal Domain Analysis
NOC:	Number of Occurrences
POD:	Proper Orthogonal Decomposition
SFMM:	Single Family Mode Model
SISO:	Single Input/Single Output
SNM:	Subset of Nominal Modes
SVD:	Singular Value Decomposition

Preface

Bladed disk is an important part of steam and gas turbines. Each blade is designed to be identical. Therefore, the structure of a bladed disk can be described as a periodic structure where a bladed disk can be divided into identical sectors. In practice, these structures are nearly periodic because of manufacturing tolerances. This loss of cyclic symmetry can lead to dramatic change in the dynamic behavior. For free vibration, mode shapes can change a great deal and there can be mode localization phenomenon. For forced response, the amplitude of one blade can be three times higher than the value based on exact periodic structure. There are thousands of papers written on this topic in the last fifty years, some of the earliest papers being Whitehead (1966), Wagner (1967), and Ewins (1969). Aspects include reasons for vibration amplification, reduced-order modeling, intentional mistuning, probabilistic analysis, robust control perspective, efficient computational procedure, and nature of excitation. This monograph presents fundamental concepts and issues in the analysis of mistuned vibration. The author has been doing research in this area for more than thirty-five years. Here, he has weaved together works done by him and other researchers.

Periodic structures are ubiquitous not only in engineering sciences but also in physical and life sciences. As mentioned before, they refer to repeating patterns and wave localization because of infinitesimally small amounts of nonlinearities or disorder that are inevitable. This Nobel Prize-winning phenomenon is known as Anderson localization (Anderson, 1958) in condensed matter physics, where it refers to electron localization. Carbon nanotubes and graphenes can have localization of both electrical and mechanical waves. In life sciences, some parts of the DNA double helix open locally at high temperatures and form the so-called denaturation bubbles, which play a role in biological function. Localized transient waves are found in cortical spreading depression in migraine. It is hoped that concepts explained in this book will help researchers in nanotechnology and life sciences as well.

In Chapter 1, free vibration characteristics of nearly periodic structures are explained on the basis of simple models that have been extensively used in the literature. First, existence of repeated eigenvalues and nonuniqueness of eigenvectors is shown for a perfectly tuned system. Then, a computational algorithm is presented to obtain mistuned frequencies and mode shapes by defining the concept of differential eigenvectors. The case of small coupling stiffness is considered to explain the

mode localization phenomenon. Next, the natural frequency veering phenomenon, which is widely used in the literature, is explained. This is followed by the analysis demonstrating stabilizing effects of mistuning on aeroelastic instability or flutter of a rotor stage. Lastly, analysis and results on vibration localization in atomic chains are presented.

In Chapter 2, forced vibration characteristics of nearly periodic structures are explored again on the basis of simple models that have been extensively used in the literature. The nature of excitation is considered to be one of the Fourier components of the periodic force caused by circumferential pressure field and constant rotational speed of the bladed rotor. These excitations are characterized by identical amplitude for each blade and constant interblade phase angles between adjacent blades corresponding to one of the natural modes of vibration of the perfectly tuned system. First, it is shown how only a single mode of a perfectly tuned system gets excited. An interesting connection with pole/zero cancellations that is related to loss of observability and controllability in control theory is established. Then, in the presence of mistuning, the phenomenon of energy transfer to many modes is explained as the reason of amplitude amplification. The statistical distribution of the peak maximum amplitude, maximum over all blades and all excitation frequencies, are examined using Monte Carlo simulation. Then, a classical analytical result on the maximum amplitude amplification along with recent numerical algorithms based on infinity norm and linear robust control theory are presented. Last, analytical approaches to compute the statistical distribution of amplitudes are presented on the basis of known statistical distribution of mistuning parameters.

In Chapter 3, reduced-order models of a bladed rotor are presented. These reduced-order models are needed for Monte Carlo simulations to determine the response statistics as the full model is extremely large. First, theory behind the cyclic symmetry analysis, a standard tool in commercially available Finite Element code, is presented, which renders the analysis of a full tuned rotor by an equivalent sector analysis resulting in a computational saving. Similar technique, known as Bloch's theorem, is presented for two-dimensional periodic structures such as graphene and carbon nanotubes. It should be noted that the cyclic symmetry is lost in the presence of mistuning, and equivalent sector analysis is not valid. First, subset of nominal modes (SNMs), single family mode model (SFMM), and component mode synthesis (CMS) approaches are presented to develop reduced-order models. Mistuning identification algorithm associated with SFMM approach is also presented. These techniques are based on the representation of mistuning as variations in blades' natural frequencies alone, known as frequency mistuning. They are unable to yield accurate results in the case of geometric mistuning, which refers to blade-to-blade variations in geometry resulting in simultaneous and dependent perturbations in both mass and stiffness matrices. The breakthrough method modified modal domain analysis (MMDA), which has been developed by the author of this monograph, is presented in detail. Numerical examples are presented to highlight the accuracies of MMDA. Next, algorithm to estimate geometric mistuning

from measured data is presented. Lastly, MMDA-like algorithm is developed for the reduced-order model of multistage rotors.

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