

# Graph Theory and Its Applications

D.L. Strelkov





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Graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects. A graph in this context is made up of vertices, nodes, or points which are connected by edges, arcs, or lines. A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to another.

The effect of graph generation on slope stability analysis based on graph theory has been investigated in first chapter. In second chapter, we consider the block matrices and 3-dimensional graph manifolds associated with a special type of tree graphs. Third chapter proposes a new method for dynamic airspace configuration based on a weighted graph model. Fourth chapter considers the real three-dimensional Euclidean Jordan algebra associated to a strongly regular graph. In fifth chapter, we investigate prime labeling for some graphs resulted by identifying any two vertices of some graphs. An original algorithm to assign graphs to each of elementary particles has been proposed in sixth chapter. Seventh chapter gives some sequence in order to generate wrapped  $\Delta$ -labellings as cluttered orderings for the complete bipartite graph. In eighth chapter, we take the shortest path problem and the minimum cost maximum flow problem in graph theory as the theoretical basis. Ninth chapter analyzes students' misconception based on rough set theory and combined with interpretive structural model (ISM) to compare students' degree of two classes. In tenth chapter, transistor level graph model is proposed to describe the behavior of CMOS circuits under predictive Nanotechnology SPICE parameters. Eleventh chapter describes a new type of graph, we call it a general-graph. Twelfth chapter proposes a graph based novel algorithm for fingerprint recognition. Thirteenth chapter presents a clustering approach based on spectral graph partitioning (SGP) for WMSN that increases the lifetime of the network. Last chapter presents an up-to-date review of the developments made in the field of rhotrix theory for a decade, starting from the year 2003, when the concept of rhotrix was introduced, up to the end of 2013.

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Strelkov

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Editor:

**D. L. Strelkov**



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# Preface

Graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects. A graph in this context is made up of vertices, nodes, or points which are connected by edges, arcs, or lines. A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to another.

The effect of graph generation on slope stability analysis based on graph theory has been investigated in first chapter. In second chapter, we consider the block matrices and 3-dimensional graph manifolds associated with a special type of tree graphs. Third chapter proposes a new method for dynamic airspace configuration based on a weighted graph model. Fourth chapter considers the real three-dimensional Euclidean Jordan algebra associated to a strongly regular graph. In fifth chapter, we investigate prime labeling for some graphs resulted by identifying any two vertices of some graphs. An original algorithm to assign graphs to each of elementary particles has been proposed in sixth chapter. Seventh chapter gives some sequence in order to generate wrapped  $\Delta$ -labellings as cluttered orderings for the complete bipartite graph. In eighth chapter, we take the shortest path problem and the minimum cost maximum flow problem in graph theory as the theoretical basis. Ninth chapter analyzes students' misconception based on rough set theory and combined with interpretive structural model (ISM) to compare students' degree of two classes. In tenth chapter, transistor level graph model is proposed to describe the behavior of CMOS circuits under predictive Nanotechnology SPICE parameters. Eleventh chapter describes a new type of graph, we call it a general-graph. Twelfth chapter proposes a graph based novel algorithm for fingerprint recognition. Thirteenth chapter presents a clustering approach based on spectral graph partitioning (SGP) for WMSN that increases the lifetime of the network. Last chapter presents an up-to-date review of the developments made in the field of rhotrix theory for a decade, starting from the year 2003, when the concept of rhotrix was introduced, up to the end of 2013.

**Editors**



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# **EFFECT OF GRAPH GENERATION ON SLOPE STABILITY ANALYSIS BASED ON GRAPH THEORY**

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## **ABSTRACT**

Limit equilibrium method (LEM) and strength reduction method (SRM) are the most widely used methods for slope stability analysis. However, it can be noted that they both have some limitations in practical application. In the LEM, the constitutive model cannot be considered and many assumptions are needed between slices of soil/rock. The SRM requires iterative calculations and does not give the slip surface directly. A method for slope stability analysis based on the graph theory is recently developed to directly calculate the minimum safety factor and potential critical slip surface according to the stress results of numerical simulation. The method is based on current stress state and can overcome the disadvantages mentioned above in the two traditional methods. The influences of edge generation and mesh geometry on the position of slip surface and the safety factor of slope are studied, in which a new method for edge generation is proposed, and reasonable mesh size is suggested. The results of benchmark examples and a rock slope show good accuracy and efficiency of the presented method.



### INTRODUCTION

Graph theory is an important branch of combinatorial mathematics. The theory originated from the Koenigsberg bridge problem, and the mathematician Euler used the theory to address this problem. After hundreds of years of development, the graph theory has been used to solve the problems of the shortest path, network flow, dynamic planning, etc. It has been widely used in engineering fields, such as the analysis of drainage pipe network system, the optimal island distribution of smart grid, the train operation plan, and the tourism route optimization (Bondy and Murty, 1976 and Wang et al., 2011).

The problem of slope stability analysis (Kim and Lee, 1997, Farias and Naylor, 1998, Sarma and Tan, 2006, Zheng et al., 2009, Guo et al., 2011, Xie et al., 2011, Zhou et al., 2011 and Shen et al., 2013) can be transformed to the shortest path one in the graph theory. The directed weighted graph is firstly constructed by analyzing mesh and vertex information of the model based on numerical calculation of stress field. Then the slip surface and safety factor can be found out by the shortest path algorithm.

There have been many researchers attempting to use the graph theory to analyze the critical slip surface and the safety factor. Cherkassky et al. (1993) and Xu et al. (2007) studied the slip surface and the safety factor using the Dijkstra algorithm on the basis of finite element results. Zhou et al. (2008) used the graph theory to evaluate the stability of slope under the condition of rainfall infiltration. Zhuang et al. (2008) developed the meshless graph theory method. Bellman (1957) developed the Bellman–Ford algorithm to search for the critical slip surface of jointed rock. Fang (2007) extended the graph theory to the three-dimensional slope stability analysis, and preliminarily applied it to tunnel slope stability analysis.

These methods are based on the current stress state, and can overcome the disadvantages of the conventional limit equilibrium method (LEM) which cannot consider the constitutive relation of rock/soil mass. In comparison, the strength reduction method (SRM) requires iterative calculations and cannot directly locate the slip surface (Zheng et al., 2005). It is a computationally desirable method for slope stability analysis.

The critical slip surface corresponds to the minimum safety factor of slope comprising the edges and vertices of the graph. In this approach, type of edge generation, mesh geometry and mesh density will influence the results of slope stability analysis. Unreasonable distribution of vertices and edges will give erroneous slip surface and safety factor. This issue has