Advances in Mechanics and Mathematics 32

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Convection with Local Thermal Non-Equilibrium and Microfluidic Effects

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Brian Straughan
Department of Mathematical Sciences
Durham University
Durham
UK

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To
Cole, Caleb and Amelie

Preface

This book is devoted to an account of theories of thermal convection which involve local thermal non-equilibrium (LTNE) effects, or are particularly important in a microfluidic situation. The term "local thermal non-equilibrium" refers to thermal convection in a fluid saturated porous material where the fluid temperature and the temperature of the solid skeleton may be different. Microfluidics refers to fluid dynamics on a small scale which may involve thermal convection in a clear fluid, or thermal convection in a fluid saturated porous medium. The areas of microfluidics and nanofluidics are very topical at present.

This is not an attempt to survey the area of convection with local thermal non-equilibrium effects, nor that of convection in a microfluidic scenario. Both topics are extremely popular research areas and such a survey would be a gargantuan task. For example, if one inserts "local thermal non-equilibrium" in the Springer query box, 12,197 entries are found, on 30th August, 2014. Likewise if one enters the same expression in the query box of Science Direct, 123,332 entries are found, on 30th August, 2014. This book is simply an account of what I believe is an appropriate collection of subjects in a very topical area.

Chapters 2–7 deal specifically with LTNE effects whereas chapter 8 contains work with LTNE effects and some microfluidic work employing a single temperature. Chapters 9–15 concentrate mostly on microfluidic situations where a single temperature field is employed although section 15.4 is concerned with LTNE. Sections 6.1, 6.2, 9.4, 12.3, 13.2, 13.3, 14.2, 14.3, 14.5, 15.3 and 15.4 contain new material and/or new numerical results which I believe are not available elsewhere.

I should like to thank three anonymous referees for pointed and very useful comments which led to improvements in this book. It is a pleasure to thank Achi Dosanjh of Springer for her advice with editorial matters. I should also like to thank Jeff Taub and Suresh Kumar of Springer for their help with Latex and production matters.

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Durham *Brian Straughan*

Contents

Chapter 1
Introduction

1.1 Microfluidics, Local Thermal Non-equilibrium

1.1.1 Applications, Examples

This book focusses on thermal convection problems which are likely to be of interest in microfluidic situations, i.e. where the dimensions of the spatial configuration of the phenomenon are very small, although not exclusively so since some of the topics will be of interest in their own right at the macro scale.

There are a variety of physical mechanisms which will undoubtedly have a major effect on thermal convection or fluid flows in general when the spatial dimensions of the problem are small. One area involving heat flow is that of second sound, the mechanism whereby temperature travels as a wave, which in itself is also a topic of increasing attention. In particular, as modern technology is creating smaller and smaller devices, the phenomenon of temperature travelling as a wave becomes increasingly important, especially in metallic-like solids. Pilgrim *et al.* [343] develop a mathematical model for finite speed heat transport in semiconductor devices and they observe that, ... the "hyperbolic description will become increasingly important as device dimensions move even further into the deep submicron regime". Since it is believed finite speed heat propagation is important in certain metallic material situations, we believe it is worthwhile considering this aspect in thermal convection flows in porous metallic foams, especially if the device dimensions are small. Various occurrences of finite speed heat transport are reviewed in the book by Straughan [425]. As he points out, most theoretical work involves the model proposed by Cattaneo in [79] to govern the behaviour of the heat flux and the temperature field. The history of the Cattaneo theory is discussed in detail in [425], chapter 1, where he also notes that a similar model, but in dielectric theory, was proposed earlier by Dario Graffi [165]. In this book we do consider second sound effects in thermal convection. Attention is paid to second sound and other physical effects for thermal convection in both a clear fluid and in a fluid saturated porous medium.

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Porous media is a subject well known to everyone. Such materials occur everywhere and influence all of our lives. There are numerous types of porous media and almost limitless applications of and uses for such media. The theory of porous media is driven by the need to understand the nature of the many such materials available and to be able to use them in an optimum way.

A key terminology in the theory of porous media is the concept of *porosity*. The porosity is the ratio of the void fraction in the porous material to the total volume occupied by the porous medium. The void fraction is usually composed of air or some other liquid and since both liquids may be described as fluids we define the *porosity* at position $\mathbf{x}$ and time t, $\varepsilon(\mathbf{x},t)$ by

$$\varepsilon = \frac{\text{fluid volume}}{\text{total volume of porous medium}}. \tag{1.1}$$

Clearly, $0 \leq \varepsilon \leq 1$. However, in mundane situations ε may be as small as 0.02 in coal or concrete, see e.g. [312], whereas ε is close to 1 in some animal coverings such as fur or feathers, [117], or in man-made high porosity metallic foams, [56, 189, 240, 497].

We include photographs of some well known porous materials. Figure 1.1 dislays wet sand which is a very well known porous material but one with a relatively low porosity. Figure 1.2 shows natural sandstone found on a beach. This natural sandstone has a higher porosity than sand. Figure 1.3 shows lava from Mount Etna in Sicily, and this lava is another type of porous rock. In figure 1.4 we show a naturally occurring aggregate found on a beach. This aggregate also has a higher porosity than sand. The photograph in figure 1.5 displays concrete which has been weathered by the sea. The concrete has a low porosity. Figure 1.6 shows animal fur which is a good example of a porous medium with high porosity, i.e. porosity close to 1. Figure 1.7 displays another type of rock but one which is highly anisotropic. Figure 1.8 displays a highly grained piece of wood (oak). This is another example which shows that a porous medium may be highly anisotropic. In fact, wood is essentially isotropic in all directions orthogonal to the grain. The anisotropy is clearly evident in the grain direction. Anisotropy such as this where one direction is very different from those directions orthogonal is known as transverse isotropy. Figure 1.9 is a schematic picture of a bidispersive or a double porosity porous medium. The large gaps between the dark objects reveal a macro porosity but the darker objects themselves are composed of small spheres and have between them a micro porosity. Such materials may be man made or can occur naturally, cf. [445] for the latter case.

In addition to these we can cite other examples of porous media, such as biological tissues, e.g. bone, skin; building materials such as sand, cement, plasterboard, brick; man-made high porosity metallic foams such as those based on copper oxide or aluminium, and other materials in everyday use such as ceramics. The types of porous materials we can think of is virtually limitless.

Applications of porous media in real life and their connection to microfluid flows are likewise very many. We could list many, but simply quote some to give an idea of the vastness of porous media theory. Use of copper based foams and other porous

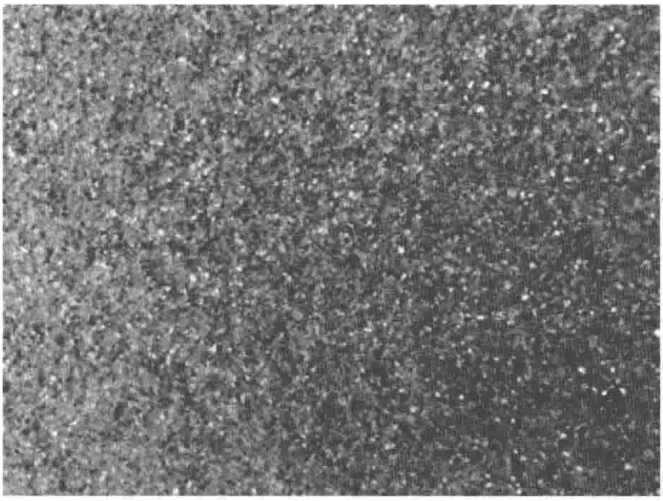

Fig. 1.1 Sand. Photograph taken on Seaham beach, March 2014.

Fig. 1.2 Natural sandstone. Photograph taken on Seaham beach, March 2014.

materials in heat transfer devices such as heat pipes used to transfer heat from such as computer chips is a field influencing everyone, see e.g. [57, 189, 269, 312, 497]. Likewise, porous media are prevalent in combustion heat transfer devices where the porous medium is employed with a liquid fuel in a porous combustion heater, see [200]. Global warming is very topical and porous media are involved there in connection with topics such as ice melting, or carbon dioxide storage, see e.g. [55, 76, 77, 183]. Many foodstuffs are porous materials. Modern technology is involved in such as microwave heating, [112], or drying of foods or other natural materials, see e.g. [500, 501]. Porous media have application in storage of energy or natural convection within the upper region of the Earth, [331, 499]. The latter areas being of particular interest in the field of renewable energy. There are many

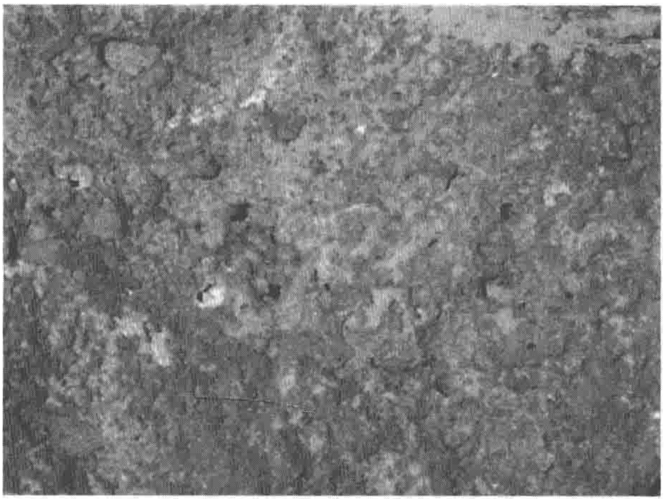

Fig. 1.3 Lava from Mount Etna, Sicily. Photograph taken at Capomulini, June 2007.

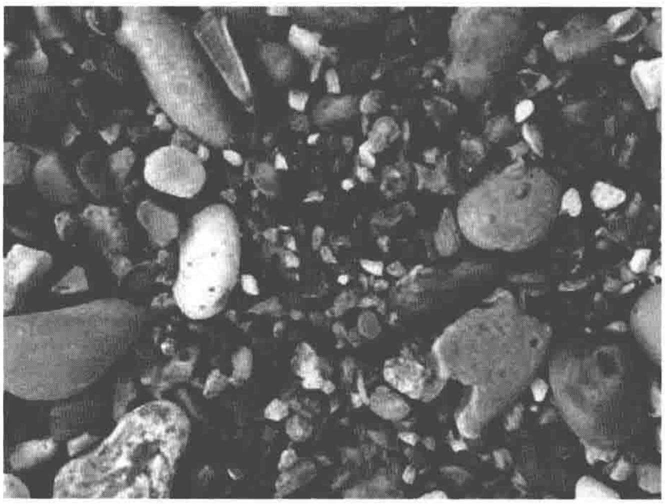

Fig. 1.4 Natural aggregate. Photograph taken on Seaham beach, March 2014.

other diverse application areas of porous materials, such as heat retention in birds or animals, [117], bone modelling, [125], or the manufacture of composite materials, increasingly in use in aircraft or motor car production, see e.g. [108].

Very much connected with thermal convection on a micro-scale are convective flow problems in a porous medium where the fluid temperature, T_f, may be different from the solid skeleton temperature, T_s. Such problems of thermal convection are being increasingly studied. This situation where the two temperatures may be different is usually referred to as local thermal non-equilibrium, abbreviated to LTNE. One of the driving reasons for the increased attention of LTNE flows in porous media is the numerous amount of applications of this area in real life. For example, there are applications in tube refrigerators in space, [19]; in nanofluid flows,

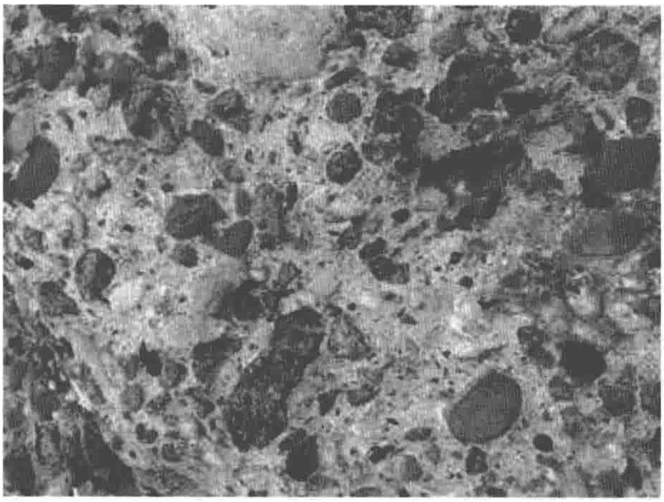

Fig. 1.5 Concrete, weathered by the sea. Photograph taken on Seaham beach, March 2014.

Fig. 1.6 Animal fur is a good example of a high porosity material, as seen in this cat.

[320, 420, 425], chapter 8; in fuel cells [102]; in resin flow, important in processing composite materials, [108]; in nuclear reactor maintenance, [137]; in heat exchangers, [107, 269]; in microwave ablation of the liver, [208]; in biological tissue analysis, [493]; in flows in microchannels, [215]; in flow and heat transfer in porous metallic foams, [189, 240, 241, 269]; in thermovibrational filtration, [388, 389], in textile transport, [486]; and in convection in stellar atmospheres, cf. [425], chapter 8, [426]. An interesting paper analysing various causes of local thermal non-equilibrium situations is that of [470].

Continuum theories for local thermal non-equilibrium effects on flow in porous materials appear to have started in the late 1990's, cf. the work of [291, 307], and [340], and instability in thermal convection taking into account LTNE effects was