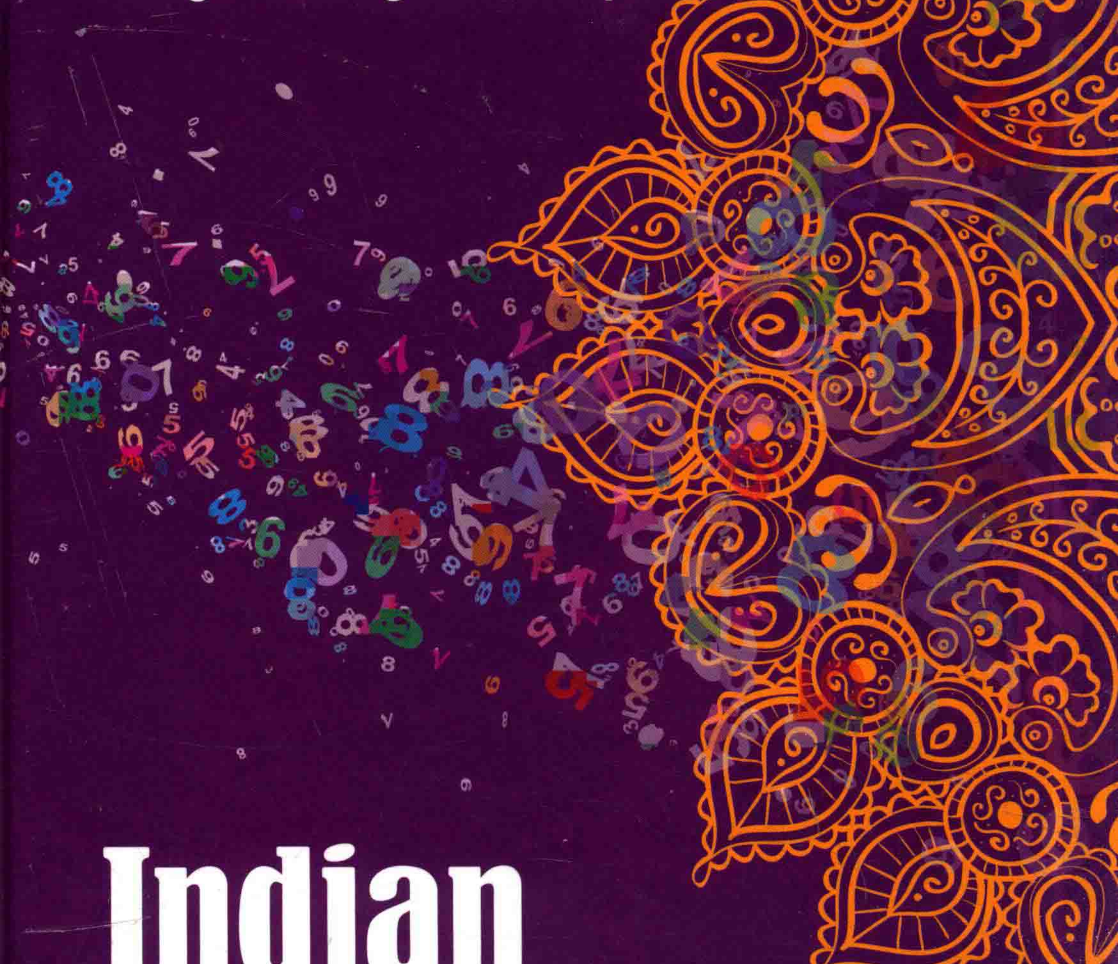
George Gheverghese Joseph



Indian Mathematics

Engaging with the World from Ancient to Modern Times

World Scientific

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To Leela

Without whom this book would have just been a pipe dream

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Dennis Almeida's contributions have been wide ranging as a member of the Research Project alluded to and an inspiration on issues relating transmissions and circulation of mathematical ideas across cultures. While researching this book, I came across an interesting initiative. The NPTEL (National Programme on Technology Enhanced Learning) has constructed a number of online courses in science and technology, including one entitled "Mathematics in India — From Vedic Period to Modern Times", by three stalwarts,

M. D. Srinivas, M. S. Sriram and K. Ramsubramaniam. Their focus is on the development of mathematical ideas and techniques with history and comparative analysis taking a secondary role. I found it a useful source of reference and illumination and wish to acknowledge my debt to the authors of this project. Finally, during the time I have been working on the book several people have given me advice, constructive criticism and encouragement. They have included Burjor Avari, Julian William, Dennis Almeida, Eddie D'Sa, Mary Searle-Chatterjee, Bill Farebrother, and the list goes on. It is appropriate given my insufficient response to some of the advice given that I exclude those mentioned above for any errors of fact and interpretation.

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Prelude

The genesis of this book can be traced to a growing recognition of three major global changes that occurred during the closing decades of the twentieth century. They are: (i) the rising economic power of the non-Western world; (ii) the growing crisis of planetary sustainability and (iii) the loss of authoritative sources of transcendence such as Marxism and religion These changes require us to revisit the paradigm of the 19th century that had sought to explain if not celebrate the unique rise of the West.

Two important concepts helped to modify the old paradigm: circulation and transcendence. 'Circulatory histories'* are now gaining acceptance following the analysis of historical data of transnational and trans-local flows. Emphasising circulatory histories over linear and bounded national or civilisational histories are now recognised as valid and need little justification. However, the acceptance of the idea of transcendence is another matter. Although it is not difficult to show that important changes in world history have in one way or another been influenced by transcendental sources of imagination, inspiration, commitment and resolve, the subject is handled with wariness in academic circles, who see them as

*Knowledge is not something to be discovered; it circulates through revelations from knowledgeable source to less knowledgeable source. Discoveries are therefore particular moments in a history of knowledge in circulation.

insufficient in themselves as explanations unless linked to societal structures and environmental conditions. It was the study of the origins, evolution and transmission of the zero that led me initially to see clearly how transcendence and circulation impinge and interact on one another.

A few years ago on a radio programme I was asked: "Where did zero originate?" Fortunately, I was allowed time to develop an answer without presuming, as many programmes are prone to do today, that listeners have the attention span of a grasshopper. Trying to gather my thoughts, I resorted to the familiar ploy of taking refuge in definitions. Now, if zero merely signifies a magnitude or a direction separator (i.e. separating 'above' the zero level from 'below' the zero level), the Egyptian zero (*nfr*), goes back at least four thousand years. If zero serves merely as a place-holder symbol, indicating the absence of a magnitude at a specified place position (as for example, the '0' in 101 shows the absence of any 'tens' in 101), then such a zero was already present in the Mesopotamian number system long before the first recorded occurrence of the Indian zero. If zero is shown by just an empty space within a well-defined positional number system, such a zero was present in Chinese mathematics some centuries before the Indian zero. The Chinese case showed that the absence of a symbol for zero did not prevent it from becoming an efficient computational tool that could handle even solutions of higher degree order equations involving fractions. However, the zero alluded to in the question asked was a multi-faceted mathematical object: a symbol, a number, a magnitude, a direction separator and a place-holder, all in one operating within a fully established positional number system. Such a zero has occurred only twice in history — the Indian zero and the Mayan zero which appeared in splendid isolation in Central America around the beginning of the Common Era.

To explain the appearance of the Indian or the Mayan zero, it is important to examine the context in which the two independent inventions occurred. From existing evidence, much of it fragmentary in the Mayan case, we surmise that both were numerate cultures with a passion for astronomy and its other half, astrology. As we will see later, the Indian culture from an early time showed interest in and even fascination for very large numbers for their own sake accompanied by a delight in calculations with these numbers. This may have been true of the Mayan culture as well. Both cultures were obsessed with the passage of time but in very different ways. The Indian interest was tied up with a widespread belief of a never-ending cycle of births and rebirths where the primary objective for individual salvation was to break this cycle. It was believed from very early time that this could be achieved in different ways, including offering sacrifices on specially constructed altars that conformed to specific shapes and sizes and carried out on auspicious days. And it was the problems of constructing these altars that gave birth to both practical and theoretical geometry in India. This is a theme explored in Chapter 3 of this book.

In the Mayan case, preoccupation with passage of time took the form of a great fear that the world would suddenly come to an end unless the gods (and especially the Sun God) were propitiated by sacrifice undertaken at auspicious times of the year dictated by specific astronomical occurrences. Human sacrifices were conducted on altars laid out for this specific purpose. The victims were chosen carefully for the possession of certain prized characteristics: beautiful virgins, high-born war captives, etc.

In both the Mayan and the Indian cases there was an essential need for accurate measurement of the passage of time, and hence the keeping of detailed calendars and an elaborate division into eras. The need for such precise calculations may have stimulated the development of efficient number systems with a fully developed zero. And it was but an accident of history and geography that the Indian zero prevailed while the Mayan zero eventually disappeared into oblivion.

The dissemination westwards (and then worldwide) of the Indian zero is an integral and well-known part of the history of Indian and later Indo-Arabic numerals. What is often emphasised is the *manner* in which the zero moved west from India to the Islamic world and then to various parts of Europe. And, given that such a movement did take place, the question arises as to whether the transmission of such a novel concept was constrained by cultural and linguistic filters operating in the recipient cultures. And did these filters subsequently inhibit our understanding of the concept of the Indian zero and the arithmetic of operations with that zero?[†] What we are aware of today is the widespread 'discomfort' of both adults and school children with the concept of zero itself and any mathematical operations involving zero. Consider, for example, what the usual response would be to the following questions:

- Is zero a positive or negative number?
- Is zero an odd or even number?
- Divide 2 by zero.

Even among university students of mathematics a discussion of these questions tends to be vague. There seems to be a singular absence of any attempt to inform students about 'calculating with zero'. This was a standard topic found in Indian texts on mathematics from the time of Brahmagupta (b. 598 CE).

In any systematic study of the history of mathematics both the methodological aspects of the transmission process as well as the pedagogical implications of an imperfect filtering process should be looked at carefully, especially wherever a claim is made that a mathematical object or method is 'borrowed' from one culture by another. It is particularly important in studying the history of Indian mathematics that any claim to Indian originality should be scrutinised both from a local and global perspective. The Indian zero provides a good example of how the concepts of 'circulation' and 'transcendence' serve as useful guides to a cross-cultural study of the history of mathematics.

This book is broadly divided into three parts for the benefit of readers with different backgrounds and interests. The first five chapters contain historical and cross-cultural emphasis. A short

[†]Cultural transmission of zero is not merely a *copying* process, but also a *reconstructive* process in which cognitive biases play an important role. A major bias that inhibits accurate transmission is a tendency for people to make different inferences regarding operations with zero and other mathematical objects (De Cruz and De Smedt, 2013).

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survey of the history of Indian mathematics and its contacts with the outside world is contained in Chapter 2. The next six chapters are more mathematical, beginning with Aryabhata I (b. 476 CE). The last two chapters culminate with the arrival of modern mathematics. More advanced mathematics is contained in the Appendices.

Chapter 1 of the book, entitled 'Rewriting the History of Indian Mathematics: Some Outstanding Issues', will hopefully set the scene. It begins with an examination of the conflicting perspectives offered on the history of Indian mathematics[‡] and how these have bedevilled a proper study of the subject. This chapter proceeds to identify some real problems in studying the history of Indian mathematics, principally the lack of a consensus on the chronology of Indian mathematics before the fifth century CE. For example, one of the earliest astronomical texts of India, the *Vedanga Jyotisa*, has been variously dated as being composed from about 28,000 years ago to about 2,500 years ago! At the centre of this incredible divergence on dating is the vexed question of the validity of finding the dates of certain texts on the basis of astronomical, literary or other even more ephemeral evidence. The other issues highlighted in this chapter include those regarding the sources and interpretations of texts and of differing characterisation of Indian mathematical traditions.

Chapter 2 provides a short survey of the history of Indian mathematics and the nature and extent of *global* interactions of different mathematical traditions, and more specifically the impact of Indian mathematics on other mathematical traditions and vice versa. Chapters 1 and 2 constitute a non-technical introductory framework within which the more substantive content of the later chapters are presented.

Chapter 3 assesses some early evidence on Indian mathematics, notably from the Harappan and the Vedic periods (*c.* 2000–500 BCE), for the suggested links between these cultures (now that this has now become a matter of public debate with politicians joining in!), and

[‡]The term 'Indian mathematics' is used throughout this book to describe mathematics of the Indian sub-continent, including countries situated geographically in that area.

their possible links with the outside world. Chapter 4 contains a detailed look at the Jain and Buddhist inputs into the mathematical and scientific traditions of ancient India. This is not only interesting because of their contribution to early number theory and pure mathematics, but also in the case of the latter it was a vehicle through which certain mathematical and astronomical ideas were taken outside India. The resulting influence of Indian mathematics not only on the West but also on China, Korea, Tibet and other cultures south and east of India are touched upon in this book.

From the earliest times, mathematics in India was closely linked to a mixed bag of astronomy, 'calendrics' (i.e. constructing calendars) and astrology. Contemporary to or slightly after the Jain and Buddhist periods of Indian history were the beginnings of the astronomical tradition, preoccupied with the charting of the solar year, including solstices, equinoxes, lunar periods, solar and lunar eclipses and planetary movements. This was a period during which transmitted Hellenistic influences were believed to have gone into the shaping of Indian astronomy. Chapter 5 will examine only briefly evidence of this transmission since early Indian astronomy is of peripheral interest in this book.

Chapters 6–9 cover the period of Indian mathematical resurgence, beginning with Aryabhata I in the second half of the fifth century CE and ending with Bhaskara II (or Bhaskaracharya) and Narayana Pandita in the early centuries of the second millennium of the Common Era. Although the mathematics of this period remains the best explored area of Indian mathematics, this book provides not only a fuller account of the mathematics of that thousand years (500–1500 CE) but also a discussion of two issues often neglected: (i) the nature and the mechanics of the transmission of ideas from the sub-continent of India to the rest of the world and vice versa; (ii) the agents and the processes of dissemination of mathematical and astronomical ideas within the sub-continent, as evidenced by the increasing production of texts and translations into regional languages. The mathematical activities uncovered will hopefully alter the perception that mathematics in India after Bhaskara II made only "spotty progress until modern times".

A commonly held view of Indian mathematics is that if the Indians made any contribution to world mathematics, it was basically in what is often termed as 'elementary mathematics'. This perception needs further scrutiny. It is well known that two powerful tools contributed to the creation of modern mathematics in the seventeenth century: the discovery of the general algorithms of calculus and the development and application of infinite series techniques. These two streams of discovery reinforced each other in their simultaneous development since each served to extend the range of application of the other. But what is less known until recently is that the origin of the analysis and derivations of certain infinite series, notably those relating to the arctangent, sine and cosine, was not in Europe, but in an area in South India that now falls within the State of Kerala. From a region covering about 500 square kilometres north of Cochin and during the period between the 14th and 16th centuries, there emerged discoveries that predate similar work of Gregory, Newton and Leibniz by about 300 years.

There are several questions worth exploring about the activities of this group of mathematicians and astronomers from Kerala, (hereafter referred to as the 'Kerala School'), apart from the mathematical content of their work. In one substantial chapter and three Appendices (Chapter 10) we examine the social and historical context in which the Kerala School developed and the mathematical motivation underlying their interest in infinite series. We conclude our discussion of the Kerala School by identifying some circumstantial and other evidence to suggest that the mathematics underlying 'the calculus' in Europe could have travelled from Kerala to Europe through different agents, including priests, navigators, craftsmen and traders.

Traditional treatment of the history of Indian mathematics is significant for what it leaves out. What is often omitted is the flourishing mathematical tradition existing or introduced into India during the medieval times where the sources were Persian and Arabic texts or texts in regional vernaculars. A brief examination of these sources of mathematics will identify major differences between these streams of mathematical activity of those working within the Sanskrit tradition and the others. By comparing the research preoccupations of different groups roughly contemporaneous with one another — for example, the Kerala mathematicians who occupied themselves with work on infinite series inspired by Aryabhata and his School and mathematicians working within the Graeco–Arab–Persian tradition whose interests were primarily in Greek geometry and Ptolemaic astronomy — it becomes possible to identify the main epistemological differences between the two mathematical traditions. The two parallel traditions did meet in a few cases involving astronomy but hardly ever on matters relating to pure mathematics. And this lack of contact was a missed opportunity that had considerable repercussions for the development of Indian mathematics and astronomy. This will be the subject matter of Chapter 12.

There has been a tendency on the part of some historians of Indian mathematics to ignore the importance of astronomy in providing both the motivation and the instruments in the historical development of Indian mathematics. To list three examples of astronomy being the prime movers in the creation of new mathematical techniques: solution of indeterminate equations, emergence of trigonometric functions (notably sine function) and the growth of spherical trigonometry, and finally the discovery of infinite series. After all, practically all the Indian mathematicians that we know of today were also were astronomers: Aryabhata, Bhaskara I, Varahamihira, Brahmagupta, Bhaskara II, Madhava, Paramesvara, Nilakantha, Jyesthadeva,... the list is many. To represent the relevance and the creative force of astronomy, a whole chapter (Chapter 11) is devoted to the handmaiden of astronomy, the subject of trigonometry.

The last chapter (Chapter 13) of the book will examine the various phases in the introduction of the modern Western mathematics into India. During the first phase, an attempt was made to incorporate ideas and concepts of modern mathematics into the indigenous mathematical tradition. However, especially after Macaulay's damning indictment of indigenous knowledge in 1835, the last vestiges of an indigenous education programme that tried to blend traditional learning with western ideas had virtually been wiped out by 1850. Western mathematics became the dominant strain acceptable to the rulers and to the ruled. But this did not prevent the emergence of exceptional individuals such as Master Ramchandra (b. 1860) who offered an approach to advanced algebra based on traditional methods, and Srinivas Ramanujan, about some 50 years later whose early interest in and approach to mathematics were probably influenced by indigenous tradition. What remains today are the remnants of traditional mathematics, mainly found today in India among practitioners of occupations such as traditional architecture and astrology.

In writing this book I am hoping to interest a wider audience, a readership hopefully as broad as that of the '*The Crest of the Peacock*'. And at the same time, the book contains a comprehensive survey of Indian mathematics exploring subjects and individuals who are often neglected. A bold historian reveals usually just as much about herself, and her driving motivations, as any actual disclosure of historical pattern. In the case of the *Crest*, it arose from a sense of outrage that people were being written out of history by a dominant culture that had imposed economic, military and psychological dominance over a vast area of the globe. In this book, which I see in some ways as a sequel, the outrage is replaced by a sense of amity to all. It is hoped that in locating Indian mathematics in a global context, drawing out parallel developments in other cultures and carefully examining the question of circulation of mathematical ideas across boundaries, the book will appeal to a wider public.

Contents

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