ELECTRICAL METHODS OF BLOOD-PRESSURE RECORDING

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By

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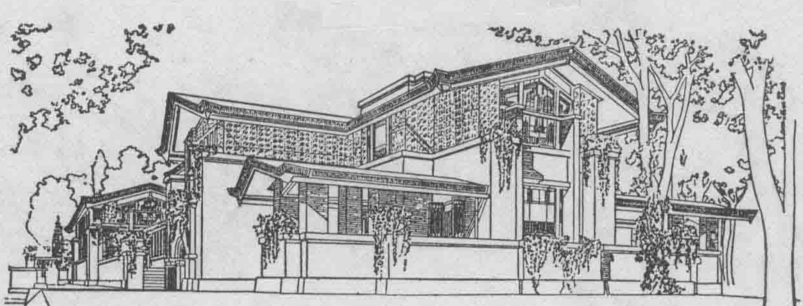

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To

Dr. Richard Parmenter

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Introduction

THE SUBJECT of dynamic blood-pressure recording has been of interest to physicians and physiologists for many years. Since the original experiment by the Reverend Stephen Hales in 1732, many improvements have been made in optically recorded mechanical manometers. The diaphragm-type manometers of Frank, Wiggers and Hamilton have provided successive improvements in the fidelity of the pulse pressure record. The present photographic type of diaphragm manometers, when properly adjusted, have excellent frequency response, linearity and stability. However, these instruments are difficult to operate because they are space-consuming, hard to fill, and their position must be rigidly maintained during recording. Electrical manometers have been constructed with very small pickup heads attached directly to needles. Connections with auxiliary amplifiers are made by means of flexible cables. They are very easy to fill, and their position may be altered during recording with no ill effects. In addition, both pressure and time scales of the record may be changed continuously over a wide range. By the use of certain electrical networks, it is possible to obtain the average pressure and the first and higher order time derivatives of pressure, and to record this information simultaneously with the pressure record. The flexibility of electrical manometers is therefore greater. The fidelity of response may be made comparable and perhaps superior.

The purpose of the writer is to discuss the hydraulic and electric systems which have been used in manometers of the diaphragm type.

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General Considerations

THE FRENCH PHYSICIST Fourier established the following theorem:

Any function f(t) which, within an interval, is single valued, finite, and continuous, or has only a finite number of discontinuities, may be expressed by the infinite series

$$f(t) = \cdot 5A_o + \sum_{n=1}^{\infty} \left[A_n \cos nt + B_n \sin nt\right] \qquad (1)$$

where:

$$\cdot 5A_o = \frac{1}{2\pi} \int_o^{2\pi} f(t)dt \qquad (2)$$

$$A_n = \frac{1}{\pi} \int_o^{2\pi} f(t) \cos nt \, dt \qquad (3)$$

$$B_n = \frac{1}{\pi} \int_o^{2\pi} f(t) \sin nt \, dt \qquad (4)$$

$$n = \text{any positive integer}$$

The pressure function of time existing in any blood vessel meets the conditions of Fourier's Theorem. The practical importance of this is that the blood pressure may be analyzed into two components:

1. An average pressure, P_a
2. A superimposed alternating pressure, $P(t)$

P(t) consists of a number of sinusoidal harmonically related waves which determine the characteristic shape of a given pressure wave. It is a simple matter to construct a pressure-measuring system which will indicate average pressure. A damped mercury manometer is such a system. However, if this system is used to obtain a dynamic pressure record, it will respond accurately to P_a, but will distort P(t) more or less badly depending upon the vibrational properties of the system.

In the dynamic measurement of blood pressure, such a wide variety of wave shapes is encountered that it is impossible to predict what harmonics of significant amplitude are actually contained in the true pressure function existing in the vessel. It is therefore desirable that a manometer respond equally well to as wide a range of frequencies as possible, i.e., that the manometer have a "flat frequency response characteristic" extending from zero to as high a frequency as possible. It is also desirable that the various harmonics be transmitted to the recorder at constant velocity. This requires that the time phase shift increase directly with frequency. Otherwise, marked distortion of the record may occur.

The hydraulic system of a manometer performs in much the same manner as a mass suspended by a spring. In order to obtain some information about the vibrational properties of manometers, it will be helpful to consider the simple mass-spring analogue.

3

An Analogue for the Diaphragm Manometer

A. Response of an Undamped Vertical Mass-spring System to a Force Step Function

WHEN A MASS is suspended by a spring (Figure 1) it will come to rest under the condition that the upward force exerted upon the mass by the spring is exactly equal and opposite to the downward force of gravity acting upon the mass. The upward force of the spring is equal to the product of the elastance S of the spring by the displacement d, i.e.,

$$F = -S\,d \qquad (1)$$

The downward force of gravity is equal to the product of the mass M by the acceleration due to gravity g, *i.e.*,

$$F = M\,g \qquad (2)$$

Therefore

$$M\,g = S\,d \qquad (3)$$

If now an additional downward force is applied to the mass to produce an additional downward displacement y, after which this additional force is suddenly removed, the mass will seek its former position of rest. We wish to find the position which the mass will occupy at any time subsequent to the release.

At the instant of release, the upward force of the spring upon the mass is

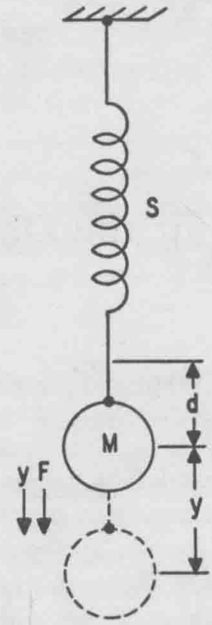

Figure 1. An analogue for a Diaphragm Manometer Operating About a Positive Average Pressure.

$$F' = -S\,(d + y) \tag{4}$$

and the downward force is M g. The net force is the difference between the upward and downward forces acting upon the mass. This net force will produce an acceleration of the mass. Thus

$$M\,\frac{d^2y}{dt^2} = M\,g - S\,(d + y) \tag{5}$$

By the use of Eq. (3), we may simplify the above equation to

$$\frac{d^2y}{dt^2} + \left(\frac{S}{M}\right) y = 0 \tag{6}$$

The general solution of Eq. (6) is

$$y = C_1 e^{-i\omega_o t} + C_2 e^{i\omega_o t} \tag{7}$$

where: $\omega_o = \sqrt{\dfrac{S}{M}}$

C_1 and C_2 are constants which may be determined from the initial conditions. Let these conditions be that when $t = 0$, $y = y_o$, and $\dfrac{dy}{dt} = 0$. The derivative of Eq. (7) is

$$\frac{dy}{dt} = i\omega_o C_2 e^{i\omega_o t} - i\omega_o C_1 e^{-i\omega_o t} \tag{8}$$

We now have the two equations

$$y_o = C_1 + C_2 \tag{9}$$

$$0 = i\omega_o C_2 - i\omega_o C_1 \tag{10}$$

Solving the above equations simultaneously for C_1 and C_2 we obtain

$$C_1 = C_2 = \frac{y_o}{2} \tag{11}$$

Therefore the solution of Eq. (6) for the initial conditions given above is

$$y = y_o \frac{e^{i\omega_o t} + e^{-i\omega_o t}}{2} = y_o \cos \omega_o t \tag{12}$$

Equation (12) indicates that the mass will oscillate in simple harmonic motion forever at constant amplitude y_o and at a frequency

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{S}{M}} \text{ cycles per second}$$

The frequency f_o is the "undamped natural frequency" of the spring-mass system.

B. Response of an Undamped Horizontal Mass-spring System to a Force Step Function

When the mass in Figure 2 is at rest, the force exerted By the use of Eq. (3), we may simplify the above equation applied to the mass in the direction shown, the mass will move to the left until an extension of the spring is reached at which the restoring force of the spring is exactly equal and opposite to the external force, at which point the mass will again come to rest. If the external force is suddenly released, the mass will be accelerated by the net force, *i.e.*, the tension in the spring. Thus

$$M \frac{d^2y}{dt^2} = -S\,y \tag{14}$$

This is identical to Eq. (6). The general solution is given in Eq. (7). If the initial conditions are specified as before, the equation of motion of this system is given by Eq. (12).

It is observed that the vertical and horizontal mass-spring systems are similar in that they both oscillate forever in simple harmonic motion about their respective positions of static equilibrium. The amplitude of the oscillation is constant and equal to the initial displacement from the position of static equilibrium. The frequency of oscillation is constant and is determined solely by the values of elastance and mass of the system.

C. Response of a Vertical Mass-spring System with Viscous Damping to a Force Step Function

Let the mass of Figure 1 be placed in a damping medium in which the force opposing motion is directly proportional

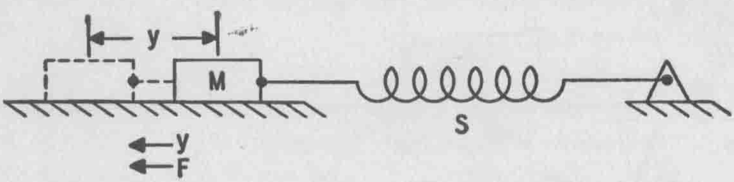

Figure 2. An Analogue for a Diaphragm Manometer Operating About Zero Average Pressure.