Second Edition

Engineering Vibrations

William J. Bottega

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Engineering Vibrations

To my mother, *Marie Bottega*

Preface to the Second Edition

This updated and expanded edition of *Engineering Vibrations* continues in the spirit of the first edition and, in addition, includes chapters on the dynamics and vibration of two-dimensional continua. The current edition also includes the effects of structural damping on the vibration of both one-dimensional and two-dimensional continua, as well as on discrete systems. Like its forerunner, the text couples thorough mathematical development with physical interpretation and emphasizes the mechanics and physics of the phenomena. As with the first edition, the current volume presents vibrations from a unified point of view, with emphasis placed on developing a connected string of ideas, concepts and techniques that are sequentially advanced and generalized throughout the text. The text naturally segues from preliminaries to vibration of single degree of freedom systems (including systems that are undamped, as well as those with viscous, Coulomb and structural/internal damping), to discrete multi-degree of free systems (including *general* viscous damping, as well as Rayleigh damping), to one-dimensional continua (rods, strings, Euler-Bernoulli beams and beam-columns, and Timoshenko beams), and ultimately to two-dimensional continua (membranes, Kirchhoff plates, von Karman plates and Mindlin plates). The comprehensive second edition can be used for a one semester course or for a two semester, three semester or four semester sequence at the advanced undergraduate and/or graduate level depending on the instructor's choice of topics.

The original eleven chapters from the first edition remain largely intact, with some minor revisions and additions. In addition, a section on the forced response of structurally damped one-dimensional continua is now included at the end of Chapter 11. The corresponding formulation and solution uses as its basis the phenomenological development (from first principle) of structural damping presented in Section 3.4. (The reader is referred to the *Preface for the First Edition* that follows, for a detailed discussion of the remainder of the contents of the first eleven chapters.) In addition, three new chapters have been added; Chapter 12 – *Dynamics of Two-Dimensional Continua*, Chapter 13 – *Free Vibration of Two-Dimensional Continua*, and Chapter 14 – *Forced Vibration of Two-Dimensional Continua*. The topics covered include characterization of three-dimensional linear and geometrically nonlinear deformation for mathematically two-dimensional structures, as well as the dynamics and vibration of various types of structures within this class. In particular, the deformation, dynamics and vibration of membranes, of Kirchhoff plates, of von Karman plates, and of Mindlin plates are covered. General analytical solutions are developed and applied for both free and forced vibration of all structures considered.

In Chapter 12, a full development for the characterization of deformation for mathematically two-dimensional continua is first presented. The governing equations, boundary and initial conditions for membranes are then developed and carefully simplified to the case of an ideal membrane. The equations of motion and the associated boundary and initial conditions are then developed for various plate structures. The common assumptions are first outlined and developed into general equations, and then specific assumptions are applied sequentially resulting in the governing equations and conditions for Kirchhoff plates, Mindlin plates and uniformly stretched von Karman plates. Separate sections of the chapter are devoted to each type of structure.

The free vibration of the various structures considered in Chapter 12 is discussed in Chapter 13. After a brief discussion of the scalar product and orthogonality of functions of two spatial variables, the general free vibration problem and its solution is outlined for structures of the class considered. Free vibration of ideal membranes is then considered, followed by free vibration of Kirchhoff plates. Free vibration of uniformly stretched von Karman plates is next considered and then free vibration of Mindlin plates is discussed. In each case, general analytical solutions are developed and then applied to illustrative examples. A discussion of the orthogonality of the modal functions for the structures of interest is presented in ensuing sections. The chapter concludes with the development and presentation of formal expressions for the evaluation of the amplitudes and phase angles for free vibration of the various two-dimensional continua considered based on the mutual orthogonality of the modes.

Forced vibration of two-dimensional continua is discussed in Chapter 14. The chapter begins with a brief discussion of the mathematical representation of point loads in a spatially two-dimensional domain. Modal analysis for structures with one dependent variable is then developed and applied to ideal membranes, Kirchhoff plates and uniformly stretched von Karman plates. The development is then extended to systems with multiple dependent variables and is applied to the forced vibration of Mindlin plates. The chapter concludes with a discussion of the steady state response of two-dimensional continua with structural damping. The stiffness operators for the damped structures of interest are first developed, and the general steady state response is then given for structurally damped Kirchhoff plates and for uniformly stretched, damped, von Karman plates. The chapter finishes with the development of the response for damped Mindlin plates.

The second edition of *Engineering Vibrations* covers a wide variety and depth of topics. Like the first edition, it endeavors to be both rigorous and readable, and to provide the student or professional with a solid and extensive background in the subject area.

To close, I'd like to thank Professor Ellis H. Dill and Mr. Peinan Ge, both of Rutgers University, for helpful discussions pertaining to the evaluation of the shape factor for Timoshenko beams. Many thanks go to Mr. Michael J. Pavlou and Ms. Amber M. McGoff for producing the many excellent drawings for Chapters 12–14. I'd also like to thank Mr. M.J. Pavlou, Ms. A.M. McGoff and Mr. J.M. Lakawicz for the drawing and/or modification of the various new and updated figures for Chapters 1–11. In addition, I'd like to express my sincere gratitude to Mr. Pavlou for performing the extensive computations for the examples in Chapters 13 and 14. Sincere thanks go to Mr. Joseph M. Lakawicz and Mr. Michael J. Pavlou, both of Rutgers University, for their valuable assistance with, discussions of, and contributions to the analytical solution for free vibration of Mindlin plates and the associated examples. Finally, I wish to thank Mr. Lakawicz and Mr. Pavlou for their extensive and meticulous proof reading, corrections and helpful comments for the entire manuscript.

William J. Bottega

Preface to the First Edition

The effects of vibrations on the behavior of mechanical and structural systems are often of critical importance to their design, performance, and survival. For this reason the subject of mechanical vibrations is offered at both the advanced undergraduate level and graduate level at most engineering schools. I have taught vibrations to mechanical and aerospace engineering students, primarily seniors, for a number of years and have used a variety of textbooks in the process. As with many books of this type, the emphasis is often a matter of taste. Some texts emphasize mathematics, but generally fall short on physical interpretation and demonstrative examples, while others emphasize methodology and application but tend to oversimplify the mathematical development and fail to stress the fundamental principles. Moreover, both types fail to stress the underlying mechanics and physics to a satisfactory degree, if at all. For these reasons, there appeared to be a need for a textbook that couples thorough mathematical development and physical interpretation, and that emphasizes the mechanics and physics of the phenomena. The book would need to be readable for students with the background afforded by a typical university engineering curriculum, and would have to be self-contained to the extent that concepts are developed, advanced and abstracted using that background as a base. The present volume has been written to meet these goals and fill the apparent void.

Engineering Vibrations provides a systematic and unified presentation of the subject of mechanical and structural vibrations, emphasizing physical interpretation, fundamental principles and problem solving, coupled with rigorous mathematical development in a form that is readable to advanced undergraduate and graduate university students majoring in engineering and related fields. Abstract concepts are developed and advanced from principles familiar to the student, and the interaction of theory, numerous illustrative examples and discussion form the basic pedagogical approach. The text, which is extensively illustrated, gives the student a thorough understanding of the basic concepts of the subject, and enables him or her to apply these principles and techniques to any problem of interest. In addition, the pedagogy encourages the reader's physical sense and intuition, as well as analytical skills. The text also provides the student with a solid background for further formal study and research, as well as for self study of specialized techniques and more advanced topics.

Particular emphasis is placed on developing a connected string of ideas, concepts and techniques that are sequentially advanced and generalized throughout the text. In this way, the reader is provided with a thorough background in the vibration of single degree of freedom systems, discrete multi-degree of freedom systems, one-dimensional continua, and the relations between each, with the subject viewed as a whole. Some distinctive features are as follows. The concept of mathematical modeling is introduced in the first chapter and the question of validity of such models is emphasized throughout. An extensive review of elementary dynamics is presented as part of the introductory chapter. A discussion and demonstration of the underlying physics accompany the introduction of the phenomenon of resonance. A distinctive approach incorporating generalized functions and elementary dynamics is used to develop the general impulse response. Structural damping is introduced and developed from first principle as a phenomenological theory, not as a heuristic empirical result as presented in many other texts. Continuity between basic vector operations including the scalar product and normalization in three-dimensions and their extensions to N-dimensional space is clearly established. General (linear) viscous damping, as well as Rayleigh (proportional) damping, of discrete multi-degree of freedom systems is discussed, and represented in state space. Correspondence between discrete and continuous systems is established and the concepts of linear differential operators are introduced. A thorough development of the mechanics of pertinent 1-D continua is presented, and the dynamics and vibrations of various structures are studied in depth. These include axial and torsional motion of rods and transverse motion of strings, transverse motion of Euler-Bernoulli beams and beam-columns, beams on elastic foundations, Rayleigh beams and Timoshenko beams. Unlike in other texts, the Timoshenko beam problem is stated and solved in matrix form. Operator notation is introduced throughout. In this way, all continua discussed are viewed from a unified perspective. Case studies provide a basis for comparison of the various beam theories with one another and demonstrate quantitatively the limitations of single degree of freedom approximations. Such studies are examined both as examples and as exercises for the student.

The background assumed is typical of that provided in engineering curricula at U.S. universities. The requisite background includes standard topics in differential and integral calculus, linear differential equations, linear algebra, boundary value problems and separation of variables as pertains to linear partial differential equations of two variables, sophomore level dynamics and mechanics of materials. MATLAB is used for root solving and related computations, but is not required. A certain degree of computational skill is, however, desirable.

The text can basically be partitioned into preliminary material and three major parts: single degree of freedom systems, discrete multi-degree of freedom systems, and one-dimensional continua. For each class of system the fundamental dynamics is discussed and free and forced vibrations under various conditions are studied. A breakdown of the eleven chapters that comprise the text is provided below.

The first chapter provides introductory material and includes discussions of degrees of freedom, mathematical modeling and equivalent systems, a review of complex numbers and an extensive review of elementary dynamics. Chapters 2 through 4 are devoted to free and forced vibration of single degree of freedom systems. Chapter 2 examines free vibrations and includes undamped, viscously damped and Coulomb damped systems. An extensive discussion of the linear and nonlinear pendulum is also included. In Chapter 3 the response to harmonic loading is established and extended to various applications including support excitation, rotating imbalance and whirling of shafts. The mathematical model for structural damping is developed from first principle based on local representation of the body as comprised of linear hereditary material. The chapter closes with a general Fourier series solution for systems subjected to general periodic loading and its application. The responses of systems to nonperiodic loading, including impulse, step and ramp loading and others, as well as general loading, are discussed in Chapter 4. The Dirac delta function and the Heaviside step function are first introduced as generalized functions. The relation and a discussion of impulsive and nonimpulsive forces follow. The general impulse response is then established based on application of these concepts with basic dynamics. The responses to other types of loading are discussed throughout the remainder of the chapter. Chapter 5, which is optional and does not affect continuity, covers Laplace transforms and their application as an alternate, less physical/nonphysical, approach to problems of vibration of single degree of freedom systems.

The dynamics of multi-degree of freedom systems is studied in Chapter 6. The first part of the chapter addresses Newtonian mechanics and the derivation of the equations of motion of representative systems in this context. It has been my experience (and I know I'm not alone in this) that many students often have difficulty and can become preoccupied or despondent with setting up the equations of motion for a given system. As a result of this they often lose sight of, or never get to, the vibrations problem itself. To help overcome this difficulty, Lagrange's equations are developed in the second part of Chapter 6, and a methodology and corresponding outline are established to derive the equations of motion for multi-degree of freedom systems. Once mastered, this approach provides the student a direct means of deriving the equations of motion of complex multi-degree of freedom systems. The instructor who chooses not to cover Lagrange's equations may bypass these sections. The chapter closes with a fundamental discussion of the symmetry of the mass, stiffness and damping matrices with appropriate coordinates.

The free vibration problem for multi-degree of freedom systems with applications to various systems and conditions including semi-definite systems is presented in Chapter 7. The physical meanings of the modal vectors for undamped systems are emphasized and the properties of the modal vectors are discussed. The concepts of the scalar product, orthogonality and normalization of three-dimensional vectors are restated in matrix form and abstracted to N-dimensional space, where they are then discussed in the context of the modal vectors. The chapter closes with extensive discussions of the free vibration of discrete systems with viscous damping. The problem is examined in both N-dimensional space and in the corresponding state space. Analogies to the properties of the modal vectors for undamped systems are then abstracted to the complex eigenvectors for the problem of damped systems viewed in state space. Forced vibration of discrete multi-degree of freedom systems is studied in Chapter 8. A simple matrix inversion approach is first introduced for systems subjected to harmonic excitation. The introductory section concludes with a discussion of the simple vibration absorber. The concepts of coordinate transformations, principal coordinates and modal coordinates are next established. The bulk of the chapter is concerned with modal analysis of undamped and proportionally damped systems. The chapter concludes with these procedures abstracted to systems with general (linear) viscous damping in both N-dimensional space and in state space.

The dynamics of one-dimensional continua is discussed in Chapter 9. Correlation between discrete and continuous systems is first established, and the concept of differential operators is introduced. The correspondence between vectors and functions is made evident as is that of matrix operators and differential operators. This enables the reader to identify the dynamics of continua as an abstraction of the dynamics of discrete systems. The scalar product and orthogonality in function space then follow directly. The kinematics of deforming media is then developed for both linear and geometrically nonlinear situations. The equations governing various one-dimensional continua are established, along with corresponding possibilities for boundary conditions. It has been my experience that students have difficulty in stating all but the simplest boundary conditions when approaching vibrations problems. This discussion will enlighten the reader in this regard and aid in alleviating that problem. Second order systems that are studied include longitudinal and torsional motion of elastic rods and transverse motion of strings. Various beam theories are developed from a general, first principle, point of view with the limitations of each evident from the discussion. Euler-Bernoulli beams and beam-columns, Rayleigh beams and Timoshenko beams are discussed in great detail, as is the dynamics of accelerating beam-columns. The various operators pertinent to each system are summarized in a table at the end of the chapter.

The general free vibration of one-dimensional continua is established in Chapter 10 and applied to the various continua discussed in Chapter 9. The operator notation introduced earlier permits the student to perceive the vibrations problem for continua as merely an extension of that discussed for discrete systems. Case studies are presented for various rods and beams, allowing for a direct quantitative evaluation of the one degree of freedom approximation assumed in the first five chapters. It further allows for direct comparison of the effectiveness and validity of the various beam theories. Properties of the modal functions, including the scalar product, normalization and orthogonality are established. The latter is then used in the evaluation of amplitudes and phase angles. Forced vibration of one-dimensional continua is discussed in Chapter 11. The justification for generalized Fourier series representation of the response is established and modal analysis is applied to the structures of interest under various loading conditions.

The material covered in this text is suitable for a two-semester sequence or a one-semester course. The instructor can choose appropriate chapters and/or sections to suit the level, breadth and length of the particular course being taught.

To close, I would like to thank Professor Haim Baruh, Professor Andrew Norris, Ms. Pamela Carabetta, Mr. Lucian Iorga and Ms. Meghan Suchorsky, all of Rutgers University, for reading various portions of the manuscript and offering helpful comments and valuable suggestions. I would also like to express my gratitude to Ms. Carabetta for preparing the index. I wish to thank Glen and Maria Hurd for their time, effort and patience in producing the many excellent drawings for this volume. Finally, I wish to thank all of those students, past and present, who encouraged me to write this book.

William J. Bottega

About the Author

William J. Bottega is Professor of Mechanical and Aerospace Engineering at Rutgers University, where he has been since 1984. He received his Ph.D. in Applied Mechanics from Yale University, his M.S. in Theoretical and Applied Mechanics from Cornell University and his B.E. from the City College of New York. He also spent several years in R&D at General Dynamics where he worked on vibration and sound-structure interaction problems. Dr. Bottega is the author of numerous archival publications on various areas of theoretical and applied mechanics. He is a Fellow of the American Society of Mechanical Engineers, an Associate Fellow of the American Institute of Aeronautics and Astronautics, and a member of the American Academy of Mechanics, the Society for Industrial and Applied Mathematics, the American Society for Engineering Education and Sigma Xi.

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