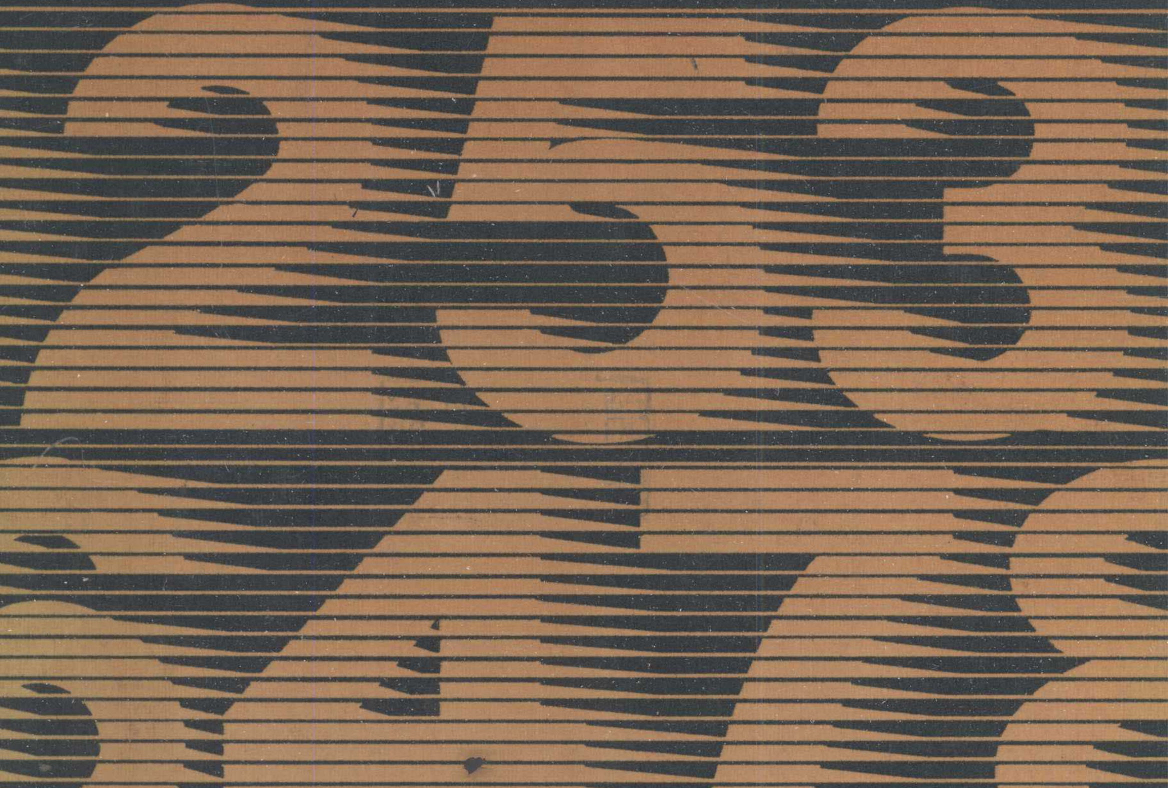
Business Mathematics

A COLLEGE COURSE

GOSSAGE



2d EDITION

Business Mathematics

A COLLEGE COURSE

2d EDITION

LOYCE C. GOSSAGE, Ed. D

Chairman, Business Administration Department
Mt. San Antonio College
Walnut, California

Published by

M08 **SOUTH-WESTERN PUBLISHING CO.**

CINCINNATI WEST CHICAGO, ILL. DALLAS PELHAM MANOR, N.Y. PALO ALTO, CALIF.

ISBN: 0-538-13080-6

Library of Congress Catalog Number: 77-75802

6 7 8 D 5 4 3

Printed in the United States of America

PREFACE

Business Mathematics – A College Course is designed to provide students with those mathematical skills and concepts that are beneficial in three ways: (1) in the study of other courses, (2) in the pursuit of a successful business career, and (3) in the everyday activity of being a consumer.

A primary objective of this book is to provide students with a firm foundation in those mathematical abilities that will enhance their success in the study of principles of accounting. Students who have studied business mathematics enjoy a greater degree of success in the study of principles of accounting because they can transfer the knowledge of how and why certain business computations are made. They are enabled to concentrate on learning the principles underlying accounting entries without undue consideration to the calculations involved. For example, after having learned in business mathematics how to calculate discounting of promissory notes, the student in accounting can concentrate on learning the principles pertaining to the recording of notes and interest. After studying business mathematics, students are better prepared for success in a number of courses in addition to accounting – courses such as data processing, retailing, finance, real estate, insurance, and statistics.

There is no escape from mathematics! Computations like those illustrated in this text permeate business and consumer activities. The business employee who understands well and can use expertly the subject matter of this text is likely to be promoted much sooner than less knowledgeable fellow employees. Furthermore, any college student, whether employed or not, can become a more intelligent consumer and investor by mastering the topics presented. From a personal-use perspective, this book is more practical and advantageous than a text in mathematics of finance or mathematical analysis.

The textbook and workbook contain a wealth of problems. There is more than a sufficient quantity of problems in the textbook alone for a two-semester or three-quarter course in business mathematics. For a shorter course of one semester or one or two quarters, the instructor may select those topics needed to meet the objectives of the course. For a short course, almost all of the topics contained in Chapters 6, 7, 8, and 9 should serve as the core around which the course is constructed. Topics from the preceding and/or following chapters should be selected in accordance with the needs of the students to be served.

The sequence of topics is such that knowledge of a subsequent topic is not needed to solve any specific problem. In each unit of work, the topic is presented through explanations, illustrations, and examples followed by exercise problems. Any basic term used in the explanation is clearly defined when initially presented. The problems in each exercise are arranged in order of difficulty. Drill problems designed to promote understanding of the topic at hand usually precede the word problems in an exercise. To reinforce learning, review problems are provided at the end of each chapter and answers to odd-numbered exercises and review problems are provided at the back of the book.

The numbers of the review problems at the end of each chapter are keyed to the exercise numbers within the chapter. More specifically, all review problems numbered *1* are similar to the problems found in the first exercise of the chapter; those numbered *2,* similar to those in the second exercise, and so on. If a specific review problem seems quite difficult to solve, the presentation preceding the exercise with the same number as the problem should be restudied.

The first few chapters provide a review of the fundamental operations in mathematics and algebra without presuming that the student has prior knowledge of the topics. These chapters may be completely or selectively omitted in advanced classes. The review problems at the end of each of the first few chapters may be used for testing purposes to determine which, if any, of the fundamental topics need to be reviewed.

In this edition, all word problems have been modernized as needed to make the text more meaningful to students. New units on ratios and proportions are presented in Chapter 4. To simplify computations and to increase the utility of the textbook, a new interest table and an amortization table for installment payments are included in Chapter 7. A new unit pertaining to net income has been added to Chapter 8. Because of the increasing popularity of the pocket calculator, use of the calculator is frequently mentioned, and alternate solutions for use with calculators are shown for discounts in Chapter 6 and markon in Chapter 8. A new chapter (17) is devoted to U.S. and metric measurements. Units pertaining to perimeter, area, and volume have been included. The metric definitions and symbols presented are in accord with the latest recommendations of the International System of Units (SI) and the United States government. Degrees Celsius and the liter, which are popular non-SI units, are also presented.

Abbreviations and symbols commonly used in business and in mathematics are contained in Appendices A and B, respectively. For easy page-number reference, in addition to the usual table of contents and index, Appendices C and D list the tables and equations used in the textbook.

The workbook consists of problems similar to those in the textbook. As the workbook does not contain any explanations, illustrations, or examples, it should be used only in conjunction with the textbook. Each unit in the workbook is numbered to correspond to the exercise in the textbook that contains similar problems. Each two-page sheet in the workbook may be treated as an individual assignment worth 100 points. Removal of an assignment sheet from the workbook does not jeopardize the solution to any preceding or following problem. To provide a comprehensive review of each chapter, a study guide of objective questions, as well as appropriate problems, follows the chapter problems in the workbook.

A key, which shows not only the answers to simple drill problems but also the detailed solutions to all word problems, is available to teachers. The key also includes test problems for each chapter.

The author hereby acknowledges the constructive suggestions made by numerous teachers who used the first edition in their classes. Special gratitude is given to Richard Korff of Mt. San Antonio College and Dean Chambers of Riverside College.

CONTENTS

CHAPTER 1

Numbers and Fundamental Operations

Modern data processing, automation, and other technological innovations have contributed greatly to the increasingly complex techniques employed in business and industry. Mathematics plays an important role in these complex procedures. Mathematics is not only a tool of great utility, it is also a system of thinking. The employee in business who is able to understand symbols and formulas; to interpret significant reports, graphs, and statistics; and, in general, to think in mathematical terms has a distinct advantage over less capable employees. Mathematics, then, is a very important part of education for employment success; therefore, study mathematics conscientiously.

NOTATION AND NUMERATION: HINDU-ARABIC SYSTEM

The writing of numbers is called *notation*. The naming or reading of numbers is called *numeration*. There are many systems of notation and numeration. The three systems most commonly encountered in business situations, however, are the Hindu-Arabic (or decimal system), the binary system, and the Roman system. Some of the basic concepts and operations in the Hindu-Arabic system are considered in this chapter. The binary and Roman systems are considered in Chapter 3.

The common number system that is most widely used today is called the *decimal system* because it is based on the number ten. The word "decimal" is derived from the Latin word *decimus* which means tenth.

The symbols used to represent numbers are called numerals. A *numeral* is a name for a number. Many symbols may be used to represent a particular number. For example, VI, 0110, $4 + 2$, $7 - 1$, 2×3, $\frac{18}{3}$, and 6 are but a few of the ways in which the name or numeral for the number six may be written. The numeral 6, however, is the most common expression for the number six. In the Hindu-Arabic system, the numerals used to write numbers are called *digits*.

Consider the numeral 333. The digit 3 in the first position on the right indicates three ones; that is, three of whatever is being counted or considered. The digit 3 in the second place from the right designates three groups of ten. In the third place from the right, the digit 3 designates three groups of "ten 10's." In this manner, the *place* the digit occupies determines the value it represents. The value of each place is ten times the value of the place to its right. Any one of the digits can occupy any one of the positions.

The number represented by the digit 4 in 457 is a product. It is the product of four and the place value assigned to the third position on the left, which is one hundred. The number represented by the digit 5 is the product of five and the place value assigned to the second position, which is ten. The 7 represents the product of seven and the place value assigned to the first position, which is one. The complete numeral represents the sum of the three products:

$$\begin{aligned} 457 &= (4 \times 100) + (5 \times 10) + (7 \times 1) \\ &= \quad 400 \quad + \quad 50 \quad + \quad 7 \\ &= \quad 457 \end{aligned}$$

In a whole number, the digit farthest to the right indicates the ones place. In a numeral such as 516.8, the decimal point indicates the starting place. The first digit to the left of the decimal point designates the ones place. The decimal point makes it possible to determine the agreed-on starting place.

In summary, the five basic characteristics of the decimal system are:

1. *Symbols or Digits.* The symbols in use have evolved historically. There are symbols for the numbers one through nine, plus a symbol for the place-holder zero.
2. *Base of Ten.* A finite group of ten symbols is used.
3. *Place Value.* When a digit is shifted one place to the left, its value is increased tenfold. Conversely, when a digit is shifted one place to the right, its value becomes 1/10 of its former value.
4. *Values Designated by the Various Digits Are Added.* For example, 368 means $(3 \times 100) + (6 \times 10) + (8 \times 1)$.
5. *Agreed-on Starting Place.* There must be a means of determining which digit occupies the ones place. The digit farthest on the right is understood to be in the ones place in a whole number. The decimal point is used to indicate the ones place when digits are used to show a decimal value smaller than one.

Reading and Writing Numbers

Many companies sell billions of dollars worth of goods and earn millions of dollars annually. A few state governments and the federal government have annual budgets that total billions of dollars. The gross national product of the United States exceeds the trillion-dollar mark. Thus, as the use of large numbers is becoming increasingly common, individuals must be able to read, write, and understand them.

To facilitate reading a number, commas should be inserted (starting from the right) to separate each group of three digits. Each three-digit group is called a *period;* each period has a name, such as thousands, millions, billions, etc. The ones, tens, and hundreds in each three-digit period, other than units, are read as an individual number followed by the period name. Study the decimal place-value names of each period in Table 1-1.

TABLE 1-1 Decimal Place-Value Names

	trillions			billions			millions			thousands			units		
	hundred trillions	ten trillions	trillions	hundred billions	ten billions	billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	units
A.													1	5	8
B.										3	3	9,	2	4	7
C.								3	1,	5	7	1,	1	1	9
D.				1	0	4,	8	0	1,	0	0	6,	0	2	0
E.		6	0,	0	5	3,	0	0	6,	0	0	0,	8	1	0

Each numeral in Table 1-1 is read as follows:

A. One hundred fifty-eight.
B. Three hundred thirty-nine *thousand,* two hundred forty-seven.
C. Thirty-one *million,* five hundred seventy-one *thousand,* one hundred nineteen.
D. One hundred four *billion,* eight hundred one *million,* six *thousand,* twenty.
E. Sixty *trillion,* fifty-three *billion,* six *million,* eight hundred ten.

Notice in the preceding examples that the hyphen (-) is used for compound numbers, such as fifty-eight. The hyphen is used when the numbers twenty-one through ninety-nine are expressed in compound words.

EXERCISE 1-1

A. Read or write the following numbers.

1. 48
2. 223
3. 2,413
4. 42,167
5. 930,930
6. 6,243,616
7. 80,078,651
8. 637,639,440
9. 86,027,065,009
10. 10,053,537,100,400
11. 22,003,060
12. 800,004,006,005
13. 73,000,025,590,058
14. 30,000,003,000,530
15. 9,000,033,000
16. 995,891,000,000,008
17. 17,004,046,069
18. 9,141,940,625
19. 7,209,069,961,000
20. 610,024,132

B. Express the following as numerals with commas inserted where applicable.

1. Forty-three
2. Eight hundred twenty-nine
3. Seven thousand, three hundred eighty-three
4. Eighteen thousand, seven hundred thirty-two
5. Eighty thousand, six hundred seventy-four
6. Two hundred forty-five thousand, two hundred one
7. One million, eight hundred twenty-two thousand, two hundred ten
8. One hundred six million, one hundred thousand, five
9. Seven billion, nine million, four thousand, three
10. Thirty-seven trillion, five hundred fifteen million, forty-eight thousand

Rounding Off Numbers

A number must sometimes be rounded off because (1) the exact number desired cannot be determined or (2) an exact number should not be used.

Follow these rules [1] to round off a number:

1. **Look at the first digit to the right of the place to which the number is to be rounded.**
2. **If the digit in Step 1 is 5 or larger, increase the last retained digit by 1 and substitute zeros for all digits to its right.**
3. **If the digit in Step 1 is 4 or smaller, substitute zeros for it and all digits to its right.**

[1] These rules are the ones commonly used in business. There are, however, other more technical rules used by scientists, actuaries, and statisticians for rounding when the part omitted is exactly 5.

Example A:	Round off 734,620 to the nearest thousand.	
	↓	
Solution:	734,620	*Explanation:* First digit to right of thousands
	+ 1	place is 5 or larger; thousands digit increased
	735,000	by 1 and zeros substituted to its right.

Example B:	Round off 486,356 to the nearest thousand.	
	↓	
Solution:	486,356	*Explanation:* First digit to right of thousands
	0	place is 4 or smaller; thousands digit remains
	486,000	the same and zeros substituted.

Example C:	Round off 56,953,830 to the nearest hundred thousand.	
	↓	
Solution:	56,953,830	*Explanation:* First digit to right of hundred-
	+ 1	thousands place is 5; hundred-thousands
	57,000,000	digit increased by 1 and zeros substituted.

EXERCISE 1-2

Round off each of the following to the place indicated.

1. 6,433 to the nearest thousand
2. 80,674 to the nearest thousand
3. 24,520 to the nearest ten thousand
4. 18,222 to the nearest ten thousand
5. 10,610,057 to the nearest hundred thousand
6. 943,751,548 to the nearest ten million
7. 619,628,663 to the nearest million
8. 39,631,404,510 to the nearest ten billion
9. 493,445,576,392,984 to the nearest million
10. 78,299,726,483,741 to the nearest trillion
11. 4,757,550,471,729,899 to the nearest hundred billion
12. 788,170,350,978,673 to the nearest ten trillion
13. 73,181,299,731,866 to the nearest hundred million
14. 404,555,898,845 to the nearest hundred billion
15. 9,651,675,336,812 to the nearest trillion
16. 317,730,958,620,080 to the nearest billion

ADDITION OF WHOLE NUMBERS

Addition is the arithmetic operation of combining numbers to obtain a single expression that is an equivalent quantity. The numbers that are added together are called the *addends*. The single expression that represents an equivalent quantity is called the *sum* or *total*.

Example A: Add the following problem.

Solution:	7	addend
	+ 8	addend
	15 ↓	total

As a general rule, only like items (chairs and chairs, ones and ones) should be added together. In the addition of long numerals using pencil and paper, ones are added to ones, tens to tens, hundreds to hundreds, etc., until the total is obtained.

Example B: Add the numerals in this problem.

Solution:

```
 3 1
 253
  92
 370
  87
 146
 ---
 948↓
```

Notice in the preceding example that the sum of the ones column is 18, which means 8 ones and 1 ten. Therefore, the 8 is written in the ones column and the 1 is carried to the tens column. After the 1 is carried, the sum of the tens column is 34, which means 4 tens and 3 hundreds. When the carried 3 is added to the hundreds column, the total of 948 is obtained and the problem is solved.

Learn to add down a column of digits unhesitatingly to improve speed and accuracy.

Checking Accuracy

Once the answer to any mathematical problem has been obtained, it should always be checked for accuracy. There are generally several methods that can be used to check the answer to any mathematical problem. Learn how to use many of these in order to select the most appropriate method for checking the answer to the specific problem at hand.

The two most frequently used methods of proving the accuracy of an addition answer are (1) the reverse-order check and (2) the casting-out-nines check.

Reverse Order. Changing the order of a set of addends does not change the sum of the addends. For this reason, the accuracy of the sum of a set of numbers may be checked by reversing the order in which the numbers were originally added. If the digits were added down the first time, they would be added up the second time. This probably is the best method to use if a non-printing calculator is being used.

Example: Add the following problem. Prove your answer by using the reverse-order check.

Solution:		*Explanation:*
	1,847 ↑	
	256	Add down to obtain sum.
	381	Add up to check accuracy.
	748	
	462	
	1,847 ↓	

EXERCISE 1-3

Add. Check each answer by adding in reverse order.

1. 882	**2.** 269	**3.** 321	**4.** 287	**5.** 792
285	877	728	149	903
263	542	861	648	488
198	55	287	773	764

6. 2,198	**7.** 4,475	**8.** 2,876	**9.** 1,689	**10.** 3,764
8,146	2,342	4,728	846	4,598
3,588	3,215	193	4,478	956
2,175	1,399	1,690	773	3,987
4,600	7,252	785	8,129	708

Casting Out Nines. Casting out nines eliminates nines or combinations that make nine and leaves a difference or excess of nines. When 9 is divided into 23, there is a remainder of 5. The remainder obtained when a number is divided by 9 is called the *excess of nines* in that number. Thus, in 23 the excess of nines is 5 $(23 - 18)$; in 28 the excess of nines is 1 $(28 - 27)$; and in 36 the excess of nines is 0 $(36 - 36)$.

As the remainder must be smaller than 9, the excess of nines in any number must be a one-digit numeral. An easy way to find the excess of nines in a number is to add the digits contained in the number and, if necessary, to add the digits in the resulting sum until a one-digit numeral other than 9 is obtained. Notice that $2 + 3 = 5$, the excess of nines in 23; that $2 + 8 = 10$, and $1 + 0 = 1$, the excess of nines in 28; and that $3 + 6 = 9$, which is "cast out," leaving 0, the excess of nines in 36.

Checking accuracy in addition by casting out nines is quite useful if a calculator or adding machine is not available and the addends are large numbers. When the nines are eliminated from an addend, it is reduced to a single digit. The remainders for the addends can be obtained mentally and jotted down beside the numbers. Adding a single column of digits (the remainders) can be accomplished in much less time than would be required to add large numbers in reverse order. The excess of nines in the total of the addition problem must equal the excess of nines in the sum of the one-digit remainders.

Example A: Add and check by casting out nines.

Solution:

Addition Problem		*Sum of Digits*		*Excess of Nines (Remainder)*
10,823 →	(1 + 0 + 8 + 2 + 3)	14 →	(1 + 4)	5
7,410 →	(7 + 4 + 1 + 0)	12 →	(1 + 2)	3
24,385 →	(2 + 4 + 3 + 8 + 5)	22 →	(2 + 2)	4
3,256 →	(3 + 2 + 5 + 6)	16 →	(1 + 6)	7
31,230 →	(3 + 1 + 2 + 3 + 0)	9 →	(9 − 9)	0
9,864 →	(9 + 8 + 6 + 4)	27 →	(2 + 7 − 9)	0
86,968 total				19 sum
→ 8 + 6 + 9 + 6 + 8 = 37				→ 1 + 9 = 10
and 3 + 7 = 10				and 1 + 0 = 1
and 1 + 0 = 1				

1 ← check numbers equal → 1

To speed up the process of casting out nines, any combination of digits making nine or a multiple of nine may be ignored or canceled in order to find the remainder. The canceling illustrated in Example B below is usually accomplished mentally.

Example B: Add and check by casting out nines.

Solution:

Addition Problem	*Explanation*	*Excess of Nines (Remainder)*
9,463	cast out 9 and 6 + 3	4
8,504	cast out 5 + 4	8
7,032	cast out 7 + 2	3
6,752	cast out 7 + 2; 6 + 5 = 11; 1 + 1 =	2
8,730	cast out 18 (2 × 9) =	0
8,712	cast out 8 + 1 and 7 + 2	0
49,193	cast out 9 and 9	17
→ 4 + 1 + 3 = 8 ←	check numbers equal	→ 8

EXERCISE 1-4

Add. Check each answer by casting out nines. Show the check number.

1.	2.	3.	4.	5.
493	754	699	5,820	9,508
151	288	565	9,232	7,096
401	160	349	9,844	4,895
717	447	486	5,974	8,070
105	487	972	8,581	2,804
			9,182	2,581

6.	7.	8.	9.	10.
13,237	52,112	88,441	479,314	4,134,795
37,470	8,158	7,783	88,137	531,885
9,094	66,914	78,613	139,172	2,173,916
18,899	7,019	66,561	83,034	34,083
3,923	96,779	3,617	4,163	361,409
43,776	4,867	93,585	494,173	3,178,442
5,802	5,690	86,123	55,467	67,455

Shortcut Methods

Make a conscientious effort to learn and use certain shortcut procedures to be able to compute more accurately and more rapidly. Accountants, clerks, and others who must add columns of numerals more frequently than most people use the following methods to save time and increase accuracy when calculators are not available.

Number Combinations That Make 10. One time-saving procedure to follow when adding is to combine two or three digits that total 10. The basic two-digit combinations that total 10 are shown below.

1	2	3	4	5
9	8	7	6	5

The basic three-digit combinations of numerals that total 10 are as follows:

1	1	1	1	2	2	2	3
1	2	3	4	2	3	4	3
8	7	6	5	6	5	4	4

Whenever these combinations of digits occur in addition, add them as 10. In the following problem, for example, using the basic combinations of 10, simply add down the column on the right by thinking 8, 18, 19, 29. After the 2 (tens) is carried to the middle column, it could be added by thinking 9, 19, 24, 34. Likewise, the column on the left could be added by thinking 7, 17, 27, 34.