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10E

Ron Larson



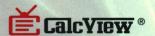
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- Videos explaining the concepts of precalculus
- Worked-Out Solution Videos for all Checkpoint problems
- Diagnostic Test with grading to determine course readiness
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Algebra and Trig with CalcChat and CalcView Tenth Edition

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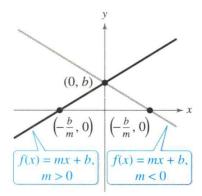
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GRAPHS OF PARENT FUNCTIONS

Linear Function

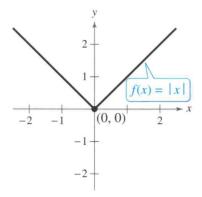
$$f(x) = mx + b$$



Domain: $(-\infty, \infty)$ Range $(m \neq 0)$: $(-\infty, \infty)$ *x*-intercept: (-b/m, 0)*y*-intercept: (0, b)Increasing when m > 0Decreasing when m < 0

Absolute Value Function

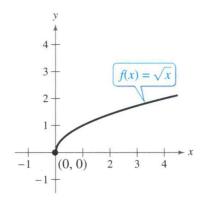
$$f(x) = |x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$$



Domain: $(-\infty, \infty)$ Range: $[0, \infty)$ Intercept: (0, 0)Decreasing on $(-\infty, 0)$ Increasing on $(0, \infty)$ Even function y-axis symmetry

Square Root Function

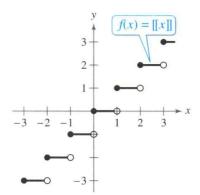
$$f(x) = \sqrt{x}$$



Domain: $[0, \infty)$ Range: $[0, \infty)$ Intercept: (0, 0)Increasing on $(0, \infty)$

Greatest Integer Function

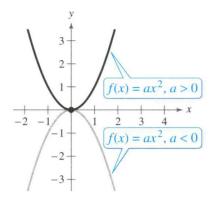
$$f(x) = [x]$$



Domain: (-∞, ∞)
Range: the set of integers
x-intercepts: in the interval [0, 1)
y-intercept: (0, 0)
Constant between each pair of consecutive integers
Jumps vertically one unit at each integer value

Quadratic (Squaring) Function

$$f(x) = ax^2$$

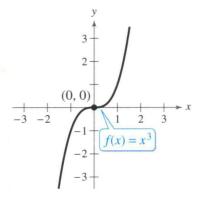


Domain: $(-\infty, \infty)$ Range (a > 0): $[0, \infty)$ Range (a < 0): $(-\infty, 0]$ Intercept: (0, 0)Decreasing on $(-\infty, 0)$ for a > 0Increasing on $(0, \infty)$ for a > 0Increasing on $(-\infty, 0)$ for a < 0Decreasing on $(0, \infty)$ for a < 0Even function y-axis symmetry Relative minimum (a > 0).

y-axis symmetry Relative minimum (a > 0), relative maximum (a < 0), or vertex: (0,0)

Cubic Function

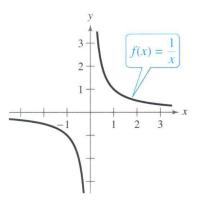
$$f(x) = x^3$$



Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Intercept: (0, 0)Increasing on $(-\infty, \infty)$ Odd function Origin symmetry

Rational (Reciprocal) Function

$$f(x) = \frac{1}{x}$$



Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$

No intercepts

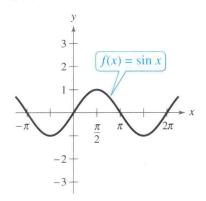
Decreasing on $(-\infty, 0)$ and $(0, \infty)$

Odd function Origin symmetry

Vertical asymptote: *y*-axis Horizontal asymptote: *x*-axis

Sine Function

$$f(x) = \sin x$$



Domain: $(-\infty, \infty)$

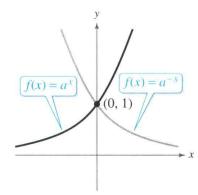
Range: [-1, 1]

Period: 2π

x-intercepts: $(n\pi, 0)$ *y*-intercept: (0, 0)Odd function Origin symmetry

Exponential Function

$$f(x) = a^x, \ a > 1$$



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Intercept: (0, 1)

Increasing on $(-\infty, \infty)$

for $f(x) = a^x$

Decreasing on $(-\infty, \infty)$

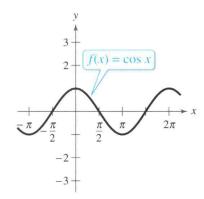
for $f(x) = a^{-x}$

Horizontal asymptote: x-axis

Continuous

Cosine Function

$$f(x) = \cos x$$



Domain: $(-\infty, \infty)$

Range: [-1, 1]

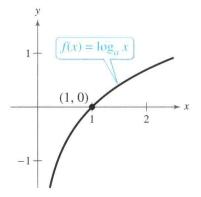
Period: 2π

x-intercepts: $\left(\frac{\pi}{2} + n\pi, 0\right)$

y-intercept: (0, 1) Even function y-axis symmetry

Logarithmic Function

$$f(x) = \log_a x, \ a > 1$$



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Intercept: (1, 0)

Increasing on $(0, \infty)$

Vertical asymptote: y-axis

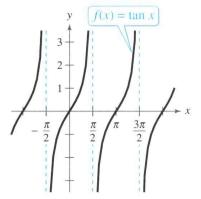
Continuous

Reflection of graph of $f(x) = a^x$

in the line y = x

Tangent Function

$$f(x) = \tan x$$



Domain: all $x \neq \frac{\pi}{2} + n\pi$

Range: $(-\infty, \infty)$

Period: π

x-intercepts: $(n\pi, 0)$ *y*-intercept: (0, 0)

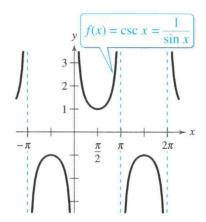
Vertical asymptotes:

$$x = \frac{\pi}{2} + n\pi$$

Odd function Origin symmetry

Cosecant Function

$$f(x) = \csc x$$



Domain: all $x \neq n\pi$

Range: $(-\infty, -1] \cup [1, \infty)$

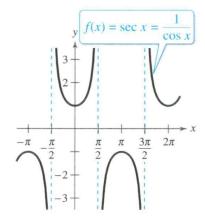
Period: 2π No intercepts

Vertical asymptotes: $x = n\pi$

Odd function Origin symmetry

Secant Function

$$f(x) = \sec x$$



Domain: all $x \neq \frac{\pi}{2} + n\pi$

Range: $(-\infty, -1] \cup [1, \infty)$

Period: 2π

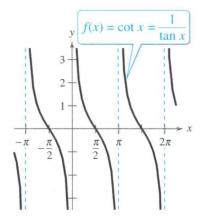
y-intercept: (0, 1) Vertical asymptotes:

$$x = \frac{\pi}{2} + n\pi$$

Even function *y*-axis symmetry

Cotangent Function

$$f(x) = \cot x$$



Domain: all $x \neq n\pi$

Range: $(-\infty, \infty)$

Period: π

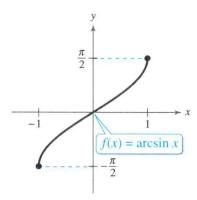
x-intercepts: $\left(\frac{\pi}{2} + n\pi, 0\right)$

Vertical asymptotes: $x = n\pi$

Odd function Origin symmetry

Inverse Sine Function

$$f(x) = \arcsin x$$



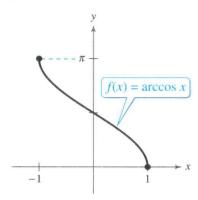
Domain: [-1, 1]

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Intercept: (0, 0) Odd function Origin symmetry

Inverse Cosine Function

$$f(x) = \arccos x$$



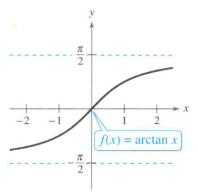
Domain: [-1, 1]

Range: $[0, \pi]$

y-intercept: $\left(0, \frac{\pi}{2}\right)$

Inverse Tangent Function

$$f(x) = \arctan x$$



Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Intercept: (0, 0)

Horizontal asymptotes:

$$y = \pm \frac{\pi}{2}$$

Odd function Origin symmetry

Preface

Welcome to *Algebra & Trig*, Tenth Edition. We are excited to offer you a new edition with even more resources that will help you understand and master algebra and trigonometry. This textbook includes features and resources that continue to make *Algebra & Trig* a valuable learning tool for students and a trustworthy teaching tool for instructors.

Algebra & Trig provides the clear instruction, precise mathematics, and thorough coverage that you expect for your course. Additionally, this new edition provides you with **free** access to three companion websites:

- CalcView.com—video solutions to selected exercises
- CalcChat.com—worked-out solutions to odd-numbered exercises and access to online tutors
- LarsonPrecalculus.com—companion website with resources to supplement your learning

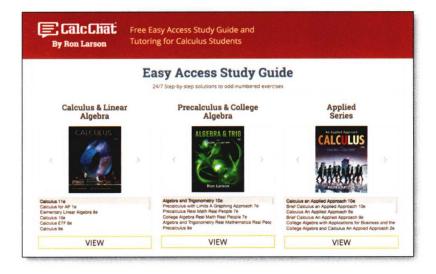
These websites will help enhance and reinforce your understanding of the material presented in this text and prepare you for future mathematics courses. CalcView® and CalcChat® are also available as free mobile apps.

Features



The website *CalcView.com* contains video solutions of selected exercises. Watch instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. The CalcView mobile app is available for free at the Apple® App Store® or Google PlayTM store. The app features an embedded QR Code® reader that can be used to scan the on-page codes and go directly to the videos. You can also access the videos at *CalcView.com*.





UPDATED 🗐 CalcChat®

In each exercise set, be sure to notice the reference to *CalcChat.com*. This website provides free step-by-step solutions to all odd-numbered exercises in many of our textbooks. Additionally, you can chat with a tutor, at no charge, during the hours posted at the site. For over 14 years, thousands of students have visited this site for help. The CalcChat mobile app is also available as a free download at the Apple® App Store® or Google PlayTM store and features an embedded QR Code® reader.

App Store is a service mark of Apple Inc. Google Play is a trademark of Google Inc. QR Code is a registered trademark of Denso Wave Incorporated.

REVISED LarsonPrecalculus.com

All companion website features have been updated based on this revision, plus we have added a new Collaborative Project feature. Access to these features is free. You can view and listen to worked-out solutions of Checkpoint problems in English or Spanish, explore examples, download data sets, watch lesson videos, and much more.

NEW Collaborative Project

You can find these extended group projects at *LarsonPrecalculus.com*. Check your understanding of the chapter concepts by solving in-depth, real-life problems. These collaborative projects provide an interesting and engaging way for you and other students to work together and investigate ideas.



REVISED Exercise Sets

The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant, and include topics our users have suggested. The exercises have been reorganized and titled so you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations. Error Analysis exercises have been added throughout the text to help you identify common mistakes.

Table of Contents Changes

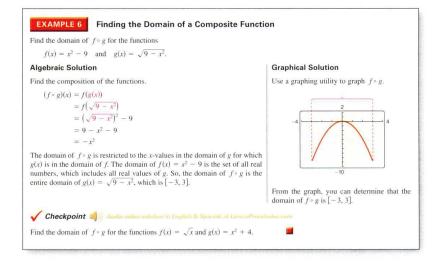
Based on market research and feedback from users, Section 8.5, The Complex Plane, has been added. In addition, examples on finding the magnitude of a scalar multiple (Section 8.3), multiplying in the complex plane (Section 8.6), using matrices to transform vectors (Section 10.2), and further applications of 2×2 matrices (Section 10.5) have been added.

Chapter Opener

Each Chapter Opener highlights real-life applications used in the examples and exercises.

Section Objectives

A bulleted list of learning objectives provides you the opportunity to preview what will be presented in the upcoming section.



Side-By-Side Examples

Throughout the text, we present solutions to many examples from multiple perspectives—algebraically, graphically, and numerically. The side-by-side format of this pedagogical feature helps you to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses many different learning styles.

Remarks

These hints and tips reinforce or expand upon concepts, help you learn how to study mathematics, caution you about common errors, address special cases, or show alternative or additional steps to a solution of an example.

Checkpoints

Accompanying every example, the Checkpoint problems encourage immediate practice and check your understanding of the concepts presented in the example. View and listen to worked-out solutions of the Checkpoint problems in English or Spanish at *LarsonPrecalculus.com*.

Technology

The technology feature gives suggestions for effectively using tools such as calculators, graphing utilities, and spreadsheet programs to help deepen your understanding of concepts, ease lengthy calculations, and provide alternate solution methods for verifying answers obtained by hand.

Historical Notes

These notes provide helpful information regarding famous mathematicians and their work.

Algebra of Calculus

Throughout the text, special emphasis is given to the algebraic techniques used in calculus. Algebra of Calculus examples and exercises are integrated throughout the text and are identified by the symbol f.

Summarize

The Summarize feature at the end of each section helps you organize the lesson's key concepts into a concise summary, providing you with a valuable study tool.

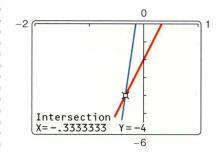
Vocabulary Exercises

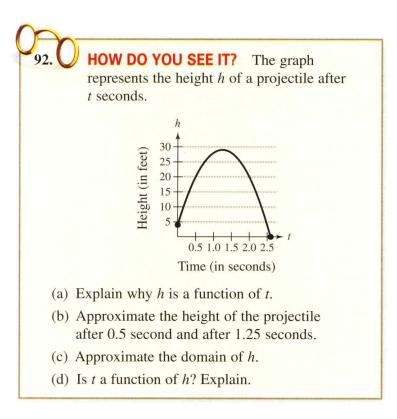
The vocabulary exercises appear at the beginning of the exercise set for each section. These problems help you review previously learned vocabulary terms that you will use in solving the section exercises. use a graphing utility to check that a solution is reasonable.
One way is to graph the left side of the equation, then graph the right side of the equation, and determine the point of intersection. For instance, in Example 2, if you graph the equations

$$y_1 = 6(x - 1) + 4$$
 The left side and

$$y_2 = 3(7x + 1)$$
 The right side

in the same viewing window, they intersect at $x = -\frac{1}{3}$, as shown in the graph below.





How Do You See It?

The How Do You See It? feature in each section presents a real-life exercise that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion or test preparation.

Project

The projects at the end of selected sections involve in-depth applied exercises in which you will work with large, real-life data sets, often creating or analyzing models. These projects are offered online at *LarsonPrecalculus.com*.

Chapter Summary

The Chapter Summary includes explanations and examples of the objectives taught in each chapter.

Acknowledgments

I would like to thank the many people who have helped me prepare the text and the supplements package. Their encouragement, criticisms, and suggestions have been invaluable.

Thank you to all of the instructors who took the time to review the changes in this edition and to provide suggestions for improving it. Without your help, this book would not be possible.

Reviewers of the Tenth Edition

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I would also like to thank the staff at Larson Texts, Inc. who assisted with proofreading the manuscript, preparing and proofreading the art package, and checking and typesetting the supplements.

On a personal level, I am grateful to my spouse, Deanna Gilbert Larson, for her love, patience, and support. Also, a special thanks goes to R. Scott O'Neil. If you have suggestions for improving this text, please feel free to write to me. Over the past two decades, I have received many useful comments from both instructors and students, and I value these comments very highly.

Ron Larson, Ph.D. Professor of Mathematics Penn State University www.RonLarson.com

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> Appendices

Appendix A: Errors and the Algebra of Calculus A1

Appendix B: Concepts in Statistics (online)*

Representing Data **B.1**

B.2 Analyzing Data

B.3 Modeling Data

Alternative Version of Chapter P (online)*

Operations with Real Numbers P.1

769

P.2 Properties of Real Numbers

P.3 Algebraic Expressions

Operations with Polynomials P.4

P.5 Factoring Polynomials

P.6 **Factoring Trinomials**

Answers to Odd-Numbered Exercises and Tests A9 Index A115 Index of Applications (online)*

^{*}Available at the text-specific website www.cengagebrain.com