NICHOLAS RESCHER

TOPICS IN PHILOSOPHICAL LOGIC

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SOCIOLOGY OF SCIENCE AND OF KNOWLEDGE,

AND ON THE MATHEMATICAL METHODS OF

SOCIAL AND BEHAVIORIAL SCIENCES

I take pleasure in dedicating this book to my students who have over the years been also my collaborators in logical research, and in particular to:

BAS VAN FRAASSEN

NEIL A. GALLAGHER

JAMES W. GARSON

BARBARA ANNE HUNT

RICHARD K. MARTIN

ROBERT K. MEYER

ANNE CROSS PELON

BRIAN SKYRMS

ERNEST SOSA

PREFACE

The aim of the book is to introduce the reader to some new areas of logic which have yet to find their way into the bulk of modern logic books written from the more orthodox direction of the mainstream of developments. Such a work seems to me much needed, both because of the intrinsic value and increasing prominence of the nonstandard sector of logic, and because this particular sector is of the greatest interest from the standpoint of philosophical implications and applications.

This book unites a series of studies in philosophical logic, drawing for the most part on material which I have contributed to the journal literature of the subject over the past ten years. Despite the fact that some of these essays have been published in various journals at different times, they possess a high degree of thematic and methodological unity. All of these studies deal with material of substantial current interest in philosophical logic and embody a fusion of the modern techniques of logical and linguistic-philosophical analysis for the exploration of areas of logic that are of substantial philosophical relevance.

Chap. VII on 'Venn Diagrams for Plurative Syllogisms' was written in collaboration with my student Mr. Neil A. Gallagher, and Chap. XIII on 'Topological Logic' was written in collaboration with my student Mr. James W. Garson. I am grateful to these gentlemen for agreeing to the inclusion of this material here. I express my appreciative thanks to Miss Dorothy Henle and Miss Judy Bazy for their help in preparing this material for the printer and in seeing it through the press. I should also like to thank Mr. Alasdair Urquhart for his help with the proofreading.

Pittsburgh, August, 1967

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CHAPTER I

RECENT DEVELOPMENTS IN PHILOSOPHICAL LOGIC

1. INTRODUCTION

The mainstream of the development of modern logic since the pioneering days from Boole to Frege has moved very decidedly in the direction of *mathematical* interests and applications. And, in fact, mathematics continues to the present day to occupy a central position on the logical stage. This may be illustrated – among many other ways – by the current prominence of what might be characterized as 'the arithmetical sector' of logic, including algorithmic theory, recursive functions, the calculi of lambda conversion, the logical theory of computability and of effective processes generally, among other components of lesser renown. The computer, and the whole host of technical issues that revolve about it, have had an enormous and reciprocally stimulative impact on recent work in logic. Results of great importance and interest continue to be obtained in this mathematical sector of logic, witness Paul J. Cohen's remarkable proof of the independence of the continuum hypothesis.

However, the continuing of this long-standing mathematical tendency has masked and obscured a highly significant cluster of developments in logic of a more recent vintage. The eventuation to which I allude is the phenomenal recent spurt of growth of logic in directions bearing on philosophical considerations. The last ten or fifteen years especially – though there were, to be sure, earlier stirrings – have seen the flourishing and accelerating growth of branches of logical theory developed specifically with such philosophical applications in mind. It is also worth noting that there is also a growing interest in the 'logic' of natural languages, particularly in the evaluation of the validity of reasoning conducted in such languages, rather than in the more formalized systems used, e.g., in mathematics. This has come about largely under the impetus of the 'ordinary language' school of philosophy.

The principal objective of the present chapter is to give a brief but synoptic survey of this important phase of the ongoing history of logic.

Moreover, I propose to offer some observations regarding the significance of these developments, and to give some consideration to the prospects that augur for the future.

2. A NOTABLE FEATURE OF THE CURRENT SITUATION IN LOGIC

In Appendix A an attempt has been made to construct a map of the terrain of logic as it appears at the present writing. In this enterprise we have not concerned ourselves with matters of detail or with the minutiae of alternative approaches, but have endeavored to give a somewhat gross overview of the 'big picture'.

The material of Appendix A can for the most part safely be left to speak for itself. However, one particular feature of the map will here be singled out for explicit consideration and discussion. I advert to the size, scope, richness, and diversity of category E ('Philosophical Developments'). This phenomenon is so striking as to warrant explicit remark all of itself. Moreover, material of substantially philosophical bearing and interest is by no means confined to this category. For virtually the whole of the subcategories A3 ('Unorthodox Modern Logic') and B ('Metalogic') cannot but also be regarded in this light, being of preeminently philosophical bearing. A very sizeable sector of current logic is thus clearly oriented in specifically philosophical directions. This fact is all the more striking when one considers it in an historico-bibliographical perspective.

In Appendix B we have given a concise and selective bibliography of recent literature of philosophical logic. In many or most cases, the works that have been listed are not only significant expository sources, but actually pioneering contributors to the specific topic at issue. This feature serves to bring out in a very forceful and striking way the *recentness* of the cultivation of the philosophical reaches of modern logic. The great bulk of work in this area has appeared in the last decade. The bibliography provides a clear indication not only of the lively activity on this particular sector of the subject, but also of the element of newness that is present here. The majority of its entries (41 of 68) represent publications of the 1960's.

3. PROSPECTS AND PORTENTS

We have noted as a significant recent tendency in the development of modern logic the extensive and energetic cultivation of philosophically oriented branches of the subject. In general terms, the prospect for the future seems clear. There is little if any room for doubt that this tendency will not only continue, but intensify and develop in the years ahead. I should like to offer a few observations as to what this means for philosophy, for logic, and for the relationship between the two.

For philosophy, the intensified cultivation of philosophical logic means, first of all, the creation of a tremendously valuable *opportunity*. With respect to a certain not insignificant class of philosophical problems, the instrumentalities are now in hand for dealing with the relevant issues in an exact, precise, incontrovertible, and essentially decisive manner. Beginning in the area of epistemology and ontology, this tendency to the precise and formally exact treatment of philosophically relevant problems has recently made its way into other areas: especially in the area of ethical and normative concepts (deontic logic, preference logic, the logic of action). In certain sectors of the subject, there is now a genuine prospect of a continuing, cumulative, and collaborative progress – of the sort that philosophy has long envied the sciences. This trend – which one cannot but regard as now established beyond retrogression – may be viewed as perhaps the major permanent heritage of logical positivism in promoting and popularizing the philosophical application of logical technique.

It should be stressed, on the other hand, that the phenomenon which we have just cast in the role of a valuable opportunity also has certain significant inherent dangers. The existence of a method of investigation that holds good promise of success in a given area of a subject exercises a potent magnetic influence in attracting attention and effort to this sector. In consequence a danger arises that attention may be diverted from those issues – generally of no less and frequently of much greater intrinsic interest and importance – that are not amenable to resolution by the instrumentalities and techniques at issue. Significant progress in the subject as a whole may thus come to be sacrificed in the course of securing minor achievements in some of its branches: workers may be diverted from fruitful efforts in the less tidy areas of philosophy only to deploy logico-analytical virtuosity on substantively trivial issues.

During the 1930's and early 1940's, a thoughtful observer might well have tended towards the conclusion that logic would break off from the ancient moorings that kept it joined to philosophy, and either link itself to mathematics, or (more probably) go its own way as an independent discipline. This development would have seemed only natural against the backdrop of the long series of special sciences which, following the lead of *philosophia naturalis*, cut themselves off from philosophy to set up as special sciences in their own right. It has by now become plain as a pikestaff that this – from the angle of philosophy much to be regretted – development will not come to pass. The phenomenon upon which our attention has here been centered, the extensive and intensive development of branches of logic of specifically philosophical applicability, will of itself assure a continuing close connection between these disciplines.

Finally we must consider the matter from the standpoint of logic itself. Here, alas, the outlook is not so unmixedly favorable as one might wish for. There is, I am convinced, nothing for it but that in the fullness of time there will increasingly come to be a fission in the subject. There will come to be an increasingly wide gap between mathematical and philosophical logic, a gulf only occasionally bridged over by a rare mind of more than ordinary capacity and versatility. I am firmly persuaded that this development, which cannot but be viewed as intrinsically unfortunate, is, in effect, inevitable in the long run. Its impact cannot be prevented: the most that can be done is to cushion against consequences of the most dire sort. Its seriousness can, I believe, be mitigated only by a resolute determination on the part of those responsible for the training of specialists in logic in departments of philosophy and of mathematics to insist that students being trained on either side of the divide also attain a thorough familiarity with the way in which things are done on the other side.

4. CONCLUSION

Our brief survey of the structure of modern logic has brought into clear focus a current trend of substantial significance: the increasingly flourishing growth of the philosophically oriented sector of logic in the recent past. We have scrutinized this trend and have endeavored to assess its significance for logic, for philosophy, and for the relationship between them. We are led inescapably to the view that – certain inherent liabilities