

A C O U R S E O N

Abstract

Algebra

Second Edition

Minking Eie • Shou-Te Chang

 World Scientific

A Course on Abstract Algebra

Second Edition

Review of the First Edition:

"The text is greatly enriched by many varied and wonderful examples, all carefully worked out, and revealing some of the more subtle points of the theories. This is the text's greatest asset ... the authors have succeeded in writing a solid and complete text with many rich and varied examples that introduces the basics of modern algebra to the undergraduate audience."

Mathematical Reviews

This textbook provides an introduction to abstract algebra for advanced undergraduate students. Based on the authors' notes at the Department of Mathematics, National Chung Cheng University, it contains material sufficient for three semesters of study. It begins with a description of the algebraic structures of the ring of integers and the field of rational numbers. Abstract groups are then introduced. Technical results such as Lagrange's theorem and Sylow's theorems follow as applications of group theory. The theory of rings and ideals forms the second part of this textbook, with the ring of integers, the polynomial rings and the matrix rings as basic examples. Emphasis will be on factorization in a factorial domain. The final part of the book focuses on field extensions and Galois theory to illustrate the correspondence between Galois groups and splitting fields of separable polynomials.

Three whole new chapters are added to this second edition. Group action is introduced to give a more in-depth discussion on Sylow's theorems. We also provide a formula in solving combinatorial problems as an application. We devote two chapters to module theory, which is a natural generalization of the theory of vector spaces. Readers will see the similarity and subtle differences between the two. In particular, determinant is formally defined and its properties rigorously proved.

The textbook is more accessible and less ambitious than most existing books covering the same subject. Readers will also find the pedagogical material very useful in enhancing the teaching and learning of abstract algebra.

A Course on Abstract Algebra

**Second
Edition**

Eie
Chang



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National Chung Cheng University, Taiwan

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Preface

During our many years of teaching the undergraduate course in abstract algebra, we have encountered many excellent textbooks at the introductory level. Why write another one?

The most important reason is that we want to provide a textbook for those students whose mother tongue is not English. Then why don't we just write one in Chinese for our own students? We hope to inspire more students into pursuing a mathematical (or other academic) career, and this will eventually involve reading books and writing papers in English. We feel it is maybe easier to start doing it in elementary courses when the mathematics is not overwhelmingly difficult. We aim to use simple and straightforward English to explain basic notions and terminologies in algebra. The process of abstraction and generalization in abstract algebra already seem foreign enough to most beginners. We don't want the language to be another barrier. Even though we write this book with our own students in mind, we believe that this book will also be beneficial to native speakers of English or other non-Chinese speaking readers who want to learn some abstract algebra.

The existing textbooks tend to be too long, often easily exceeding five or six hundred pages. Some authors put in a lot of information, either in explaining in great detail, or adding historical notes, or even providing

additional chapters on applied algebra. As instructors, we greatly appreciate and have benefited from these efforts. However, we seldom find time during the semesters to convey to our students all this extra material. Most of our students, being not fluent in English, are intimidated by the long explaining sections. They never really seem to notice that how much precious knowledge is near at hand. This is why we want to write one that is not so densely packed with extra information, but do contain the most essential reading for any beginning student of algebra. We hope that the size of this book will not be so daunting to our readers. Mastering this book will more than adequately prepare anyone who wants to continue into more advanced courses in any field of algebra.

In our experience of studying mathematics, there is nothing more important than doing the exercises. Hardworking students often find reading explanations and proofs easy. However, one cannot really learn mathematics without doing the exercises. (This does not include reading other people's solutions of the exercises.) Students are encouraged to try the exercises immediately after finishing reading the text for a full learning experience. Exercises will help test if one really has acquired enough understanding of the material. Exercises will also help students discover new problems for themselves, which is an essential process in higher learning. Indeed, we cannot stress enough how important doing exercises is! Especially in this book, some details are left as exercises, so that students will have a feeling of participation in "doing" this book. You do while you learn, and you learn while you do. Some of the exercises are designed to practise the concepts just discussed in the text, while some exercises are designed to introduce new concepts which we may or may not discuss in the coming chapters. Whichever the case these materials will still be important and interesting in their own right. Most of the exercises are designed to help students develop a deeper understanding of the content of the book. Finally, there are a small amount of exercises which are meant to be challenging! We are sure students will feel greatly rewarded after they have solved these more difficult problems.

In this book there are a fair amount of exercises which we hope will not be intimidating and repetitive to our readers. Well designed exercises are extremely helpful to students in becoming familiar with the material.

They also stimulate students' interests in wanting to learn more. We hope that our problem sets are well chosen and not excessive so that they fill the purpose of well designed exercises.

This textbook is designed for a fast-paced two-semester (one-year) course or a leisurely three-semester course. Depending on the pace of the course and certainly on the length of the school terms, the instructor might choose to skip some of the materials. If so, §10.3, Chapter 18, Chapter 19, §20.2 and §22.3 can be skipped without affecting the understanding of the rest of the book. Chapters 10–12, 18, 19, 23, 24 cover more advanced materials which are better suited for a third semester. Under time constraints, the proofs of some of the major theorems, for example, Sylow's Theorems, Structure Theorem of Finite Abelian Groups and Fundamental Theorem of Galois Pairing, can definitely be skipped.

The authors of this book work in analytic number theory and in commutative algebra, respectively. We hope we have contributed in this book a little bit of flavor in each discipline. We also hope our readers will have a great experience in using this book and enjoy learning abstract algebra as we did when we were students ourselves.

For this second edition, three new chapters are added. There are some changes and corrections made to the old chapters and the problem sets. We are sure this book still contains typos and small errors no matter how hard we try to avoid them. Please send us emails if you have any questions or comments. Any reader's input will be greatly appreciated. We can be reached at minking@math.ccu.edu.tw and shoute.chang@gmail.com.

At last, we would like to thank our editor at World Scientific, Ms Kwong Lai Fun, for being kind, helpful and tolerant. We would also like to dedicate this book to our families for their love and support.

Minking Eie
Shou-Te Chang
June 6, 2017

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