

Advances in Mechanics and Mathematics 32

Brian Straughan

Convection with Local Thermal Non-Equilibrium and Microfluidic Effects

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ISSN 1571-8689

ISSN 1876-9896 (electronic)

Advances in Mechanics and Mathematics

ISBN 978-3-319-13529-8

ISBN 978-3-319-13530-4 (eBook)

DOI 10.1007/978-3-319-13530-4

Library of Congress Control Number: 2015930520

Mathematic Subject Classification: 76E06 76E30 76S05 76T30 76R50 35Q30 35Q35

Springer Cham Heidelberg New York Dordrecht London

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Printed on acid-free paper

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Advances in Mechanics and Mathematics

Volume 32

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Driven by elaborate modern technological applications, the relationship between mathematics and mechanics is continually developing. The burgeoning number of specialized journals has generated an ever growing duality gap between the partners. *Advances in Mechanics and Mathematics* is a series intending to bridge the gap by providing a platform for the publication of interdisciplinary content with rapid dissemination of monographs, graduate texts, handbooks, and edited volumes, on the state-of-the-art research in the broad area of modern mechanics and applied mathematics. Topics with multi-disciplinary range, such as duality, complementarity and symmetry in mechanics, mathematics, and physics, are of particular interest.

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To
Cole, Caleb and Amelie

Preface

This book is devoted to an account of theories of thermal convection which involve local thermal non-equilibrium (LTNE) effects, or are particularly important in a microfluidic situation. The term “local thermal non-equilibrium” refers to thermal convection in a fluid saturated porous material where the fluid temperature and the temperature of the solid skeleton may be different. Microfluidics refers to fluid dynamics on a small scale which may involve thermal convection in a clear fluid, or thermal convection in a fluid saturated porous medium. The areas of microfluidics and nanofluidics are very topical at present.

This is not an attempt to survey the area of convection with local thermal non-equilibrium effects, nor that of convection in a microfluidic scenario. Both topics are extremely popular research areas and such a survey would be a gargantuan task. For example, if one inserts “local thermal non-equilibrium” in the Springer query box, 12,197 entries are found, on 30th August, 2014. Likewise if one enters the same expression in the query box of Science Direct, 123,332 entries are found, on 30th August, 2014. This book is simply an account of what I believe is an appropriate collection of subjects in a very topical area.

Chapters 2–7 deal specifically with LTNE effects whereas chapter 8 contains work with LTNE effects and some microfluidic work employing a single temperature. Chapters 9–15 concentrate mostly on microfluidic situations where a single temperature field is employed although section 15.4 is concerned with LTNE. Sections 6.1, 6.2, 9.4, 12.3, 13.2, 13.3, 14.2, 14.3, 14.5, 15.3 and 15.4 contain new material and/or new numerical results which I believe are not available elsewhere.

I should like to thank three anonymous referees for pointed and very useful comments which led to improvements in this book. It is a pleasure to thank Achi Dosanjh of Springer for her advice with editorial matters. I should also like to thank Jeff Taub and Suresh Kumar of Springer for their help with Latex and production matters.

This research was in part supported by a grant from the Leverhulme Trust, “Tipping points: mathematics, metaphors and meanings”, reference number F/00128/BF.

Durham

Brian Straughan

Contents

1	Introduction	1
1.1	Microfluidics, Local Thermal Non-equilibrium	1
1.1.1	Applications, Examples	1
1.1.2	Notation, Definitions	8
1.1.3	Overview	14
1.2	The Navier–Stokes Equations for Incompressible Fluid Flow	15
1.2.1	The Balance of Energy and the Boussinesq Approximation	16
1.3	The Darcy Model	21
1.3.1	The Porous Medium Equation	22
1.4	Anisotropic Darcy Model	23
1.5	The Forchheimer Model	25
1.5.1	Quadratic Forchheimer Model	26
1.5.2	Cubic Forchheimer Model	26
1.5.3	Generalized Forchheimer Model	26
1.5.4	Teng–Zhao Model	27
1.6	The Brinkman Model	28
1.6.1	Brinkman’s Equation	28
1.6.2	Alternative Brinkman Equation	28
1.6.3	Generalizations	29
1.7	Inertia Coefficient	30
1.7.1	Darcy Equations	31
1.7.2	Quadratic Forchheimer Equations	31
1.7.3	Cubic Forchheimer Equations	31
1.7.4	Brinkman Equations	32
1.7.5	Anisotropic Inertia Coefficient	32
1.8	Equations for Other Fields	34
1.8.1	Temperature Equations	34
1.8.2	Average Temperature	35
1.8.3	Darcy Equations for Thermal Convection	36
1.8.4	Forchheimer Equations, Thermal Convection	36
1.8.5	Cubic Forchheimer Thermal Convection	37

1.8.6	Brinkman Equations for Thermal Convection	37
1.8.7	Salt Field	38
1.8.8	Soret Effect	40
1.8.9	Many Salt Fields	42
1.9	Boundary Conditions	43
1.9.1	Slip Boundary Conditions	45
2	Thermal Convection with LTNE	49
2.1	Stability and Symmetry	49
2.1.1	Classical Bénard Convection	49
2.1.2	Symmetric Operators	51
2.2	Darcy Theory	53
2.2.1	Linear Instability	54
2.2.2	Nonlinear Stability	57
2.3	Anisotropy	59
2.4	Forchheimer Theory	61
2.4.1	Global Nonlinear Stability	62
2.5	Brinkman Theory	64
2.5.1	Global Nonlinear Stability	66
3	Rotating Convection with LTNE	69
3.1	Rotating Thermal Convection, Single Temperature	69
3.2	Rotation with Darcy Theory	72
3.2.1	Linear Instability	72
3.2.2	Nonlinear Stability	75
3.3	Rotation with Vadasz–Darcy Theory	76
3.4	Rotation with Vadasz–Brinkman Theory	77
4	Double Diffusive Convection with LTNE	79
4.1	Darcy Theory with Inertia	80
4.2	Brinkman Theory with Inertia	82
4.3	Double Diffusion with Rotation	83
4.4	Double Diffusion with Reaction	85
5	Vertical Porous Convection with LTNE	87
5.1	Vertical Layer	88
5.1.1	Global Stability	90
5.1.2	Nonlinear Stability for All Rayleigh Numbers	92
6	Penetrative Convection with LTNE	95
6.1	Quadratic Density Model	97
6.2	Internal Heat Sources	99
6.2.1	Uniform Heat Generation	99
6.2.2	Internal Heat Source Penetrative Convection	100

7	LTNE and Multi-layers	103
7.1	Local Thermal Non-equilibrium Equations	103
8	Other Convection and Microfluidic Scenarios	113
8.1	Natural Convection Polymerase Chain Reaction	113
8.2	Heat Flux Boundary Conditions	114
8.2.1	Heat Flux Both Boundaries	114
8.2.2	Heat Flux One Boundary	116
8.3	LTNE Convection and Horizontal Flow	117
8.3.1	Horizontal Pressure Gradient	117
8.3.2	Uniform Horizontal Flow	118
8.4	Viscoelastic LTNE Convection	119
8.4.1	Oldroyd-B Fluid, Darcy Material	119
8.4.2	Oldroyd-B Fluid, Double Diffusive Convection	120
8.4.3	Model Justification	121
8.5	LTNE Thermovibrational Convection	121
8.6	Hot Fluid Injection	123
9	Convection with Slip Boundary Conditions	127
9.1	Thermal Convection, Slip Boundary Conditions	127
9.2	Poiseuille Flow, Slip Boundary Conditions	132
9.3	Poiseuille Flow in a Porous Medium	134
9.4	Thermal Slip	136
9.5	Other Slip Problems	141
10	Convection in a Porous Layer with Solid Partitions	143
10.1	Multilayer Convection	143
10.1.1	Darcy Porous Layers	143
10.1.2	Brinkman Layers	149
11	Convection with Protruding Baffles	155
11.1	Vertical Partition	155
11.2	Horizontal Partition	159
12	Anisotropic Inertia Effect	161
12.1	Introduction	161
12.2	Thermosolutal Convection	162
12.2.1	Heated Below, Salted Above	165
12.3	Heated and Salted Below	167
12.3.1	Deductions	171
12.3.2	Deductions When $\ell = 1$	172
12.3.3	Deductions When $\ell \neq 1$	173

13	Bidispersive Porous Media	183
13.1	Double Porosity	183
13.2	Thermal Convection in Bidispersive Porous Media	185
13.2.1	Instability	188
13.2.2	Nonlinear Stability	190
13.2.3	Numerical Results	192
13.2.4	Variation of σ_f and Da_f	192
13.2.5	Variation of σ_f and H	192
13.3	Darcy Bidispersive Porous Media	193
13.3.1	Linear Instability for Darcy Theory	196
13.3.2	Nonlinear Stability for Darcy Theory	198
13.3.3	Numerical Results for Darcy Theory	200
13.4	Tridispersive Media	203
14	Resonance in Thermal Convection	205
14.1	Resonant Penetrative Convection in a Fluid	205
14.2	Resonant Penetrative Convection in a Porous Medium	208
14.2.1	Nonlinear Density, Heat Source Model	208
14.2.2	Linear Instability Analysis	211
14.2.3	Global Nonlinear Stability Analysis	212
14.2.4	Oscillatory Behaviour Observed	214
14.3	Resonance Linear Heat Source	216
14.3.1	Linear Instability	219
14.3.2	Global Nonlinear Stability	219
14.3.3	Cellular Instability Structure	220
14.4	Triple Resonance	227
14.4.1	The Model	227
14.4.2	Instability	229
14.4.3	Global Nonlinear Stability	230
14.5	Resonance with Variable Gravity	231
15	Thermal Convection in Nanofluids	237
15.1	Heat Transfer Enhancement in Nanofluids	237
15.2	The Tzou Nanofluid Model	238
15.3	Convection with Heat Wave Theories	240
15.3.1	Cattaneo–Fox Law	241
15.3.2	Cattaneo–Christov Law	246
15.3.3	Cattaneo Theories and Porous Materials	251
15.4	LTNE Cattaneo Solid, Fourier Fluid	253
15.4.1	LTNE–Cattaneo Model	254
15.4.2	Linear Instability	256
15.4.3	Global Nonlinear Stability	257
15.4.4	Conclusions for LTNE–Cattaneo Convection	259
15.5	Cattaneo Double Diffusion	264
15.5.1	Cattaneo–Christov Model	265
15.5.2	Linear Instability	269
15.6	Green–Naghdi Nanofluid Model	271

15.7 Nield–Kuznetsov Nanofluid Model 277

15.8 Generalizations 279

 15.8.1 Extended Green–Naghdi Model 279

 15.8.2 Slip Boundary Conditions 280

References 281

Index 311

Chapter 1

Introduction

1.1 Microfluidics, Local Thermal Non-equilibrium

1.1.1 Applications, Examples

This book focusses on thermal convection problems which are likely to be of interest in microfluidic situations, i.e. where the dimensions of the spatial configuration of the phenomenon are very small, although not exclusively so since some of the topics will be of interest in their own right at the macro scale.

There are a variety of physical mechanisms which will undoubtedly have a major effect on thermal convection or fluid flows in general when the spatial dimensions of the problem are small. One area involving heat flow is that of second sound, the mechanism whereby temperature travels as a wave, which in itself is also a topic of increasing attention. In particular, as modern technology is creating smaller and smaller devices, the phenomenon of temperature travelling as a wave becomes increasingly important, especially in metallic-like solids. Pilgrim *et al.* [343] develop a mathematical model for finite speed heat transport in semiconductor devices and they observe that, ... the “hyperbolic description will become increasingly important as device dimensions move even further into the deep sub-micron regime”. Since it is believed finite speed heat propagation is important in certain metallic material situations, we believe it is worthwhile considering this aspect in thermal convection flows in porous metallic foams, especially if the device dimensions are small. Various occurrences of finite speed heat transport are reviewed in the book by Straughan [425]. As he points out, most theoretical work involves the model proposed by Cattaneo in [79] to govern the behaviour of the heat flux and the temperature field. The history of the Cattaneo theory is discussed in detail in [425], chapter 1, where he also notes that a similar model, but in dielectric theory, was proposed earlier by Dario Graffi [165]. In this book we do consider second sound effects in thermal convection. Attention is paid to second sound and other physical effects for thermal convection in both a clear fluid and in a fluid saturated porous medium.

Porous media is a subject well known to everyone. Such materials occur everywhere and influence all of our lives. There are numerous types of porous media and almost limitless applications of and uses for such media. The theory of porous media is driven by the need to understand the nature of the many such materials available and to be able to use them in an optimum way.

A key terminology in the theory of porous media is the concept of *porosity*. The porosity is the ratio of the void fraction in the porous material to the total volume occupied by the porous medium. The void fraction is usually composed of air or some other liquid and since both liquids may be described as fluids we define the *porosity* at position \mathbf{x} and time t , $\epsilon(\mathbf{x}, t)$ by

$$\epsilon = \frac{\text{fluid volume}}{\text{total volume of porous medium}}. \quad (1.1)$$

Clearly, $0 \leq \epsilon \leq 1$. However, in mundane situations ϵ may be as small as 0.02 in coal or concrete, see e.g. [312], whereas ϵ is close to 1 in some animal coverings such as fur or feathers, [117], or in man-made high porosity metallic foams, [56, 189, 240, 497].

We include photographs of some well known porous materials. Figure 1.1 displays wet sand which is a very well known porous material but one with a relatively low porosity. Figure 1.2 shows natural sandstone found on a beach. This natural sandstone has a higher porosity than sand. Figure 1.3 shows lava from Mount Etna in Sicily, and this lava is another type of porous rock. In figure 1.4 we show a naturally occurring aggregate found on a beach. This aggregate also has a higher porosity than sand. The photograph in figure 1.5 displays concrete which has been weathered by the sea. The concrete has a low porosity. Figure 1.6 shows animal fur which is a good example of a porous medium with high porosity, i.e. porosity close to 1. Figure 1.7 displays another type of rock but one which is highly anisotropic. Figure 1.8 displays a highly grained piece of wood (oak). This is another example which shows that a porous medium may be highly anisotropic. In fact, wood is essentially isotropic in all directions orthogonal to the grain. The anisotropy is clearly evident in the grain direction. Anisotropy such as this where one direction is very different from those directions orthogonal is known as transverse isotropy. Figure 1.9 is a schematic picture of a bidisperse or a double porosity porous medium. The large gaps between the dark objects reveal a macro porosity but the darker objects themselves are composed of small spheres and have between them a micro porosity. Such materials may be man made or can occur naturally, cf. [445] for the latter case.

In addition to these we can cite other examples of porous media, such as biological tissues, e.g. bone, skin; building materials such as sand, cement, plasterboard, brick; man-made high porosity metallic foams such as those based on copper oxide or aluminium, and other materials in everyday use such as ceramics. The types of porous materials we can think of is virtually limitless.

Applications of porous media in real life and their connection to microfluid flows are likewise very many. We could list many, but simply quote some to give an idea of the vastness of porous media theory. Use of copper based foams and other porous

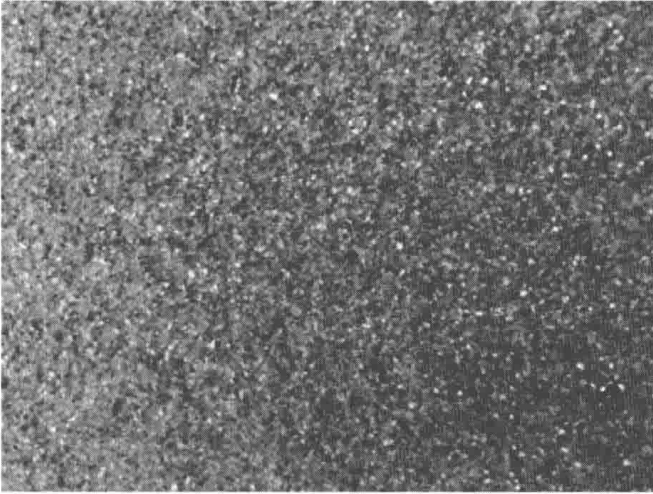


Fig. 1.1 Sand. Photograph taken on Seaham beach, March 2014.

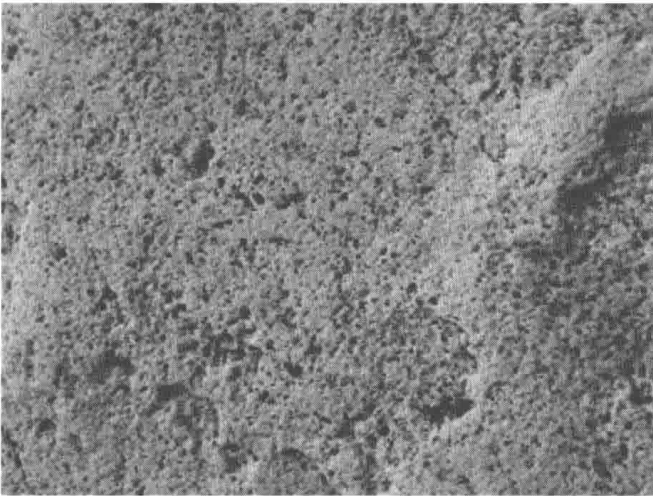


Fig. 1.2 Natural sandstone. Photograph taken on Seaham beach, March 2014.

materials in heat transfer devices such as heat pipes used to transfer heat from such as computer chips is a field influencing everyone, see e.g. [57, 189, 269, 312, 497]. Likewise, porous media are prevalent in combustion heat transfer devices where the porous medium is employed with a liquid fuel in a porous combustion heater, see [200]. Global warming is very topical and porous media are involved there in connection with topics such as ice melting, or carbon dioxide storage, see e.g. [55, 76, 77, 183]. Many foodstuffs are porous materials. Modern technology is involved in such as microwave heating, [112], or drying of foods or other natural materials, see e.g. [500, 501]. Porous media have application in storage of energy or natural convection within the upper region of the Earth, [331, 499]. The latter areas being of particular interest in the field of renewable energy. There are many

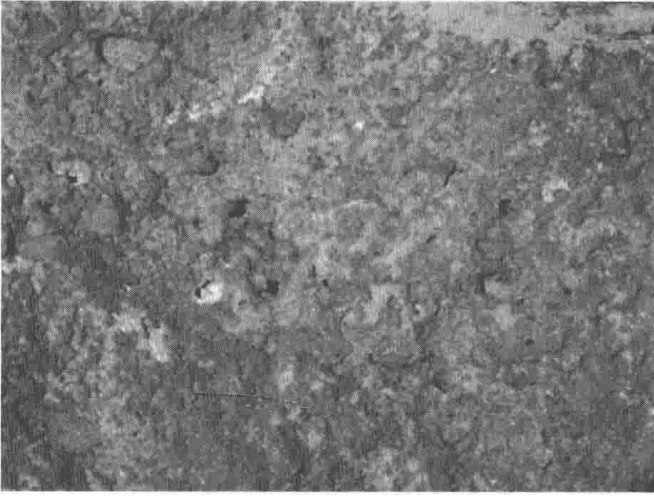


Fig. 1.3 Lava from Mount Etna, Sicily. Photograph taken at Capomulini, June 2007.



Fig. 1.4 Natural aggregate. Photograph taken on Seaham beach, March 2014.

other diverse application areas of porous materials, such as heat retention in birds or animals, [117], bone modelling, [125], or the manufacture of composite materials, increasingly in use in aircraft or motor car production, see e.g. [108].

Very much connected with thermal convection on a micro-scale are convective flow problems in a porous medium where the fluid temperature, T_f , may be different from the solid skeleton temperature, T_s . Such problems of thermal convection are being increasingly studied. This situation where the two temperatures may be different is usually referred to as local thermal non-equilibrium, abbreviated to LTNE. One of the driving reasons for the increased attention of LTNE flows in porous media is the numerous amount of applications of this area in real life. For example, there are applications in tube refrigerators in space, [19]; in nanofluid flows,

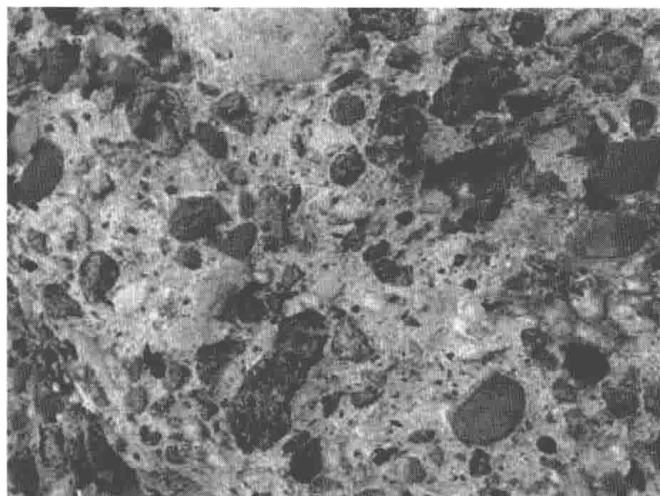


Fig. 1.5 Concrete, weathered by the sea. Photograph taken on Seaham beach, March 2014.



Fig. 1.6 Animal fur is a good example of a high porosity material, as seen in this cat.

[320, 420, 425], chapter 8; in fuel cells [102]; in resin flow, important in processing composite materials, [108]; in nuclear reactor maintenance, [137]; in heat exchangers, [107, 269]; in microwave ablation of the liver, [208]; in biological tissue analysis, [493]; in flows in microchannels, [215]; in flow and heat transfer in porous metallic foams, [189, 240, 241, 269]; in thermovibrational filtration, [388, 389], in textile transport, [486]; and in convection in stellar atmospheres, cf. [425], chapter 8, [426]. An interesting paper analysing various causes of local thermal non-equilibrium situations is that of [470].

Continuum theories for local thermal non-equilibrium effects on flow in porous materials appear to have started in the late 1990's, cf. the work of [291, 307], and [340], and instability in thermal convection taking into account LTNE effects was