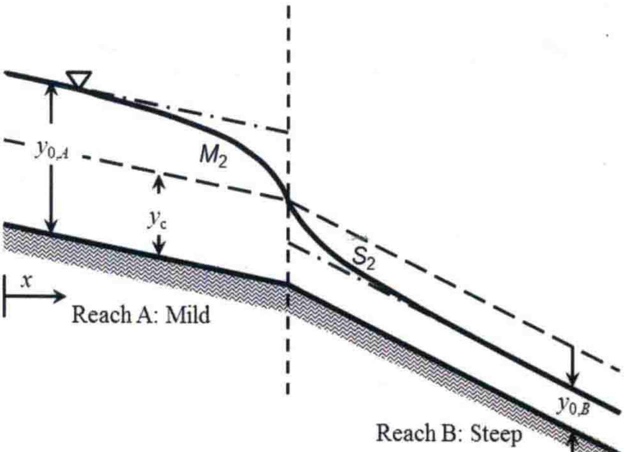
Fundamentals of OPEN CHANNEL FLOW



Glenn E. Moglen

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Fundamentals of OPEN CHANNEL FLOW

To my wife,

Jenny, and to my children,

Rachel and Richard

Preface

As with many students of open channel flow from my generation, I first was exposed to this topic as an undergraduate senior using the excellent and wonderful book authored by Henderson (1966). With a further nod toward Henderson, the alert reader will notice that the sequence of material presentation in this text is the same as in the early chapters of his book. So a fair question is, "What do I feel I have to offer to this topic that Henderson did not already address?" My answer is twofold.

First, Henderson's book was broader than this one, presenting the fundamentals of open channel flow along with more advanced topics. This book, by contrast, is intended for exclusive use in a first course on this topic, as it appears in the curriculum of most undergraduate civil engineering programs. Advanced courses will necessarily draw on other material beyond this text. I have written this text with the intention of making up for limited scope with what I hope is a thorough and clear examination of the nuances of the most fundamental concepts.

Second, I have endeavored to bring this topic into the twenty-first century by emphasizing tools and programs that did not exist in Henderson's time. Tools such as the "Goal Seek" function in Excel, computer animations of basic open channel phenomena, and an exploration of surface water profiles as modeled using both spreadsheet applications and the US Army Corps of Engineers Hydrologic Engineering Center River Analysis System (HEC-RAS) program should serve the current reader well. These items reduce mindless repetition, tap into the reader's innate ability to understand complex concepts visually, and expose the reader to the industry-standard tools of the day.

This book offers a few other new contributions. At this writing, I am unaware of any text that presents an analytical solution relating one alternate depth to the other in a rectangular channel. While the analog of this equation relating conjugate depths has existed since before Henderson's time, the rather simple equation for alternate depths has remained elusive. Ironically, I developed this relationship directly from the mathematics offered by Henderson. In his book, he not only presented the equation relating one conjugate depth to the other, but he showed the surprising duality that exists between y' and $1/y'$ in dimensionless forms of the energy and momentum equations. I reasoned that if these two equations could be made equivalent in a dimensionless presentation, then the conjugate depth relationship must likewise have an alternate depth counterpart. That derivation appears in this text.

Another theme I have emphasized in this book is graphical interpretation of the E-y and M-y relationships. I have had the benefit of presenting this material to students over many years. While many students capably manipulate the relevant equations when solving open channel flow problems, a fair number of others struggle. I have found that by focusing the problem interpretation into a shift along either or both of the E-y and M-y curves, students are able to gain the insights necessary to solve the problem at hand. Those insights then guide the mathematics. Many of the examples and animations provided here reinforce the correspondence between these graphical and mathematical realms.

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As I write this, I feel strongly aware of my place in the teaching and learning continuum of those who have studied open channel flow. I have learned immensely from three people who merit special recognition. First, I owe a debt of gratitude to my original teacher of this subject, Dr. Yaron Sternberg at the University of Maryland. Ron's classroom was a joy to enter because he so skillfully encouraged quiet contemplation as the source of true understanding. I remember his classroom as being an almost magical sanctuary, where deep and careful thought could prevail over any problem. Second, I owe the trajectory of my life in academia to Dr. Richard McCuen, also at the University of Maryland. It was Rick who so impressed me as an undergraduate as to what was possible in my life's work. It was Rick who planted the seed and helped me believe my future was in academics. And it is Rick who has steadfastly supported my efforts to complete this book. Third, I would like to recognize Dr. Michael Casey for his unique role in my life. Unlike Ron and Rick, Mike was once my student, never formally my teacher. But he has taught me so much more than I ever taught him. Mike was my student when I was a struggling assistant professor. Intellectually, he taught me GIS, how to be a clever programmer, and how to integrate apparently disparate tools into a unique whole. And he was the person who actually suggested I write this book. Mike has been a remarkable friend to me. He is the younger brother I never had.

There is no better way to learn a subject than to teach it. In keeping with my earlier statement of the continuum, this work has benefitted enormously from the hundreds of students whom I've had the good fortune to lead through this material, both at the University of Maryland and now at Virginia Tech. These students have patiently sat through numerous iterations of past material that now appears in this book. They have asked insightful questions and pointed out errors in algebra, in computation, and in logic. They have forced me to hone presentations, to pare down needless and confusing examples, and to demonstrate what might have been obvious to me but not to them. This book exists because of these students.

Finally, I would like to thank my family and friends for their support. My wife and children have seen me labor on this book at all hours and in all places. There have been occasions when they have sacrificed my time with them so that I could pursue the completion of this work. They have never complained and have remained supportive and encouraging to the end.

Reference

Henderson, F.M. (1966). *Open Channel Flow*, Macmillan, New York.

Glenn Moglen
September 2014
Manassas, Virginia

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Contents

Chapter 1

Introductory Material

CHAPTER OBJECTIVES

1. Define open channel flow.
2. Define basic quantities relevant to open channel flow.
3. Present and discuss fundamental governing equations.
4. Identify different classes of open channel flow problems.
5. Emphasize the value and importance of critical thinking.

1.1 INTRODUCTION: WHAT IS OPEN CHANNEL FLOW?

As used in this text, the term *open channel flow* covers the range of natural and artificial conveyances of water in settings that are open to atmospheric pressure. Some examples of open channels include water flowing in creeks and rivers, through irrigation canals, within open air conveyances in wastewater treatment plants, and along the curb and gutter at the edges of streets and parking lots. The channel, therefore, varies from a natural surface lined with naturally occurring sediment to a man-made surface made of concrete, metal, or graded soil and sediment.

To properly study open channel flow, we first need to define the basic quantities that are at the heart of the physical system, to enumerate the most fundamental governing equations, and to develop a classification system for different kinds of problems that will be addressed. These items are discussed in this chapter. The chapter concludes with a brief discussion on the value of critical thinking. This last item is a nod toward the author's experience teaching this subject for many years.

1.2 QUANTIFICATION OF OPEN CHANNEL FLOW

Figure 1.1 presents a longitudinal view of flow in an open channel. The figure shows many of the basic quantities that govern solutions to open channel flow problems.

The flow depth, y, is arguably the most important quantity when approaching problems in open channel flow. Depth is measured in the vertical plane, from channel bottom to the water surface. The reader may be concerned that depth is apparently not measured perpendicular to the channel bottom. While this is true, the channel slope, S_0, is generally very small (see Problem 1.1) so that the difference between measuring the channel depth perpendicular to channel bottom and measuring the depth in the vertical plane is miniscule. By basic geometry Figure 1.1 also shows that $\theta = \tan^{-1}(S_0)$. The quantity, z, is generally used to represent any topography presented by the channel bottom itself, relative to the horizontal datum.

The flow velocity, v, is also of primary importance to solving open channel flow problems. In reality, velocity varies in the vertical from essentially zero at the channel bottom to generally a maximum value at or near the water surface. Velocity also varies across the width of the channel in potentially very complex ways depending upon the channel shape. In this text, however, we rarely consider this variation. Instead we work with a single, aggregate value of velocity that represents a mean value across both the depth and width of the channel cross-section.

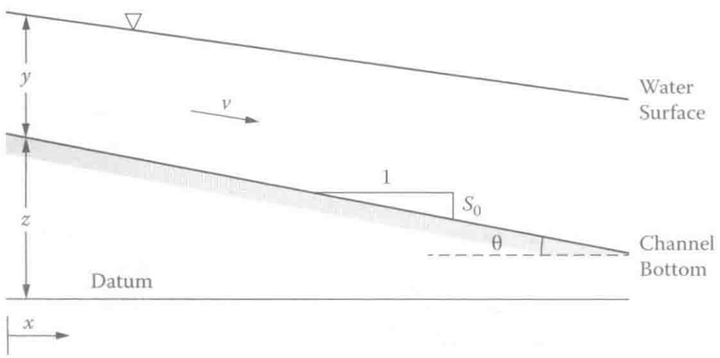

FIGURE 1.1 Profile view of basic open channel flow quantities.

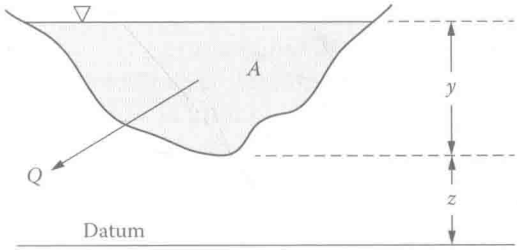

FIGURE 1.2 Cross-sectional view of basic open channel flow quantities.

Figure 1.2 presents a cross-sectional view of flow in an open channel. Like velocity, the channel cross-section is potentially quite complex in shape. It should be apparent to the reader that the cross-sectional area of the channel, A, is dependent on the flow depth, y. More than anything else, this observation is what distinguishes channel flow from flow in a closed conduit. In a closed conduit such as a pipe, assumed to be flowing full, the cross-sectional area is fixed and constant. Thus, the physics of the flow are constrained largely to understanding the flow velocity, issues of friction, and of total energy possessed by the flow. In open channel flow, the depth plays a dual role: (1) it controls the energy possessed by the flow, and (2) it determines the cross-sectional area of the flowing volume. This is the challenge of solving open channel flow problems. It is also the beauty of solving such problems. Although it should be understood by the reader that the natural world offers an endless range of possible channel shapes, this text focuses almost exclusively on regular channel shapes: rectangular, trapezoidal, and circular. An emphasis is placed on rectangular channels, not because they are widely used, but because the mathematics of flow in a rectangular channel are as simple as possible and allow the reader to focus attention on other more pressing open channel flow phenomena.

Discharge, Q, is the last of the basic open channel flow quantities the reader must consider. Dimensionally, Q has units of L^3T^{-1} such as cubic meters per second or cubic feet per second. Like velocity, Q also varies continuously in both the vertical and horizontal planes. But like velocity, Q is generally treated in this text as an aggregate average over the cross-sectional area of the flow.

Not shown in either Figure 1.1 or Figure 1.2 but still relevant in some open channel problems are several properties of water. One fundamental property of water is density, ρ (in ML^{-3}). The density of water is roughly 1000 kg/m^3. Closely related is the unit weight of water, γ (in FL^{-3}), which is simply the product of density and gravitational acceleration:

$$\gamma = \rho \cdot g \tag{1.1}$$

A typical value of γ is about 9810 N/m^3 in metric units or 62.4 lb/ft^3. Density and unit weight vary slightly with temperature.

Another fundamental quality of water is its viscosity, which is reported in two different ways. The kinematic viscosity of water, ν (in L^2T^{-1}) is roughly 1.8×10^{-6} m^2/s at 0°C, but diminishes considerably (by more than a factor of two) as it warms to say, 30°C. The other way viscosity is measured and reported is as dynamic viscosity, μ (in FTL^{-2}). Dynamic and kinematic viscosity are related according to the following equation:

$$\mu = \rho \cdot \nu \tag{1.2}$$

The dynamic viscosity of water is approximately 1.8×10^{-3} N-s/m^2 at 0°C, and also diminishes by a factor of two as it warms to 30°C. Chapter 7 deals most closely with issues of water density, unit weight, and viscosity. Table 7.1 (Chapter 7) shows the variation of these quantities with temperature.

1.3 FOUNDATIONAL EQUATIONS

This text is premised on essentially four foundational equations that quantify the concepts of continuity, energy, momentum, and friction. Energy and momentum are the immediate focus of Chapters 2 and 3, respectively. Chapter 4 first introduces friction, but it requires that chapter, along with Chapters 5 and 6, to more fully examine both qualitatively and quantitatively the nuances of friction. Of course, these concepts do not exist separate from one another, so the full development of friction concepts in Chapters 4 through 6 draws heavily on continuity, energy, and momentum. All must be understood in order to adequately address most problems relating to open channel flow.

We defer the proper discussion of energy, momentum, and friction to subsequent chapters. For now, we assume the reader is already familiar with continuity and Bernoulli's energy equations and use these to motivate the central concept of the Froude number by deriving the speed of shallow wave propagation in an open channel.

Inspired by Henderson (1966), Figure 1.3 provides a definition sketch of a shallow wave moving at velocity, c, that is set in motion by a wall pushing a flow of water at velocity Δv. At location 1, to the left of the shallow wave front, the velocity of the flow is Δv. At location 2, to the right of the wave front, the water is stationary (i.e., $v = 0$). The depth is slightly greater at location 1, $y + \Delta y$, than at location 2 where the depth is simply, y. We now make use of continuity and Bernoulli's equation to determine the velocity at which the shallow wave is propagating. Taking the perspective of an observer on the banks of the

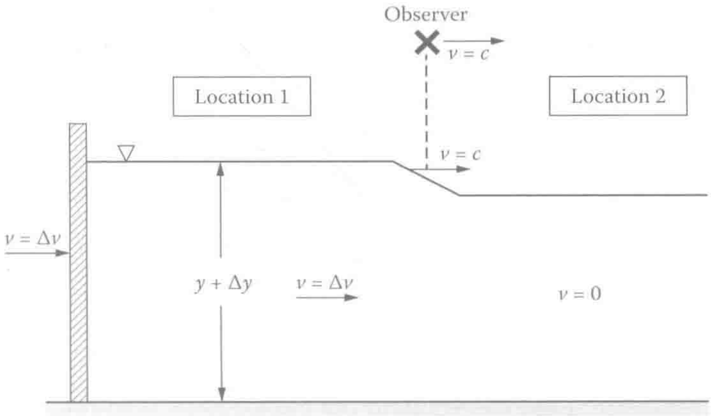

FIGURE 1.3 A shallow wave moving at velocity, *c*, is set up by a wall traveling at velocity, Δ*v*, to the right.

stream, the observer moves in synchronization with the shallow wave so that from the observer's perspective the wave is stationary. Because the observer is moving at velocity c to the right, the apparent velocity (velocity taken as positive moving from left to right in Figure 1.3) of the flow at location 1 is $v_1 = c - \Delta v$. Similarly, at location 2, the apparent velocity of the flow is $v_2 = c$.

Applying continuity and assuming a rectangular channel with width, w, we have

$$Q_1 = Q_2 \tag{1.3}$$

$$v_1 \cdot y_1 \cdot w = v_2 \cdot y_2 \cdot w \tag{1.4}$$

Dividing through by w and substituting for y_1 and y_2, we get

$$(c - \Delta v) \cdot (y + \Delta y) = c \cdot y \tag{1.5}$$

$$c \cdot y + c \cdot \Delta y - \Delta v \cdot y - \Delta v \cdot \Delta y = c \cdot y \tag{1.6}$$

By definition, Δv is much smaller than c, and Δy is small relative to y. Thus the product, $\Delta v \cdot \Delta y$, is very small and can be neglected. Equation 1.6 can be simplified and rearranged to

$$\frac{c}{y} = \frac{\Delta v}{\Delta y} \tag{1.7}$$

The wave height, Δy, is small, so the energy dissipated by the wave is treated as negligible. Using Bernoulli's equation between locations 1 and 2 and noting that the pressure term (i.e., $p_1 = p_2$) is unchanged between these locations,

$$y + \Delta y + \frac{(c - \Delta v)^2}{2g} = y + \frac{c^2}{2g} \tag{1.8}$$

$$y + \Delta y + \frac{c^2 - 2c \cdot \Delta v + \Delta v^2}{2g} = y + \frac{c^2}{2g} \tag{1.9}$$

As we previously neglected $\Delta v \cdot \Delta y$ as small in equation 1.6, we similarly neglect Δv^2 as small. Canceling common terms on both sides of equation 1.9 and dismissing Δv^2 as small, we get

$$\Delta y - \frac{c \cdot \Delta v}{g} = 0 \tag{1.10}$$

Equation 1.10 can be rearranged to

$$\frac{g}{c} = \frac{\Delta v}{\Delta y} \tag{1.11}$$

The right-hand sides of both equations 1.7 and 1.11 are identical, so we equate the left-hand sides of these two equations which yields

$$\frac{c}{y} = \frac{g}{c} \tag{1.12}$$

Finally, solving for c in equation 1.12 gives

$$c = \sqrt{gy} \tag{1.13}$$

Thus, the velocity at which a shallow wave propagates in a rectangular channel is given by equation 1.13. The ratio of the bulk velocity of the water, v, to this wave velocity is known as the Froude number, F_r

$$F_r = \frac{v}{\sqrt{gy}} \tag{1.14}$$