Third Edition

Advanced Engineering Fluid Mechanics

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Alpha Science

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Alpha Science International Ltd.
Oxford, U.K.

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Revised Edition 1999
Second Edition 1999
Third Edition 2015

ALPHA SCIENCE INTERNATIONAL LTD.
7200 The Quorum, Oxford Business Park North
Garsington Road, Oxford OX4 2JZ, U.K.

www.alphasci.com

Printed from the camera-ready copy provided by the Authors.

ISBN 978-1-84265-912-0

Preface to the third edition

Apart from corrections in the text, this edition includes substantial revision of Chapter 2. We have explicitly brought out the link between a control volume approach to problem solving in fluid mechanics and a boundary-value formulation stemming from Navier-Stokes equations. The utility of momentum and energy equations for analysis at the scale of a control volume has been highlighted. The fact that Bernoulli equation is a special form of the more general energy equation is discussed. Examples leading to limiting cases of flow rates and forces are included. Additional examples have been introduced and details of calculations are presented. The list of unsolved examples has been expanded to reflect the new text of Chapter 2.

The authors are conscious that the subject of fluid mechanics continues to be important and of interest. Accordingly, comments received on previous editions are gratefully acknowledged.

As always, we welcome response from students and colleagues.

K. Muralidhar

G. Biswas

Preface to the first edition

This book is an outcome of a collection of notes prepared by the authors for teaching second level undergraduate and first level postgraduate courses on fluid mechanics at Indian Institute of Technology, Kanpur and Kharagpur. Over the past few years, the notes have been upgraded on at least two occasions. The authors feel that the notes have reached a level where the material could be crystallized into the form of a book whose appeal and utility is felt by a cross-section of students in engineering.

During the course of preparation of the class notes, the authors have observed that there is paucity of second level fluid mechanics texts that would be suitable for the IIT curriculum. International books, even those recently published tend to be specialized and in general, inaccessible to the student community. Hence, the authors have set for themselves the task of preparing a text covering advanced topics in fluid mechanics in a text book format. There is also a second reason for producing a book on advanced topics in fluid mechanics. The post-graduate programme in most countries has been rejuvenated owing to rapid advances being made in the defence, aerospace, environment and energy sectors. The subject of fluid mechanics plays an important role in all these areas. An analytical background of the subject has now become a necessity for engineers in the context of these developments.

The thrust of the book is towards the mathematical formulation of fluid mechanics problems in the form of differential equations and description of strategies available for solving them. A considerable amount of emphasis is subsequently placed on interpretation of the results obtained as well as extraction of meaningful data of engineering importance. In the present form, the book should be useful for students in mechanical, civil, chemical and aerospace engineering. It should also be of interest to students in applied mathematics and classical physics who would like to know what aspects of fluid mechanics are important in applications.

The book contains major chapters on derivation of Navier-Stokes equations, exact solutions, potential theory, boundary-layer theory and turbulent flows. There are shorter chapters on stability and compressible flows. To make the book well-rounded, an introduction to numerical methods for boundary-layer equations and a chapter on experimental techniques is included. All chapters include a fairly large set of worked out problems. These are followed by unsolved problems that are of a good amount of complexity. Taken together, the text, the solved and unsolved problems are intended to train an engineer to take up rather quickly the analysis of difficult fluid mechanics problems.

The preparation of this book would not have been possible without support from various quarters. The authors thank the curriculum develpment cell of QIP, Indian Institute of Technology Kanpur for providing the initial financial support for preparation of notes. They thank Professor S G Dhande, ex-Head, Department of Mechanical Engineering and Professor B Sahay, the present Head for providing a cheerful and stimulating working environment. An earlier draft of the book was read by Professor Raminder Singh, IIT Kanpur and Dr Vijay K Garg, NASA Lewis Research Center. Their enlightening and critical remarks led to a thorough revision. We record our sense of gratitude to them. Thanks are due to the students at Indian Institute of Technology Kanpur who have used the present book in the form of notes and made constructive suggestions. It is because of Professor Milton Van Dyke's marvelous book, *An Album of Fluid Motion*, (1982), we were motivated to include some important photographs. Professor Thomas C Corke and Professor Hassan Nagib of Illinois Institute of Technology, Chicago permitted us to use a photograph from their work. Permission for using other photographs has been obtained by Frau Ursula Beitz of Ruhr Universität, Bochum and Ms Mariko Takabatake of Physical Society of Japan. The authors express their gratitude to them. The authors are thankful to Mrs Suguna Sathyamurthy for enthusiastically undertaking the massive task of preparing the camera-ready manuscript. Assistance provided by Dr P M V Subbarao in preparation of the final version of the text is acknowledged. Mr S S Kushwaha and Mr G K Shukla are to be commended for preparing the neat drawings. The authors would like to express their gratitude to Mr N.K. Mehra and Mr M.S. Sejwal of Narosa Publishers, New Delhi for their interest in our venture. Finally, the work would not have been possible but for the patience and splendid support of the families of the respective authors. The book is the joint effort of both authors; however, the first author was primarily responsible for Chapters 1, 2, 3, 4, 6, 8 and 9 and the second author for Chapters 5 and 7.

We welcome suggestions and critical comments from our readers.

K. Muralidhar

July 1999 **G. Biswas**

Contents

1 Introduction

1.1 Preliminaries

Fluid mechanics deals with the analysis of motion of liquids and gases. The fluid state of matter is predominantly found in nature and this in itself is reason enough to study its behaviour in detail. Information regarding fluid flow is also required in a variety of engineering applications. Some examples where the knowledge of fluid mechanics is essential in determining system performance are:

Process equipment and heat exchangers in chemical and power plants
Turbomachinery (hydraulic, steam, gas)
Combustion chamber design in furnaces and IC engines
Material processing, casting of metals, injection molding of plastics
Geophysical flows, atmospheric turbulence, groundwater movement
Aerospace technology

The goal of the present book is to develop the formulation and tools necessary to analyze fluid flow phenomena in a unified manner. The text is organized along the following lines: Mathematical equations which govern fluid motion are first derived. Attention is restricted to a class of flows of fluids that are called Newtonian. General solutions of these equations do not exist at this time. The bulk of this subject concerns obtaining special solutions of these equations that have relevance to engineering practice.

The set of equations defining the motion of a fluid is complete if it satisfies the following physical laws:

1. Law of conservation of mass
2. Newton's second law of motion
3. Law of conservation of energy

For an isothermal fluid, the conservation of energy principle reduces to conserving mechanical energy alone. Under these conditions, it can be shown that Newton's second law of motion is equivalent to the energy principle. Hence, in isothermal flows, the law of conservation of energy need not be explicitly brought into the formulation for the prediction of flow behaviour.

The physical laws of motion give rise to equations which are not in terms of easily measurable quantities. In a flow field, the measurable quantities are the components of the velocity vector and thermodynamic pressure. However, the laws of motion are expressible in terms of stresses on a fluid element. In engineering analysis, it is common to relate the stresses to the velocity components through empirical equations called *constitutive relations*. These equations contain undetermined constants, which must be independently known from laboratory measurements. While a variety of constitutive relations may be allowed depending on the choice of the fluid, the general form of these relations is restricted by the constraint of the second law of thermodynamics. The second law imposes certain conditions on the constants used to relate stress to the velocity components in such a way that there is a one-way transfer of dissipated, irreversible portion of mechanical energy in the flow field to thermal energy.

The purpose of solving the equations of motion set up by the physical laws is to obtain the velocity field and pressure at every point in the flow field and for all time. With this data, one may then extract specific useful information such as fluid forces acting on a structure, heat transfer rates from heated bodies and travel times of pollutants.

1.2 Definition of a Fluid

A fluid is defined as the state of matter which cannot sustain a shear stress, however small. Figure 1.1 shows an element of a material subject to a shear stress τ and whose response is the angular displacement δ. Figure 1.2 shows the strain δ plotted as a function of time. For an elastic solid, the strain increases with time and attains a steady value. For a fluid, it increases indefinitely with time. For this reason, a fluid is said to deform continuously with time under the influence of a shear stress. The response of a fluid material to stress may however be defined as strain-rate, rather than strain alone. For a variety of fluids, the strain rate $d\delta/dt$ has a unique value for a given shear stress τ and can be obtained from experiments.

Consider Newton's parallel plates experiment, (Figure 1.3). Two parallel plates of area A, a small separation h with a fluid in-between, are moved relative

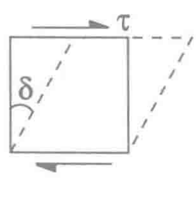

Figure 1.1: Deformation of a Fluid Element in Shear.

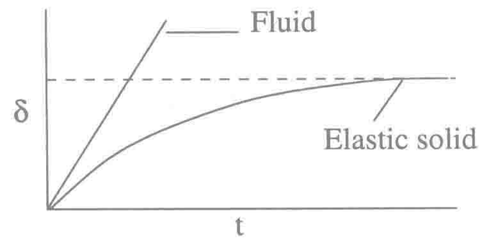

Figure 1.2: Angular Deformation as a Function of Time.

to each other at a velocity U. Experiments show that the force F required to sustain this motion is given as

$$F = \frac{CUA}{h}$$

for a variety of fluids. Here C is a constant that depends on the choice of the fluid. Identifying F/A as shear stress and U/h as the associated strain rate, the above equation can be cast in the form

Shear stress is proportional to rate of shear strain

The class of fluids that obey this rule is called *Newtonian*. It turns out that the most commonly occurring fluids such as air, water and oils obey this relationship. The proportionality constant in this relation is called *dynamic viscosity* and is denoted by μ. The ratio of μ to fluid density ρ is called *kinematic viscosity* and is denoted by ν.

Figure 1.4 shows curves relating shear stress and strain rate for a large variety of fluids. For a fluid such as type (a), this curve is a line passing through the origin. These are Newtonian fluids defined above. All others are called non-Newtonian. Blood, polymeric fluids, inks and slurries display non-Newtonian behaviour. A non-Newtonian fluid is occasionally viewed as a substance which has a viscosity dependent on the strain rate itself. This simplistic notion can explain the non-linearities of curves c and d in Figure 1.4. More complicated models are required if complex behaviour (as in (b)) is realized in practice.

The dynamic viscosity μ plays the role of an internal friction coefficient since it resists relative motion between fluid surfaces. A portion of the work done in overcoming this friction is converted entirely into the internal energy of the fluid, resulting in a rise of its temperature. (The rest of the external work done

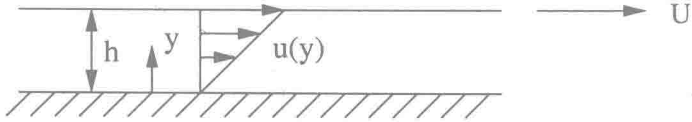

Figure 1.3: Newton's Parallel Plates Experiment.

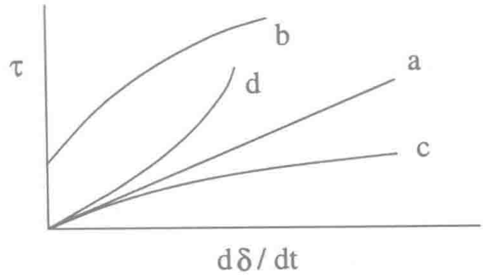

Figure 1.4: Shear Stress-Strain Rate Curves.

goes towards increasing the kinetic energy of the fluid). The kinematic viscosity ν $(= \mu/\rho)$ has a different physical meaning altogether. It is usually identified as the coefficient of momentum transport.

Consider Newton's experiment, where the spacing h is sufficiently large. As shown in Figure 1.5, the distance into the fluid affected by the motion of the plate is called as δ. The distance δ is the penetration depth in the fluid arising from a disturbance on its boundary, the plate movement in this example. As will be shown in the Chapter 3, $\delta \sim \sqrt{\nu}$. This provides an interpretation of ν as a fluid property that determines the extent of transport of motion in a stationary fluid medium.

The force required to sustain the plate motion in the example given above is

$$F \sim \frac{\mu U A}{\delta} \sim \sqrt{\mu}\, U A$$

and hence dependent on μ, the dynamic viscosity. Properties μ and ν (besides the density ρ) are fundamental fluid properties encountered in the study of fluid mechanics.

Magnitudes of ρ, μ and ν for commonly encountered Newtonian fluids are given in Table 1.1 at an average temperature of 30°C and 40°C. Also given are values of thermal conductivity k, specific heat C_p and thermal diffusivity α, as these appear prominently in heat transfer problems.

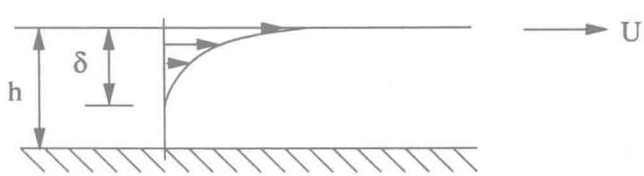

Figure 1.5: Transient Parallel Plate Experiment.