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Variational Methods in Image Processing

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CHAPMAN & HALL/CRC MATHEMATICAL AND COMPUTATIONAL IMAGING SCIENCES

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Preface

This manuscript is devoted to variational models, their corresponding Euler–Lagrange equations and numerical implementations for image processing. Such techniques allow us to solve many inverse problems by minimization and regularization using rigorous tools from function spaces, calculus of variations, numerical analysis, and scientific computing. The most important problems in image processing are studied here, such as image restoration and image segmentation. Other related problems and applications are also presented and analyzed in detail. The variational approach offers an optimal and elegant solution in many cases, given knowledge about the image formation model, constraints, and a priori information. The variational method by regularization has been proven to be one of the most powerful techniques for solving many image processing tasks. This book covers numerous methods and applications, with accompanying tables, illustrations, algorithms, exercises, and online electronic material. It seeks to balance the theory with practice and the use of computational approaches.

Our goal is to offer a general textbook on variational approaches for image processing. Several topics are discussed in detail and may appeal to a larger audience. Instead of minimizing overlap with existing textbooks, the aim is for a more comprehensive and up-to-date presentation.

Each chapter includes the presentation of the problem, its mathematical formulation as a minimization operation, discussion and analysis of its mathematical well-posedness, derivation of the associated Euler–Lagrange equations, numerical approximations and algorithm descriptions, several numerical results, and a list of exercises.

In line with a desire for accessibility, this book attempts to be a self-contained guide to variational models in image processing, providing a synopsis of the required mathematical background necessary to understand the presented methods.

There are a number of successful advanced texts, including textbooks, on variational models and partial differential equations for image processing and related topics from image analysis and computer vision. This is a proof of the strong and continuing interest in these areas. This textbook focuses specifically on the principles and techniques for variational image processing and applications, balancing the traditional computational models with the more modern techniques developed to answer new challenges introduced by the new image acquisition devices.

The audience

This text is intended primarily for advanced undergraduate and graduate students in applied mathematics, scientific computing, medical imaging, computer vision, computer science, engineering and related fields and for engineers, professionals from academia, and the image processing industry. The manuscript can be used as a textbook for a graduate course or for a graduate summer school. It will serve as a self-contained handbook and detailed overview of the relevant variational models for image processing. The general area of image processing and its state-of-the-art methods have become essential in many fields, including medical imaging, defense, surveillance, Internet, television, image transmission, special effects, physics, astronomy, and other fields that require image acquisition for further processing and analysis.

Topics not covered

There are many other important topics in variational image processing that could not be covered here. These include higher-order models, wavelet and statistical methods, convexification algorithms for image segmentation and partition, proximal point methods, and other relevant applications from computer vision and medical imaging.

Additional resources

Electronic resources accompany the manuscript. These can be found at the online link given below, and include MATLAB® codes for the main models and algorithms presented in the manuscript. The electronic resources will be updated when necessary. Other useful resources to students, instructors, researchers and practitioners will be found as well at this link:
http://www.math.ucla.edu/~lvese/VMIP

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