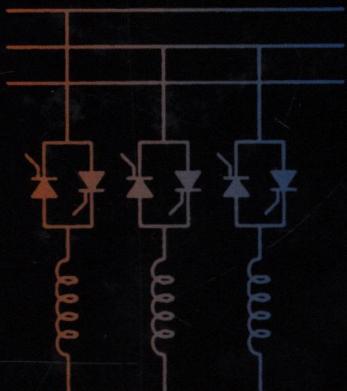
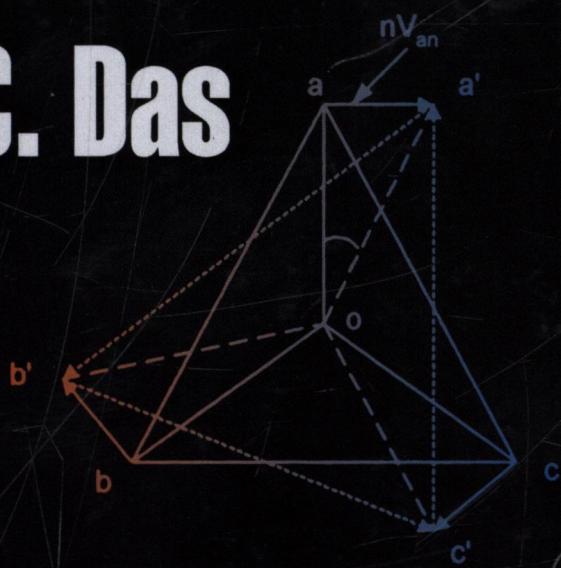


**Power Systems Handbook - Volume 2**

# Load Flow Optimization and Optimal Power Flow

**J.C. Das**



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## Volume 2

J.C. Das



CRC Press

Taylor & Francis Group

Boca Raton London New York

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Taylor & Francis Group, an **informa** business

CRC Press  
Taylor & Francis Group  
6000 Broken Sound Parkway NW, Suite 300  
Boca Raton, FL 33487-2742

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Printed at CPI on sustainably sourced paper

International Standard Book Number-13: 978-1-4987-4544-4 (Hardback)

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# Power Systems Handbook

## Load Flow Optimization and Optimal Power Flow

Volume 2

# Power Systems Handbook

Series Author

**J.C. Das**

*Power System Studies, Inc., Snellville, Georgia, USA*

Volume 1: Short-Circuits in AC and DC Systems:  
ANSI, IEEE, and IEC Standards

Volume 2: Load Flow Optimization and Optimal Power Flow

Volume 3: Harmonic Generation Effects Propagation and Control

Volume 4: Power System Protective Relaying

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## *Series Preface*

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This handbook on power systems consists of a set of four volumes. These are carefully planned and designed to provide the state-of-the-art material on major aspects of electrical power systems, short-circuit currents, load flow, harmonics, and protective relaying.

An effort has been made to provide a comprehensive coverage, with practical applications, case studies, examples, problems, extensive references, and bibliography.

The material is organized with sound theoretical base and its practical applications. The objective of creating this series is to provide the reader with a comprehensive treatise, which could serve as a reference and day-to-day application guide for solving the real-world problem. It is written for plasticizing engineers and academia, level of education upper undergraduate and graduate degrees.

Though there are published texts on similar subjects, this series provides a unique approach to the practical problems that an application engineer or consultant may face in conducting the system studies and applying it to varied system problems.

Some parts of the work are fairly advanced on a postgraduate level and get into higher mathematics. Yet, the continuity of the thought process and basic conceptual base are maintained. A beginner and advanced reader will equally benefit from the material covered. An undergraduate level of education is assumed, with fundamental knowledge of electrical circuit theory, rotating machines, and matrices.

Currently, power systems, large or small, are analyzed on digital computers with appropriate software packages. However, it is necessary to understand the theory and basis of these calculations to debug and decipher the results.

A reader may be interested only in one aspect of power systems and may like to purchase only one of the volumes of the series. Many aspects of power systems are transparent between different types of studies and analyses—for example, knowledge of short-circuit currents and symmetrical component is required for protective relaying, and knowledge of fundamental frequency load flow is required for harmonic analysis. Though appropriate references are provided, the material is not repeated from one volume to another.

The series is a culmination of the vast experience of the author in solving real-world problems in the industrial and utility power systems for the last more than 40 years.

Another key point is that the solutions to the problems are provided in Appendix D. A reader should be able to independently solve these problems after perusing the contents of a chapter, and then look back to the solutions provided, as a secondary help. The problems are organized, so that these can be solved with manual manipulations, without the help of any digital computer power system software.

It is hoped that the series will be a welcome addition to the current technical literature.

The author thanks Ms. Nora Konopka of CRC Press for her help and cooperation throughout the publication effort.

—J.C. Das



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## **Preface to Volume 2: Load Flow Optimization and Optimal Power Flow**

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This volume discusses the major aspects of load flow, optimization, optimal load flow, and culminating in modern heuristic optimization techniques and evolutionary programming.

In the deregulated environment, the economic provision of electrical power to consumers requires knowledge of so many related aspects—maintaining a certain power quality and load flow being the important aspects. Chapter 1 provides basic structures: security assessments, load estimation, forecasting, and nonlinear nature of load flow, requiring iterative techniques for optimum solutions. This is followed by Chapter 2 which goes into AGC and AFC concept, controls, and underfrequency load shedding.

Load flow over AC and HVDC transmission lines is somewhat unique in the sense that it much depends upon line length and in HVDC system the system configurations.

Nodal analysis techniques that have developed over the past many years for solution of the load flow problem are covered in Chapters 5 and 6. These discuss and compare almost all available load flow algorithms in practical use on commercial software packages.

Maintaining an acceptable voltage profile on sudden load impacts and contingency load flow problems, large induction and synchronous motor starting, their models and characteristics, and stability considerations are discussed in Chapter 7.

The related topic of reactive power control and voltage instability and impact on the acceptable voltages in the electrical power system is the subject of Chapter 8, followed by FACTS and SVC applications for transmission and distribution systems in Chapter 9.

The distribution systems can have phase unbalances that cannot be ignored. The symmetrical component analysis cannot be applied with prior phase unbalance. The techniques of phase coordinate methods for three-phase modeling and advanced models of three-phase transformers, with optimum locations of capacitor banks using dynamic modeling concepts, are provided in Chapter 10.

Chapters 11 and 12 cover the classical methods of optimization. Gradient methods, linear programming, dynamic programming, barrier methods, security- and environmental-constrained OPF, generation scheduling considering transmission losses, and unit commitment are discussed.

Finally, Chapter 13 provides an introduction to evolutionary programming, genetic algorithms, particle swarm optimization, and the like.

Thus, the subject of load flow, optimization, and optimal load flow is completely covered.

Many case studies and practical examples are included. The problems at the end of a chapter can be solved by hand calculations without resort to any computer software. Appendix A is devoted to calculations of line and cable constants and Appendix B provides solutions to the problems.

—J.C. Das



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## **Author**

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**J.C. Das** is an independent consultant, Power System Studies, Inc., Snellville, Georgia, USA. Earlier, he headed the Power System Analysis Department at AMEC Foster Wheeler for the last 30 years. He has varied experience in the utility industry, industrial establishments, hydroelectric generation, and atomic energy. He is responsible for power system studies, including short circuit, load flow, harmonics, stability, arc flash hazard, grounding, switching transients, and protective relaying. He conducts courses for continuing education in power systems and has authored or coauthored about 70 technical publications nationally and internationally. He is the author of the following books:

- *Arc Flash Hazard Analysis and Mitigation*, IEEE Press, 2012.
- *Power System Harmonics and Passive Filter Designs*, IEEE Press, 2015.
- *Transients in Electrical Systems: Analysis Recognition and Mitigation*, McGraw-Hill, 2010.
- *Power System Analysis: Short-Circuit Load Flow and Harmonics*, Second Edition, CRC Press, 2011.
- *Understanding Symmetrical Components for Power System Modeling*, IEEE Press, 2017.

These books provide extensive coverage, running into more than 3000 pages and are well received in the technical circles. His interests include power system transients, EMTP simulations, harmonics, passive filter designs, power quality, protection, and relaying. He has published more than 200 electrical power system study reports for his clients.

He has published more than 200 study reports of the power systems analysis addressing one problem or the other.

Mr. Das is a Life Fellow of the Institute of Electrical and Electronics Engineers, IEEE (USA), a Member of the IEEE Industry Applications and IEEE Power Engineering societies, a Fellow of the Institution of Engineering Technology (UK), a Life Fellow of the Institution of Engineers (India), a Member of the Federation of European Engineers (France), a Member of CIGRE (France), etc. He is a registered Professional Engineer in the States of Georgia and Oklahoma, a Chartered Engineer (C.Eng.) in the UK and a European Engineer (Eur. Ing.) in Europe. He received a meritorious award in engineering, IEEE Pulp and Paper Industry in 2005.

He earned a PhD in electrical engineering at Atlantic International University, Honolulu, an MSEE at Tulsa University, Tulsa, Oklahoma, and a BA in advanced mathematics and a BEE at Panjab University, India.



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