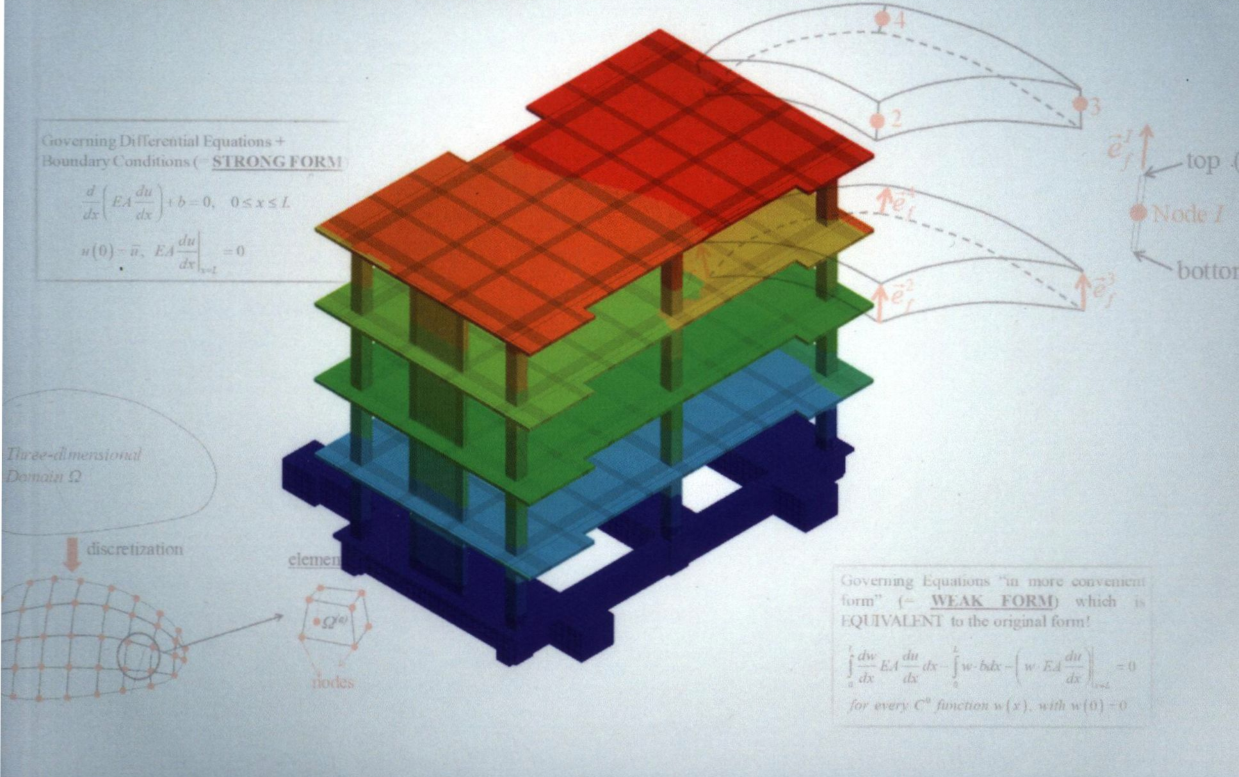
Ioannis Koutromanos



Fundamentals of Finite Element Analysis

Linear Finite Element Analysis

with website

WILEY

An Introductory Textbook Covering the Fundamentals of Linear Finite Element Analysis (FEA)

This book constitutes the first volume in a two-volume set that introduces readers to the theoretical foundations and the implementation of the finite element method (FEM). The first volume focuses on the use of the method for linear problems. A general procedure is presented for the finite element analysis (FEA) of a physical problem, where the goal is to specify the values of a field function. First, the strong form of the problem (governing differential equations and boundary conditions) is formulated. Subsequently, a weak form of the governing equations is established. Finally, a finite element approximation is introduced, transforming the weak form into a system of equations where the only unknowns are nodal values of the field function. The procedure is applied to one-dimensional elasticity and heat conduction, multi-dimensional steady-state scalar field problems (heat conduction, chemical diffusion, flow in porous media), multi-dimensional elasticity and structural mechanics (beams/shells), as well as time-dependent (dynamic) scalar field problems, elastodynamics and structural dynamics. Important concepts for finite element computations, such as isoparametric elements for multi-dimensional analysis and Gaussian quadrature for numerical evaluation of integrals, are presented and explained. Practical aspects of FEA and advanced topics, such as reduced integration procedures, mixed finite elements and verification and validation of the FEM are also discussed.

- Incorporates quantitative examples on one-dimensional and multi-dimensional FEA.
- Provides an overview of multi-dimensional linear elasticity (definition of stress and strain tensors, coordinate transformation rules, stress-strain relation and material symmetry) before presenting the pertinent FEA procedures.
- Discusses practical and advanced aspects of FEA, such as treatment of constraints, locking, reduced integration, hourglass control, and multi-field (mixed) formulations.
- Includes chapters on transient (step-by-step) solution schemes for time-dependent scalar field problems and elastodynamics/structural dynamics.
- Contains a chapter dedicated to verification and validation for the FEM and another chapter dedicated to solution of linear systems of equations and to introductory notions of parallel computing.
- Accompanied by a website hosting an open-source finite element program for linear elasticity and heat conduction, together with a user tutorial.

Fundamentals of Finite Element Analysis: Linear Finite Element Analysis is an ideal text for undergraduate and graduate students in civil, aerospace and mechanical engineering, finite element software vendors, as well as practicing engineers and anybody with an interest in linear finite element analysis.

IOANNIS KOUTROMANOS, PhD, is an Assistant Professor in the Department of Civil and Environmental Engineering at the Virginia Polytechnic Institute and State University. His research primarily focuses on the analytical simulation of structural components and systems under extreme events, with an emphasis on reinforced concrete, masonry and steel structures under earthquake loading. He has authored and co-authored research papers and reports on finite element analysis (element formulations, constitutive models, verification and validation of modeling schemes). He is a voting member of the joint ACI/ASCE Committee 447, *Finite Element Analysis of Reinforced Concrete Structures.*

with website

A companion website with additional resources is available at **www.wiley.com/go/koutromanos/linear**

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Fundamentals of Finite Element Analysis

Linear Finite Element Analysis

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Fundamentals of Finite Element Analysis

To my family

Preface

This text grew out of a set of notes, originally created for a graduate class on Finite Element (FE) analysis of structures that I teach in the Civil and Environmental Engineering department of Virginia Tech. The book attempts to provide an understanding or the salient features of linear FE analysis, beginning with the mathematics describing an actual physical problem and continuing with the introduction of the FE approximation and the calculation of pertinent arrays and vectors.

The mathematical description of various physical problems in this book (primarily elasticity and heat conduction) employs the *"strong form–weak form" paradigm.* Given a physical problem, the strong form (governing differential equations and boundary conditions) is established. Then, a weak form is obtained, into which the finite element approximation is subsequently introduced. I believe the strong form–weak form paradigm is a more efficient way of teaching the method than, for example, variational principles, because it is fairly straightforward and also more powerful from a teaching point of view. For example, the derivation of the weak form for solid and structural mechanics provides a formal proof of the principle of virtual work.

The book contains a total of 19 chapters and 4 appendices.

Chapter 1 constitutes an introduction, with an explanation of the necessity of numerical simulation, the essence of FE approximations and a brief presentation of the early history of FE analysis. Chapters 2 and 3 present the conceptual steps required for setting up and solving the FE equations of a one-dimensional physical problem. Specifically, Chapter 2 describes the process by which we can establish the governing differential equations and boundary conditions, which we collectively call the *strong form of a problem,* and how to obtain the *corresponding weak form,* which is an alternative mathematical statement of the governing physics, fully equivalent to the strong form. The weak form turns out to be more convenient for the subsequent introduction of the FE approximation. The latter is described in Chapter 3, wherein we see how the *discretization* of a domain into a finite element mesh and the stipulation of a *piecewise approximation* allows us to transform the weak form to a system of linear equations. Computational procedures accompanying finite element analysis, such as Gaussian quadrature, are also introduced in Chapter 3.

Chapter 4 establishes some necessary mathematical preliminaries for multidimensional problems, and Chapters 5 and 6 are focused on the strong form, weak form, and FE solution for scalar-field problems. Emphasis is laid on heat conduction, but other scalar field problems are discussed, such as flow in porous media, chemical diffusion, and inviscid, incompressible, irrotational fluid flow. The similarity of the mathematical structure of these problems with that of heat conduction and the expressions providing the various arrays and vectors for their FE analysis is established.

Chapter 7 introduces fundamental concepts for multidimensional linear elasticity. Any reader who wants to focus on multidimensional solid and structural mechanics should read this chapter. Chapters 8 and 9 present the FE solution of two-dimensional and three-dimensional problems in elasticity (more specifically, elastostatics, wherein we do not have inertial effects). Chapter 10 is devoted to some important practical aspects of finite element problems, such as the treatment of constraints, field-dependent natural boundaries (e.g., spring supports), and how to take advantage of symmetry in analysis. Chapters 11 and 12 are primarily focused on the application of the method to solid mechanics. Chapter 11 discusses advanced topics such as the use of incompatible modes to alleviate the effect of parasitic shear stiffness, and volumetric locking and its remedy through use of uniform- or selective-reduced integration. The side-effect of spurious zero-energy (hourglass) modes for uniform-reduced integration is identified, and a procedure to establish *hourglass control*, by means of artificial stiffness, is described. Chapter 12 is an introduction to multifield (mixed) finite element formulations, wherein the finite element approximation includes more than one field. The two-field weak form for solid mechanics, corresponding to the Hellinger-Reissner variational principle, and the three-field weak form, i.e. the Hu-Washizu principle, are also derived. Specific two-field finite element formulations are briefly discussed. A derivation of the *inf-sup condition* for displacement-pressure (*u-p*) mixed formulations is also provided. The chapter concludes with an introductory description of the family of assumed-enhanced strain methods.

Chapters 13 and 14 are focused on FE analysis of structural mechanics—beam problems and shell problems. Chapter 13, focused on beams, begins with the simplest type of beam (i.e., a member with a sectional law) and the two most popular kinematic theories for beams, the Euler-Bernoulli and the Timoshenko theory. The chapter then proceeds with more advanced—yet popular —approaches, such as the continuum-based beam concept, and its application to the treatment of curved structural members. A similar approach is followed for analysis of shells in Chapter 14.

The first 14 chapters are all focused on steady-state (i.e., static) problems, wherein the solution and governing equations are constant with time and there is no reason to introduce time as a variable. This assumption is abandoned in Chapter 15, which provides a treatment of time-dependent (dynamic) problems. The process to obtain the strong form and weak form, as well as the *semi-discrete FE equations*, is presented. The chapter heavily emphasizes elastodynamics and structural dynamics, but a treatment of time-dependent scalar field problems is also provided.

Chapters 16 and 17 are focused on the solution of time-dependent scalar-field (i.e., heat conduction, flow in porous media etc.) and vector-field (i.e., solid and structural dynamics) problems, respectively. The notion of transient (step-by-step) integration is introduced, and some popular step-by-step schemes for scalar field problems are presented in Chapter 16. Chapter 17 presents the most popular schemes for solid and structural dynamics. A discussion of accuracy and numerical stability is also provided for the various schemes.

Chapter 18 provides a discussion of verification and validation principles, as applied to FE analysis. Finally, Chapter 19 constitutes an overview of algorithms for the solution of linear systems of algebraic equations that arise in FE analysis.

Understanding the FE method requires a solid knowledge of calculus and matrix algebra. I assume that the reader has good familiarity with one-dimensional calculus (differentiation, integration, integration by parts). Several notions like Green's formula, divergence theorem, and Green's theorem for multidimensional analysis are briefly explained in Chapter 4. A brief Appendix A on Vector/Matrix algebra is also provided. I need to acknowledge that the text is occasionally characterized by an abuse of notation for the case of vectors. Specifically, the two alternative notations of a vector $\vec{b}$ (which is more appropriate for calculus) and $\{b\}$ (which is more appropriate for matrix algebra) are considered perfectly equivalent and may even appear together in several expressions. Hopefully, no confusion will arise from this notation abuse.

The analysis of *discretized systems* obtained after the introduction of a FE approximation involves a set of standard procedures, such as the gather-scatter operation and the associated assembly of the global coefficient array and right-hand-side vector. A separate appendix (Appendix B) is provided to elucidate the matrix analysis of discrete systems. Readers are strongly encouraged to read Appendix B before they proceed with any chapter involving the FE solution of physical problems. As a minimum, familiarity with the material in Sections B.1, B.2, and B.3 is required.

This book may at times adopt a "hands-on, practical approach," that is, focused on providing an understanding of how to formulate and implement finite elements for linear analysis, rather than emphasizing formal mathematical proofs. One example where a rigorous discussion may be lacking is the explanation of the convergence behavior of the FE method. I think that a perfectly rigorous approach would immensely increase the size of the book. Furthermore, there already are excellent references with a rigorous mathematical treatment. My own personal favorite is the book by T. Hughes (*The Finite Element Method—Linear Static and Dynamic Finite Element Analysis*, Dover, 2000). The book by Hughes can be used as a follow-up, supplementary reference to the present book. Another reference that is worth mentioning here is the book by J. Fish and T. Belytschko (*A First Course on Finite Elements*, Wiley, 2007), whose presentation style is—in my opinion—the best for introducing fundamental notions of FE analysis to students. The content of the Fish and Belytschko (F&B) book primarily corresponds to Chapters 1–6 of the present text. I have pursued a similar notation and approach to those of F&B, to ensure that a person who has already read the F&B book should be able to directly focus on Chapters 7 to 19 of this text.

Contrary to several existing textbooks on FE analysis, I have avoided relying on a description of the weak form as a "variational principle." Still, a brief discussion of variational principles for linear elasticity and the identification of the principle of minimum potential energy with the weak form of the elasticity problem is provided in Appendix C. Additionally, as mentioned above, several multifield (weak) forms in Chapter 12 are identified with corresponding variational principles.

The final appendix (Appendix D) provides the derivation of the transformation equations in the presence of rigid-body constraints. The equations pertain to transformation of nodal displacement vectors, coefficient (stiffness) matrices, and nodal force vectors. These equations are useful for Chapters 10, 13, and 14.

Finite element analysis is integrally tied with computational programming. For this reason, a companion webpage has been created for the book, providing a standalone FE analysis program, called *VT-FEA*, for elasticity and heat conduction problems. A tutorial (Appendix E) for the program (including the use of visualization tools for post-processing), together with sample input files and with the source code, are also provided in the webpage. The source code is written in Fortran, a very old language that is still very popular for scientific computation. To facilitate understanding of the source code, the companion webpage also includes a concise tutorial on Fortran programming.

The chapters in the book can be read sequentially, depending on the needs of the reader. A number of different courses on FE analysis can be supported with selected chapters. Several example, graduate-level courses that can be taught, together with the corresponding chapters, are listed below.

- Introduction to FE analysis (one semester): Chapters 1–6, parts of Chapter 7, Chapters 8–9, parts of Chapters 10 and 13.
- Finite elements for heat and mass transfer (one semester): Chapters 1–6, parts of Chapter 10, Chapters 15 and 16.
- Finite elements for applied elasticity and structural mechanics and dynamics (one semester): Chapters 1–4, Chapters 7–9, Parts of Chapters 10 and 11, Chapters 13–15, and Chapter 17.
- Advanced topics on linear FE analysis (one semester): Chapters 10–19.

I would like to thank the Civil and Environmental Engineering Department at Virginia Tech for giving me the opportunity to teach a course that supported the development of this text. Furthermore, I wish to thank Professor Chris Roy and Dr. James McClure for contributing one chapter each (Chapters 18 and 19, respectively) for the textbook, in areas of their expertise. I am grateful to the editorial office of Wiley, for supporting this effort and providing invaluable guidance whenever necessary. I also wish to thank my former student, Dr. Reza Moharrami, for preparing the Fortran tutorial provided in the companion webpage of this text. Special thanks are due to my graduate student Mehrnoush Farhadi for collaborating with me in the creation of the analysis program *VT-FEA*, which is included in the companion webpage, and to Mr. Japsimran Singh, for providing help with examples in Chapters 7, 11, and 13, and in Appendix B. Thanks are due to researchers and colleagues who kindly provided figures with examples of their finite element analyses, which are presented in Chapter 1. Additionally, I would like to thank Mr. Konstantinos Anagnostopoulos, Mr. Georgios Deskos, Dr. Alexia Kottari, Mr. Andreas Koutras, and Dr. Marios Mavros for reviewing early drafts of several chapters and for detecting typos and making helpful suggestions to improve the content of these chapters. Of course, any remaining errors in the text are my own responsibility.

I have been fortunate to be taught, advised, and mentored by several extraordinary instructors and researchers with expertise in FE analysis. I think it is my obligation to mention two of them, namely, Professors P.B. Shing (my doctoral advisor) and D. Benson, for helping me obtain a grasp of many of the topics presented in this text. In fact, early drafts of several of the descriptions provided in this text have originated from lecture notes that I took in classes taught by Professors Shing and Benson at UC San Diego. Obviously, notes of mine may not necessarily reflect the teaching philosophy and value of these classes, but rather my own (potentially inaccurate) interpretation of the various topics taught therein.

Finally, I wish to thank my parents, Antonios and Panagiota, and my siblings, Kostas, Christina, and Alexandros, for their support throughout the duration of graduate my studies and career in the United States. It is to them that I dedicate this book.

About the Companion Website

Don't forget to visit the companion website for this book:

www.wiley.com/go/koutromanos/linear

The companion website of this text contains a standalone, open-source, finite element analysis program, called VTFEA, for one-dimensional and multidimensional elasticity and heat conduction problems. The source code of the program (written in Fortran) and brief tutorials and sample input files are also provided in the website. These tutorials include the use of a graphical post-processor for generating plots of the obtained results. A brief tutorial on Fortran programming is also provided, to facilitate understanding of the source code.

Scan this QR code to visit the companion website.

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