


volume 132

lecture notes in pure and applied mathematics



geometry and complex variables

edited by  
Salvatore Coen

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### about the editor . . .

SALVATORE COEN is Professor of Higher Geometry at the University of Bologna, Italy, where he has worked since 1976. The author of two books and several papers, and director and editor of the University of Bologna's *Seminars in Geometry* series, Dr. Coen serves as a reviewer for numerous journals, including *Mathematical Reviews*. A member of the American Mathematical Society, French Mathematical Society, and Italian Union of Mathematics, Dr. Coen received a degree (1967) in mathematics from the University of Pisa, Italy.

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# geometry and complex variables

proceedings of an international meeting on the occasion  
of the IX centennial of the University of Bologna

edited by  
Salvatore Coen  
*University of Bologna*  
*Bologna, Italy*



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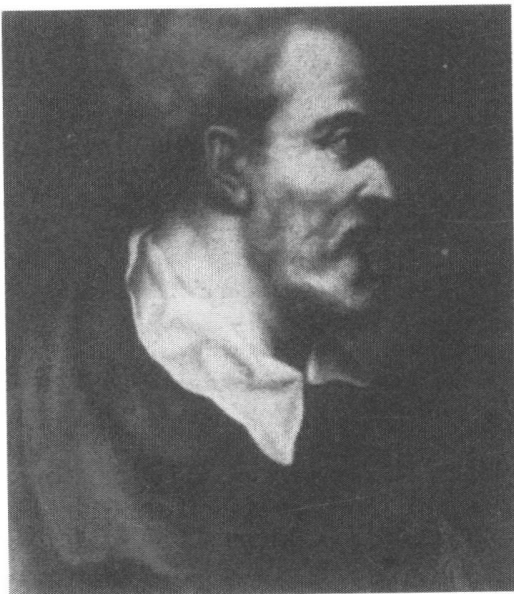
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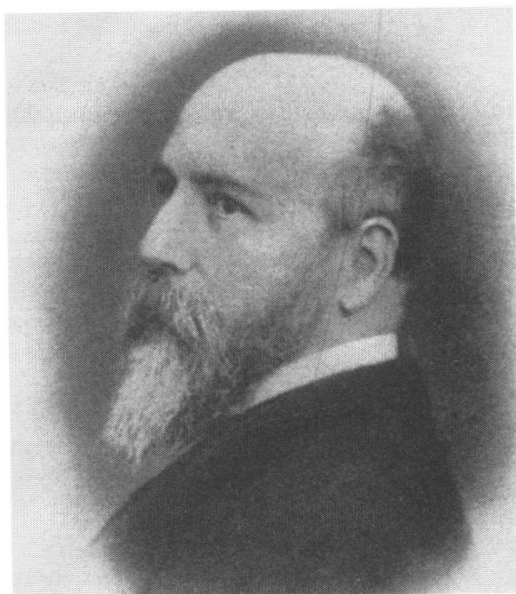
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**Girolamo Cardano ( 1501- 1576 )**



**Luigi Cremona ( 1830 - 1903 )**



**Federigo Enriques ( 1871 - 1946 )**



**Luigi Fantappiè ( 1901 - 1956 )**

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## Preface

In the year 1988, the University of Bologna celebrated its 900 years of existence dating back to 1088, the acknowledged founding date of this prestigious university known to its students and professors as “*Alma Mater Studiorum*.”

The commemoration lasted approximately two years and climaxed in September of 1988 when in Bologna’s Piazza Maggiore, the Magna Charta delle Università Europee was solemnly signed in the presence of more than 350 deans from some of the most important universities in the world. The festivities were marked with a series of events: scientific meetings, exhibits, artistic performances, conferring of honorary degrees, restoration of museums, grants, and awards.

It was, therefore, in this commemorative spirit that a committee\* was established for the purpose of organizing this meeting. The scientific committee consisted of professors M. Heins (University of Maryland, College Park), G.-C. Rota (Massachusetts Institute of Technology, Cambridge), E. Vesentini (Scuola Normale Superiore, Pisa), and S. Coen (Università di Bologna). The meeting was for the most part carried on within a period of approximately 12 months, from July 1988 to June 1989 and consisted of a series of 38 conferences, with a more intensive series in February 1989 and a historical session in March of the same year.

At the happy conclusion of the meeting on May 15, 1990, the Science Faculty of the University of Bologna conferred an honorary degree *honoris causa* in mathematics on Prof. J. J. Kohn of Princeton University for his outstanding scientific work.

### Purpose of the meeting

It was the aim of the meeting to bring together experts to present and discuss research based on the works of great mathematicians who have taught at the University of Bologna in the field of geometry and complex variables.

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\*The organizing committee consists of S. Coen, President (Università di Bologna), M. Manaresi, Vice President (Università di Salerno), G. Bolondi, Secretary (Università di Camerino), G. Forni, Assistant Secretary (Università di Bologna), U. Bottazzini (Università di Bologna), G. Menichetti (Università di Bologna), R. Musti (Università di Bologna), P. Salmon (Università di Bologna), C. Tinaglia (Università di Bologna), and A. Vaz Ferreira (Università di Bologna).

For a better understanding of the meeting's theme, it seems opportune to briefly recall some of the professors who have over the centuries been associated with the subject of mathematics in Bologna.

### Historical notes on mathematics in Bologna

Among the earliest names of importance, we find that of Luca Pacioli (Borgo San Sepolcro 1445, Roma 1514), lecturer *ad Mathematicam* in Bologna 1501-1502, and author of the famous work *De divina Proportione* and (in his time) the widely circulated *Summa de Arithmetica, Geometria, Proportioni et Proportionalità*. We must also mention Domenico Maria Novara (Ferrara 1454, Bologna 1504), astronomy lecturer in the period 1483-1504, famous for his accurate astronomical observations and his ability to interpret them, as well as for his role in guiding Copernicus during his Bologna period.

We must also bear in mind that for centuries (from the twelfth century) mathematics was being taught in Bologna by abacus teachers (*Maestri d'Abbaco*) and astrology lecturers.

One of the best periods for the study of mathematics in Bologna is that of the *Grandi Algebristi*, who worked in Bologna during the sixteenth century: Scipione dal Ferro (Bologna 1465, Bologna 1526), mathematics lecturer from 1496 to 1526, the fascinating personality Girolamo Cardano (Pavia 1501, Roma 1576), mathematician, philosopher, and lecturer in medicine at Bologna from 1561 to 1570, Ludovico Ferrari (Bologna 1522, Bologna 1565) lecturer in 1564-1565, and, finally, Rafael Bombelli, about whom we know very little and who died around 1575. The above mentioned, together with Tartaglia and others, had the courage and ability to look for and find solution formulas for third- and fourth-degree algebraic equations. In line with our interest in complex variables, it is worthwhile to take a moment to consider an important problem that came up at that time in the solution of the cubic equations  $x^3 = px + q$ . Let  $\Delta = q^2/4 - p^3/27$ , the discriminant of this equation. Scipione del Ferro and Tartaglia's solution formula

$$\sqrt[3]{q/2 + \sqrt{\Delta}} + \sqrt[3]{q/2 - \sqrt{\Delta}}$$

was without meaning if  $\Delta < 0$ . This situation, called the "irreducible case," is particularly important since it produces real distinct solutions. Cardano made some important observations on the subject, but it is to Bombelli that credit is generally assigned for the definitive solution of the difficult problem obtained by introducing imaginary numbers.

In the seventeenth century, the University of Bologna would once more boast of its great mathematicians: these, of course, dedicated much of their work to developing infinitesimal analysis and geometry. Pietro Antonio Cataldi (Bologna 1552, Bologna 1626), lecturer in mathematics from 1583 to 1626, is famous for having introduced and studied continued fractions. The most representative figure, however, is Bonaventura Cavalieri (Milano 1598, Bologna 1647), a follower and student of Galileo since his student days in Pisa, who was lecturer in mathematics at Bologna from 1629 to 1647. Recent important studies on Cavalieri, who is most famous for his work *Geometria indivisibilibus continuorum nova quadam ratione promota*, have come up with a complete picture of all his work. Pietro Mengoli (Bologna 1625 or 1626, Bologna 1686), a student of Cavalieri's and lecturer from 1650 to 1686, is usually remembered for his work *Geometria speciosa* and for contributions to series theory, to the computation of special integrals, and to the theory of logarithms. E. Giusti [G] wrote in reference to *Novae Quadraturae Arithmeticae* that Mengoli pushes the preciseness of concepts to and often above the limits of pedantry, defining each term, even the most commonly known and used, and never leaving the safe terrain of geometrically demonstrated propositions.

Even at the risk of seeming to deviate somewhat from our principal subject area, we must note that in the seventeenth century some great astronomers were *ad Mathematicam* lecturers in Bologna. Among these we find: Giovanni Antonio Magini (Padova 1555, Bologna 1618), lecturer from 1588 to 1618, about whom it is interesting to remember that he was chosen for Bologna over Galileo, whose theories he always implacably opposed, and Giandomenico Cassini (Pirinaldo, Imperia, 1625, Paris 1712), head of a well-known family of astronomers and geodesicists, lecturer from 1652 to 1669, when he transferred to Paris. Let us also remember Domenico Guglielmini (Bologna 1655, Padova 1710), the hydraulic engineering expert who from 1689 to 1699 was lecturer *ad Mathematicam*, which title was later changed to *ad Mathematicam hydrometricam*.

Among the mathematicians of the 1700s, we find Gabriele Manfredi (Bologna 1681, Bologna 1761) and Vincenzo Riccati (Castelfranco Veneto 1707, Castelfranco Veneto 1775), the first being well known for his studies on homogeneous differential equations of differential geometry and hydraulic problems, and the second belonging to an illustrious family of Italian mathematicians and recognized for his research on hyperbolic functions as well as for a famous mathematics treatise (on which he collaborated). The name of Maria Gaetana Agnesi (Milano 1718, Milano 1799) should also be mentioned since Pope Benedict XIV, secularly the Prospero Lambertini who did so much for studies in Bologna, offered her a mathematics lectureship in Bologna after she published her *Istituzioni analitiche*. There is no record, however, of her ever having taught at the university. While discussing the development of science in Bologna during the eighteenth century, it is important to mention the founding of the Istituto di Bologna through the work and patronage of Luigi Ferdinando Marsigli (1658-1730), a man of great intelligence, a soldier, and a scholar.

At this point, we find ourselves in a period considered critical for mathematics in Bologna. While we must await publication of recent research on the history of mathematics during the 1800s to give us more information, we can proceed to the situation of mathematical studies in Bologna immediately after the unification of Italy.

In 1860, Luigi Cremona (Pavia 1830, Roma 1903) was named Professor of Higher Geometry (Geometria Superiore) at the University of Bologna. We can, in fact, point out that Cremona was nominated personally by the then Minister of Education, Terenzio Mamiani della Rovere (Pesaro 1799, Roma 1885), who, with a view to improving studies in Bologna, at the same time nominated as professors the geologist Giovanni Capellini (Spezia 1833, Bologna 1922), Giosuè Carducci (Val di Castello, Lucca 1835, Bologna 1907; Nobel prize winner for literature in 1906), and the philosopher Bertrando Spaventa (Bomba, Chieti 1817, Napoli 1882). Then in 1862, we have Eugenio Beltrami (Cremona 1835, Roma 1900) as complementary algebra professor. The two young, well-qualified, and intelligent professors, Cremona and Beltrami, brought mathematical studies in Bologna to a turning point. Unfortunately, their time in Bologna was short. Cremona in 1867 went to the Istituto Tecnico Superiore (presently the Politecnico) in Milan and soon afterward to Rome, while Beltrami remained in Bologna for only a year, after which he went to Pisa for three years, returning to Bologna until 1873, with a professorship in *Meccanica Razionale*. Their time in Bologna, however, was scientifically productive for both. Cremona published the monograph *Preliminari di una teoria geometrica delle superficie* in 1866, following the *Introduzione ad una teoria geometrica delle curve piane* (1862), while Beltrami produced the work *Teoria degli spazi a curvatura costante* in 1868. From 1888 until their deaths, the University of Bologna included them as *Professori Onorari*. A rather difficult period followed for the study of mathematics in Bologna owing to the scarcity of professorships in this field; we must nevertheless remember Riccardo De Paolis (Roma 1854, Roma 1892), who taught algebra and analytic geometry in the years 1878-1880.

It is only toward 1880-1881, during what we can call the mathematics renaissance at the university, that three young, bright, and enthusiastic mathematicians in their thirties, namely, Salvatore

Pincherle (Trieste 1853, Bologna 1936), Cesare Arzelà (Santo Stefano di Magra 1847, Bologna 1912), and Luigi Donati (Fossombrone 1846, Bologna 1932) came to Bologna from the school of Pisa. None of these three transferred elsewhere. In 1896, Federigo Enriques (Livorno 1871, Roma 1946) was assigned the geometry professorship; in 1908, Pietro Burgatti (Cento, Ferrara 1868, Bologna 1938) was called to *Meccanica Razionale* and in 1922, Leonida Tonelli (Gallipoli 1885, Pisa 1946) was called to *Analisi Superiore*. In 1923 Enrico Bompiani (Roma 1889, Roma 1975) succeeded Enriques in Bologna until 1927. We can say that at the beginning of the nineteenth century, Bologna finally realized an esteemed position in mathematics. In 1924, at the International Congress of Mathematicians in Toronto, Salvatore Pincherle was named president of the International Mathematical Union. Later, Pincherle was assigned the task of organizing in Bologna the first international mathematicians' congress after World War I including the participation of German mathematics scholars. Pincherle, who had already founded the *Istituto di Matematica* a few years earlier as well as the *Unione Matematica Italiana* in 1923, becoming its first president, carried out his work for the international meeting very well. His work was faced with serious difficulties, but the Bologna International Congress was a success.

It seems noteworthy that years earlier, in 1911, the mathematician/philosopher Federigo Enriques had been assigned the organizing task for the Fourth International Philosophy Congress. We might state that by then Bologna had reached a position of high acclaim. Successors were all of high caliber. To replace the retiring Pincherle in 1928, we find Beppo Levi (Torino 1875, Rosario, Argentina 1961). For Tonelli, in 1930, Giuseppe Vitali (Ravenna 1875, Bologna 1932) arrived but stayed a very short time, passing away unexpectedly two years later in 1932; and in 1932, to replace Vitali we find Luigi Fantappiè (Viterbo 1901, Bagnaia, Viterbo 1956). In 1931 Beniamino Segre (Torino 1903, Roma 1977) was called. In this brief listing we have omitted assistants and assigned professors, some of whom had noteworthy names. Among the many we can mention for their work in geometry and complex variables: Ugo Amaldi (Verona 1875, Roma 1957), a student of Pincherle, assistant professor of algebra and analytic geometry in 1899-1900 and then of projective and descriptive geometry in 1900-1901, Francesco Severi (Arezzo 1879, Roma 1961), assistant professor of projective and descriptive geometry in 1902-1903, Annibale Comessatti (Udine 1866, Padova 1945), assigned professor for several years starting 1936, Oscar Chisini (Bergamo 1889, Milano 1967), being graduated from the University of Bologna and assigned projective geometry professor in the years 1921-1922. Concluding this brief study, we note that we must not undervalue the difficulties encountered periodically by mathematics professors in Bologna owing to the, at times dramatic, scarcity of teachers.

In the academic year 1937-1938, the *Professori Ordinari* of the *Istituto di Matematica* of Bologna were Pietro Burgatti, Luigi Fantappiè, Beppo Levi, and Beniamino Segre. Only an external malefic event could destroy such a solid institution. And it is with the year 1938, the eight hundred and fiftieth anniversary of the university's founding, a year of trauma and change for the *Istituto di Matematica*, that we end this brief account.

Of the splendid period 1880-1938, we have only very briefly traced the university's development in mathematics, since it is amply and very well described in the various papers of this meeting.

For the history of mathematics at the University of Bologna up to the scientific activity in Bologna by Pincherle, Arzelà, and Donati, one can refer to the work of Ettore Bortolotti [B].

### Outline of the contents of this volume

Many of the contributions in this book are works of pure mathematical research, while others represent research on the history of mathematics, and some can be considered substantial survey papers.