Analysis and Behavior of Structures

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Influence Lines for Statically Determinate Structures

In establishing mathematical models to evaluate the integrity of a proposed design, the uncertainties of the material properties and the general construction procedures must be assessed and a judgment made of their significance in the analysis of structural response. To an extent the magnitude of these uncertainties can be controlled through testing programs for the structural materials and careful monitoring of the construction process.

An equally important part of the analysis of the behavior of a structure is the loading to which the structure is subjected. Not only is the magnitude of the loads needed, but also the distribution and location of those loads on the structure. A great deal of uncertainty is associated with both the magnitude and the location of live loads on a structure. The designer must envision the worst possible combination of load magnitude and location to which the structure will be subjected during its lifetime. Only then can a realistic mathematical model be established to review the safe performance of the structure.

In the design of bridges and buildings the placement of live loading to produce the maximum stress in a member, or at a point along a member, is part of the analysis process. Configurations of live loads that produce maximum deflections are also important to that process. Experience and intuition are valuable tools of the structural engineer when undertaking these analyses, but the understanding and use of influence lines is also a great aid. In this chapter influence lines are defined and their use in assessing maximum structural response in statically determinate structures is explored. Influence lines are also used in the design of statically indeterminate structures, but the concept and use of influence lines are presented more easily with determinate structures. These ideas are extended to indeterminate systems in Chapter 13.

8.1 Definition of an Influence Line

In Chapter 1 the different types of loads to which structures are subjected were introduced and discussed. In Chapters 2 to 5 techniques were presented to enable analysis of statically determinate beams, trusses, and frames under the action of a fixed combination and configuration of concentrated and distributed loads. These analyses produced the magnitudes of the reactions of structure and the distribution of internal actions required by static equilibrium. The important and interesting question of how the magnitude of a reaction or an internal action changes because of a change in position of a single concentrated load acting on a structure is answered through the establishment and use of influence lines.

A simple definition of an influence line can be stated as follows:

> An influence line is a graphical presentation of the variation of the magnitude of a force, moment, or deflection at a single fixed point in a structure as a function of position of an applied unit load on a structure.

The two key concepts of the definition are that the force, moment, or deflection is to be measured *at a fixed point* in that structure and that the *unit load varies in position* over a specified path in the structure. Implied in the definition and use of the influence line concept is the assumption that the displacements of the structure are so small that its geometry is unchanged due to any loading. This definition is also valid for statically indeterminate structures, so for all structures:

> Influence lines for a structure are based on the assumption that the geometry of the structure is essentially unchanged by the actions of all loads that are applied to the structure.

Influence lines for deflections are not used very frequently and the remainder of this chapter is devoted to influence lines for force or moment actions.

The creation and use of influence lines is best introduced and illustrated for statically determinate beams. After developing the technique for drawing influence lines for reactions, moment at a point, and shear at a point in a beam, the use of influence lines to obtain reactions, moments, or shears at a point due to a general loading is explained. Finally, the drawing of influence lines for trusses, beams, and girder structures and the positioning of groups of loads for maximum effects is illustrated.

8.2 Influence Lines for Beams

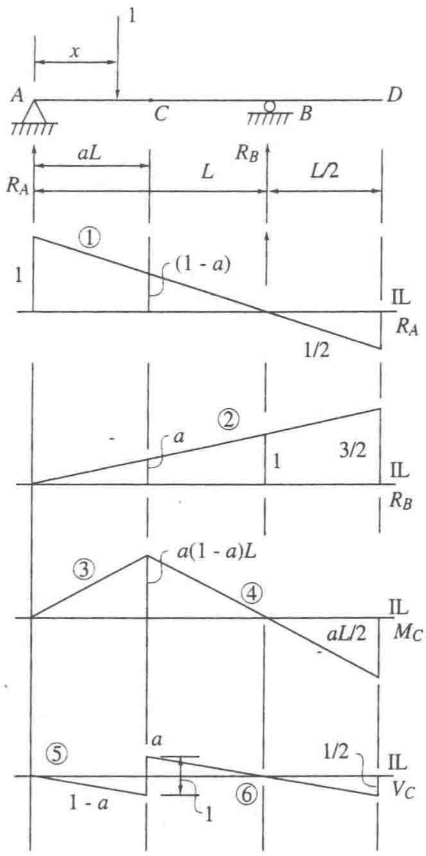
Example 8.1 shows how the influence lines for the reactions and the moment and shear at a specified internal point for a simply supported beam are obtained. While the influence line for any force or moment action always can be written in mathematical form, a presentation of the influence line in graphical form is preferable. The influence line diagram can be constructed simply by noting the manner of variation of the force or moment action and then drawing it accordingly. The concept of obtaining influence lines follows directly from equilibrium considerations in the free body, which isolates the desired action.

Example 8.1 The influence lines for the reactions at A and B of the beam are obtained simply by summing moments about the B and A reaction points, respectively, of the whole beam. The location of the unit load is defined by the coordinate, x, which can have any value from 0 to $3L/2$. The resulting expression for the reactions are in fact functions of x, but only the nature of the variation of the reaction with position of the unit load is noted. Since in this example the variation of the reactions with position of the unit load is linear, the influence line is drawn simply by taking two convenient positions of the unit load (say, $x = 0$ and $x = L$), and computing the magnitude of the reaction of those points, plotting them in the influence line diagram, and drawing a straight line through them over the length of the load path on the structure, which is from A to D. Expressions (1) and (2) in steps 1 and 2 are plotted as the influence lines for the reactions and are labeled with numbers (1) and (2).

In drawing an influence line diagram, it is important to note the limits of applicability of each expression derived from the free-body diagram. This concept is introduced when the influence lines for the moment and shear at C are obtained. The free body for the moment at C, for example, is not of the entire structure. The free-body diagram for M_C is for the A–C part of the beam and also shows the unit load as present. Since the unit load can act anywhere on the structure, it will actually only appear in this free-body diagram when it is positioned somewhere between A and C. However, when the unit load's position is to the right of C it will

Example 8.1

Obtain the influence line for the reaction at A and B and the shear and moment at C.



STEP 1 *Influence line for R_A*: Use the entire structure as a free body and sum moments about B.

$$\curvearrowleft \Sigma M_B: -R_A L + 1(L - x) = 0$$

$$R_A = 1 \cdot \frac{L - x}{L} \quad \therefore R_A \text{ linear} \tag{1}$$

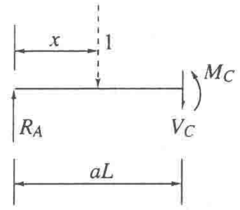
Example 8.1 (continued)

STEP 2 *Influence line for R_B*: Use the entire structure as a free body and sum moments about A.

$$\overset{\curvearrowright}{\Sigma M_A}: 1 \cdot x - R_B L = 0$$

$$R_B = 1 \cdot \frac{x}{L} \qquad \therefore R_B \text{ linear} \qquad (2)$$

STEP 3 *Influence line for M_C*: Cut the beam at C and isolate the free body to the left. Note that the unit load may not be in the free body. Assume first that the unit load is *in* the free body. Show M_C and V_C in a positive sense and use equilibrium equations.



$$\overset{\curvearrowleft}{\Sigma M_{\text{cut}}}: M_C + 1 \cdot (aL - x) - R_A aL = 0$$

$$M_C = R_A \cdot aL - 1 \cdot (aL - x) \qquad \therefore M_C \text{ linear unit load } \textit{in} \text{ free body } (A \rightarrow C) \qquad (3)$$

When the unit load is *outside* the free body, it simply vanishes from the equilibrium expression.

$$M_C = R_A aL \qquad \therefore M_C \text{ linear unit load } \textit{outside} \text{ free body } (C \rightarrow D) \qquad (4)$$

STEP 4 *Influence line for V_C*: Use the same free body as for M_C above. Obtain an expression for V_C when the unit load is *in* the free body and *outside* the free body.

$$\Sigma F\uparrow: R_A - 1 - V_C = 0 \qquad V_C = R_A - 1 \qquad \therefore V_C \text{ linear unit load } \textit{in} \text{ free body } (A \rightarrow C) \qquad (5)$$

$$V_C = R_A \qquad \therefore V_C \text{ linear unit load } \textit{outside} \text{ free body } (C \rightarrow D) \qquad (6)$$

not appear in the free-body diagram and can be said to be outside the free-body diagram. Thus the equilibrium expression for M_C has two forms, one when the unit load is *in* the free body [expression (3)] and a second when it is *outside* the free body [expression (4)]. The limits and form for each expression are noted and the appropriate expression used when drawing the influence line. A similar procedure is used with the same free-body diagram to obtain the influence line for V_C, which yields expressions (5) and (6). The changes in the influence lines for M_C and V_C occur because the unit load passes from the left of C to the right of C as it moves from A to B along the structure. The consequence of this is that two different equilibrium expressions are obtained for M_C and V_C, which depend on the location of the unit load with respect to point C.

The four influence lines presented in Example 8.1 give a complete picture of the variation of the reactions at A and B and the internal shear and moment at C for any position of the unit load on the structure. The units associated with the influence lines are the force unit of the unit load for the reaction and the shear influence lines and the force unit of the unit load multiplied by the length unit used in the analysis for the moment influence line. Consider, for example, the reaction R_A. As a unit load moves across the structure from A to C to B to D, the magnitude of R_A, which has the same force unit as the unit load, varies from 1 to $(1 - a)$ to 0 to $-\frac{1}{2}$. The $-\frac{1}{2}$ indicates that the magnitude of R_A is $\frac{1}{2}$ but that the direction of the reaction is down rather than up as shown in the sketch of the complete structure.

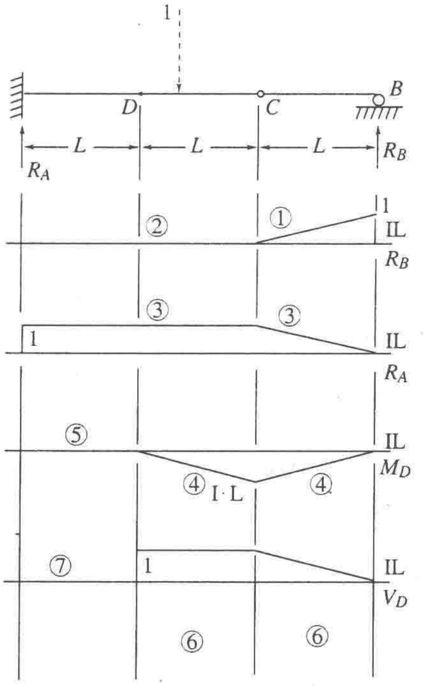
A single look at the influence line diagrams indicates how one would place a concentrated load on the structure to obtain the maximum magnitude of some particular action. To obtain the largest reaction at A due to a concentrated load, the load would be placed at A. The largest reaction at B occurs when the load is placed at D. The largest moment at C would occur when the load is placed at C or D, depending on the value of a. The largest shear occurs when, depending on the value of a, the load is placed just to the left of C, just to the right of C, or at D. Truly it can be said that an influence line provides the structural engineer with valuable insight and information about the response of a structure to loading.

A more complicated structure is analyzed for influence lines in Example 8.2. Here the presence of the hinge introduces additional changes in the shape of the influence line. A general step-by-step analysis procedure is presented in the example that is helpful in the calculations necessary for obtaining and drawing influence lines.

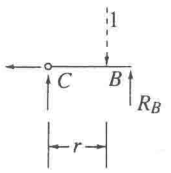
Example 8.2 The structure is first divided into regions, each region being defined by the ends of the member, presence of a reaction component, a continuity release (such as a hinge), or a point in a member where a stress quantity is to be computed. This division of a structure into regions is different from the division used for shear and moment diagrams in Chapter 4. There the regions that define the variation of shear and mo-

Example 8.2

Obtain the influence line for the vertical reactions at A and B and the moment and shear at D.



STEP 1 *Influence line for R_B*: Take the free body C to B.



$$\Sigma M_C: R_B \cdot L - 1 \cdot r = 0$$

$$R_B = 1 \cdot \frac{r}{L} \quad \therefore R_B \text{ linear unit load } in \text{ free body } (C \to B) \qquad (1)$$

$$R_B = 0 \text{ unit load } outside \text{ free body } (A \to C) \qquad (2)$$

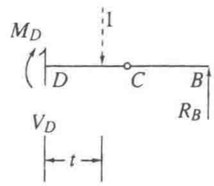
Example 8.2 (continued)

STEP 2 *Influence line for R_A:* Use the entire structure as the free body.

$$\Sigma F\uparrow: R_A - 1 + R_B = 0$$

$$R_A = 1 - R_B \quad \therefore R_A \text{ linear } (A \rightarrow B) \tag{3}$$

STEP 3 *Influence line for M_D:* Use the free body to the right of the cut through D.



$$\overset{\curvearrowright}{\Sigma M_{\text{cut}}}: M_D + 1t - R_B \cdot 2L = 0$$

$$M_D = 2LR_B - 1t \quad \therefore M_D \text{ linear unit load } in \text{ free body } (D \rightarrow B) \tag{4}$$

$$M_D = 2LR_B \quad \therefore M_D \text{ linear unit load outside free body } (A \rightarrow D) \tag{5}$$

STEP 4 *Influence line for V_D:* Take the same free body as that used for M_D and sum forces vertically.

$$\Sigma F\uparrow: V_D - 1 + R_B = 0$$

$$V_D = 1 - R_B \quad \therefore V_D \text{ linear unit load } in \text{ free body } (D \rightarrow B) \tag{6}$$

$$V_D = -R_B \quad \therefore V_D \text{ linear unit load } outside \text{ free body } (A \rightarrow D) \tag{7}$$

ment are determined directly by the nature of the loading diagram, which is fixed by the specified configuration of the applied loading. In the present case the loading is not fixed, but variable, due to the unspecified position of the unit load. In this example there are three regions in which influence line diagrams can take different forms, shown by the vertical lines running through the diagrams. They are defined by the ends of the member, the location of the hinge, and point D, where the internal moment and shear are required. In Example 8.1 there are also three regions, defined by the ends of the member, the reaction at B, and point C, where the internal moment and shear are required. Again they are delineated by the vertical lines running through the diagrams.

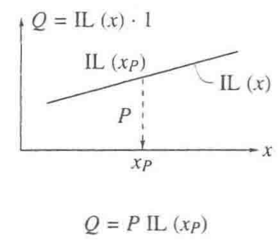
The next step is to obtain the influence line for the reactions. This is a set of influence lines that usually requires very little effort to draw. In the present example the equation of condition for the hinge at C is used immediately to obtain the influence line for the reaction at B. In step 1 the influence line for R_B is determined to be piecewise linear, defined by the two expressions labeled (1) and (2) that come from the free-body diagram of C–B.

In step 2 the influence line for the reaction at A is obtained from a free body of the entire structure and is labeled (3). Note that the expression (3) for R_A is called linear, but is actually piecewise linear due to the piecewise linear nature of the influence line for R_B. The influence line for R_A is constructed in two stages. First it is drawn for the region A–D–C, where R_B is zero in the equation for R_A, which makes R_A constant and equal to 1. The second stage is for region C–B, where R_B, which varies linearly from 0 at C to 1 at D, is subtracted from 1 in expression (3). As can be seen, the influence lines for the reactions have different linear variations in regions A–D–C and C–B which affect the form of the influence lines for the shear and moment at D. The influence lines for the reactions will have the same force unit as that of the unit load.

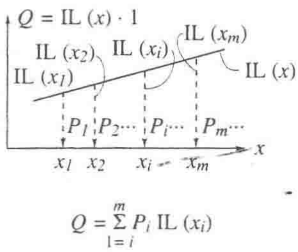
Finally, the free body D–C–B is taken to isolate the desired force or moment actions and equilibrium is used to obtain their variation as a function of position of the unit load. The presence or absence of the unit load in the free body is noted for the appropriate range of its positions. The expression labeled (4) for M_D is called linear but is actually piecewise linear because of the piecewise linear character of the influence line for R_B in regions D–C and C–B. The final portion of the influence line for M_D in the region A–D is obtained from expression (5) for the condition of the unit load outside the free body. The units of the influence line are the force unit of the unit load and the units of length of L. For the same reasons stated for the influence line for the moment, M_D, the influence line for V_D is also piecewise linear in the same regions. Both influence lines for shear and moment at D have been obtained in terms of reaction (R_B in this example), which shows that the influence lines for reactions are required as part of any influence line calculation. For numerical computation in U.S. units, take $L = 6$ ft; for SI units, take $L = 2$ m.

In Examples 8.1 and 8.2, the influence lines are all linear or piecewise linear. This is a consequence of the fact that these structures are statically determinate. As shown later, statically determinate structures, be they beams, frames, or trusses, will all have piecewise linear influence lines for all force or moment actions. Indeterminate structures will have influence lines that are generally curved in nature, as shown in a later chapter.

8.3 Use of the Influence Line to Obtain the Magnitude of a Force or Moment Action for a General Loading



(a) Single concentrated load



(b) Several concentrated loads

Figure 8.1a–b
Calculation of a force or moment action due to concentrated loads.

Having an influence line for a force or moment action at a specific point significantly reduces the effort required to compute the magnitude of that force or moment action for a general loading. As seen in Fig. 8.1a, due to a single concentrated load of magnitude P, the value of a force or moment action, Q, is simply the product of P and the ordinate of the influence line for Q, IL(x), at the point of application of the load, x_P.

$$Q = P\text{IL}(x_P) \quad \text{single load} \tag{8.1}$$

This follows from the definition of the influence line that is based on a concentrated load of unit magnitude. It is assumed in Eq. (8.1) that the unit loads used to obtain IL(x) and P have the same force unit and direction of action. In Fig. 8.1b a number of loads, m, of magnitude P_i act at the m points x_i. The magnitude of Q due to all loads acting simultaneously is simply the sum of the magnitude of each load acting alone, as given in Eq. (8.1). This yields the mathematical statement

$$Q = \sum_{i=i}^{m} P_i IL\,(x_i) \quad m \text{ loads} \tag{8.2}$$

The effect of an arbitrary disturbed load $q(x)$ over a range of x from a to b can be obtained by direct integration. In Fig. 8.2 a differential slice of the distributed load, $q(x)\,dx$, is equivalent to a concentrated load, and hence the concept of summation of effects expressed by Eq. (8.2) can be used to obtain Q if the summation is replaced by integration over the range of x from a to b. This becomes

$$Q = \int_a^b q(x) IL(x)\,dx \quad \text{arbitrary distributed load} \tag{8.3}$$

The force unit of Q in Eq. (8.3) is the same as the force unit of $q(x)$. Although Eq. (8.3) defines the most general case, it is instructive to look at the common case where the distributed load $q(x)$ is of constant magnitude w. As can be seen in Eq. (8.3), if $q(x)$ is replaced with w, it can be tak across the integration sign since it is constant. The resulting integral is simply the area under the influence line diagram. Thus

$$Q = \int_a^b wIL\,(x)\,dx = w \int_a^b IL\,(x)\,dx$$
$$= w\ [\text{area under IL}(x)\text{ between } a \text{ and } b] \qquad (8.4)$$

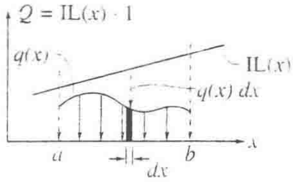
as shown in Fig. 8.3.

Now that the method of calculation of a force or moment action, Q, has been established using influence lines, it is a simple matter to extend the ideas to include the calculation of the absolute maximum possible value of Q for a given loading. For a single concentrated load, the maximum value of Q is given when the load acts on the structure at a point where the ordinate to the influence line for Q is an absolute maximum.

Distributed loads can act over any length of the structure. In design where the distributed load is treated as a live load (i.e., it can act over one or more different portions of the structure), the assumption is made that at some time in the life of the structure the load will be distributed on the structure in such a manner that it will cause the maximum magnitude of a force or moment action. For a uniform load, w, the maximum value of Q is obtained when the distributed load is placed on those portions of a structure such that the area, either positive or negative, under the influence line for Q is a maximum. The maximum value of Q due to the combination of a concentrated load and a uniform load of variable length is simply the sum of the maxima of the same sign due to each load type computed separately.

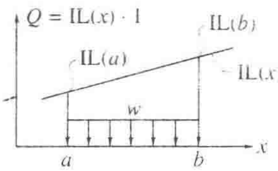
The discussion above shows how the structural engineer can tell exactly where to place loads and combinations of loads to achieve a maximum effect once an influence line for that effect has been established. The placement for maximum effect of only a single concentrated load has been discussed above, but the placement of multiple concentrated loads is more complicated and will be considered in a later section. The great advantage of influence lines is that once they are established, the magnitude and sense of a stress quantity can be obtained for any arbitrary loading with relative ease.

A simple example of the application of influence lines to compute maximum values of reactions and the internal moment in a structure due to both concentrated and distributed loads is shown in Example 8.3.



$$Q = \int_a^b IL\,(x) q(x)\,dx$$

Figure 8.2 Calculation of a force or moment action due to an arbitrary distributed load



$$Q = \int_a^b IL(x)w\,dx = w \int_a^b IL(x)\,dx$$

$$Q = \frac{w\,(b-a)}{2}\left[IL(a) + IL(b)\right]$$

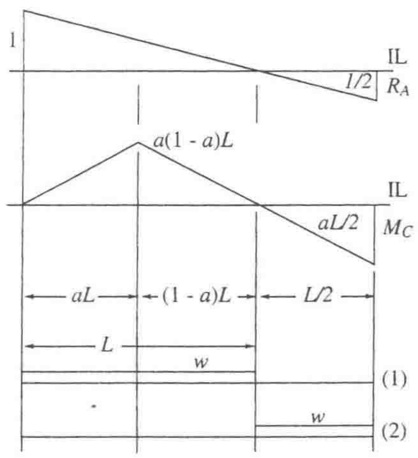
Figure 8.3 Calculation of a force of moment action due to a uniformly distributed load.

Example 8.3 The influence lines for the reaction, R_A, and the moment, M_C, of the structure of Example 8.1 are shown. A concentrated load of magnitude P, and a uniform distributed load of magnitude w and variable length are placed on the structure to create maximum positive and negative values of these two force actions.

In part (1) the concentrated load, P, is placed at the maximum positive and negative ordinates of the influence lines to obtain the maximum positive and negative values of R_A and M_C. In part (2), the uniform load, w,

Example 8.3

Use the influence lines for R_A and M_C from Example 8.1 for the following calculations:



1. For a concentrated load, P, obtain the maximum positive and negative R_A and M_C. Use Eq. (8.1).

$$(R_A)_{\text{max. pos.}} = P \cdot 1 = P$$

$$(R_A)_{\text{max. neg.}} = P\left(-\frac{1}{2}\right) = -\frac{P}{2}$$

$$(M_C)_{\text{max. pos.}} = P\left(a(1 - a)L\right) = Pa(1 - a)L$$

$$(M_C)_{\text{max. neg.}} = P\left(\frac{-aL}{2}\right) = -P\frac{aL}{2}$$

Note: For concentrated loads the maximum positive and negative ordinate of each influence line are used in Eq. (8.1).

Example 8.3 (continued)

2. For a uniform load w of variable extent, obtain the maximum positive and negative values of R_A and M_C. Use Eq. (8.4).

$(R_A)_{\text{max. pos.}}$: Use load distribution configuration (1) above

$$(R_A)_{\text{max. pos.}} = \frac{1}{2} \cdot w \cdot 1 \cdot L = \frac{wL}{2}$$

$(R_A)_{\text{max. neg.}}$: Use load distribution configuration (2) above

$$(R_A)_{\text{max. neg.}} = \frac{1}{2} \cdot w \cdot \frac{L}{2} \cdot \left(\frac{-1}{2}\right) = \frac{-wL}{8}$$

$(M_C)_{\text{max. pos.}}$: Use load distribution configuration (1) above

$$(M_C)_{\text{max. pos.}} = \frac{1}{2} wLa\,(1 - a)L = wa(1 - a)\frac{L^2}{2}$$

$(M_C)_{\text{max. pos.}}$: Use load distribution configuration (2) above

$$(M_C)_{\text{max. neg.}} = \frac{1}{2} w \frac{L}{2} \cdot \left(\frac{-aL}{2}\right) = \frac{-waL^2}{8}$$

is assumed to act on the structure in one of the two distributions, (1) o (2) shown. Load distribution (1) corresponds to the regions in the influ ence line diagrams that have positive ordinates. Load distribution (2 corresponds to the regions in the influence line diagrams that have nega tive ordinates. To obtain the maximum positive value of R_A due to w, must be distributed over that portion of the influence line for R_A that i positive, which corresponds to distribution (1). Using Eq. (8.4), th maximum positive value of R_A is $wL/2$, as indicated. For the maximu negative value of R_A due to w, distribution (2) is used. Again, Eq. (8.4 yields the value $-wL/8$ for R_A, which corresponds to distribution (2 Similar considerations yield the maximum positive and negative valu of the moment M_C.

8.4 Influence Lines for Trusses

In Chapter 3 it was established that loads are applied to the panel points trusses by means of a system of stringers and floor beams. Because loads a applied only at panel points of trusses, the drawing of any influence line fo truss can be undertaken simply by applying the unit load successfully at ea panel point of the truss and then computing the force action. This technique tedious and unnecessary if the load-carrying actions of a truss are understoo

Figure 3.3 shows the floor system for a highway truss bridge, which is ty ical of the construction of trusses to carry loads over long spans, although t floor system may connect to the top rather than bottom panel points of t truss. The floor system of the truss is constructed from floor beams that perpendicular to the two main trusses and connect the panel point of o truss to the corresponding panel point of the other truss. The connection moment free, so that the floor beam can be considered to be simply suppor at the two panel points. Running parallel to the trusses between the fl beams are stringers which are connected in a moment-free manner to floor beams. The stringers are considered to be simply supported beams re ing on the floor beams. The deck of the structure then rests on the stringe

The action of the floor system can be described in the following mann Any load applied to a floor beam is carried to each of the two trusses in p portion to its location on the floor beam. The load appears at the pa points of each truss. Any load applied to a stringer is carried by proport to each of the floor beams at the ends of the stringer and then to the f panel points of the trusses to which the two floor beams are connected. A load applied to the deck between stringers and floor beams is assumed to by the action of the deck proportionately to the stringers, then proporti ately to the floor beams, and finally proportionately to the panel point the truss. For every point on the deck of the truss, there is a load path f concentrated load to follow to panel points of the two main trusses.

The process just described shows how a three-dimensional pattern of loading is resolved into a vertical loading on the two main trusses of a bridge. The design of these trusses for the vertical loading is then considered to be a two-dimensional problem. In drawing influence lines for a truss, it is convenient to take the unit loads to act in the plane of the truss.

The action of the floor system provides a linearly varying change in the magnitude of the load that appears at a panel point as the load moves parallel to the axis of the truss between that panel point and adjacent panel points. For example, if a unit load acts on the floor beam that connects to panel point L_1 in Fig. 3.3, it appears in the truss as a load at L_1 only. As the load moves from the floor beam at L_1 to the floor beam at L_2, the effect is for the floor beam at L_1 to carry a linearly decreasing proportion of the unit load while the floor beam at L_2 carries simultaneously a linearly increasing proportion of the load. When the unit load arrives at the floor beam connected to L_2, all of the load enters at panel L_2. These observations are very important to the process of drawing influence lines for trusses.

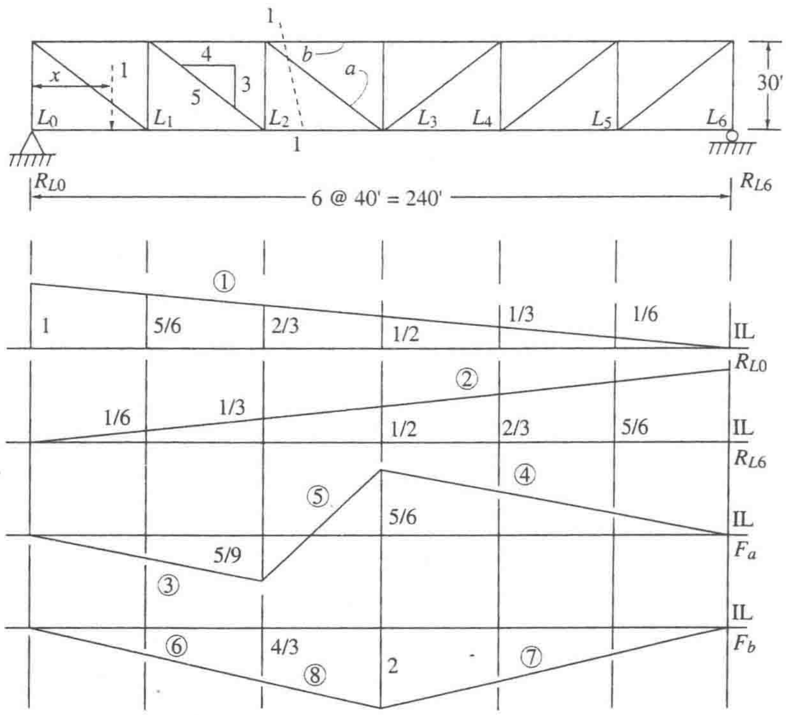
In design the influence lines for the forces in different members of a truss are wanted. The procedure that is followed to obtain these influence lines is nearly the same as that for obtaining the influence lines for shear and moment at some point in a beam. An important difference in developing the influence lines for trusses is the treatment of the unit load in the free body, which is used to isolate the force in a particular truss member. Conceptually, the unit load can either be in the free body or outside the free body as with beams, but since the free body of the truss does not include the floor system, an important refinement must be introduced. For a position of the unit load on the floor system between panel points, the effect of the unit load will enter the truss at two panel points, as discussed above. Since a free body of the truss is created by a section between panel points, the unit load may be positioned on the floor system so that the panel points where it enters the truss are both in the free body, both outside the free body, or with one in the free body and one outside it. In these situations, the unit load is said to be entirely in the free body, entirely outside the free body, or partially in the free body, respectively. These ideas are shown in Example 8.4.

Example 8.4 In the first step the influence lines for the reactions of the truss are obtained. The structure can be considered to act as a single rigid member in obtaining those influence lines. Next, the panel points are identified as dividing points for each of the six regions of the structure. Finally, using the method of sections technique presented in Chapter 3, a section is passed through the structure, isolating a free body with the member force of interest.

The influence line for F_a is obtained from the free body containing the panel points L_0, L_1, and L_2. This free body will have the unit load entirely in it as long as the unit load is acting on the floor system between L_0 and L_2 because the floor system will always transmit the effect of the unit

Example 8.4

For the truss shown, obtain influence lines for R_{L0}, R_{L6}, F_a, and F_b.



STEP 1 Obtain the influence line for R_{L0} and R_{L6} by using the entire structure as a free body.

$$\overset{\curvearrowright}{\Sigma M_{L6}}: 1 \cdot (240 - x) + R_{L0} \cdot 240 = 0 \rightarrow R_{L0} = 1 \cdot \frac{240 - x}{240}$$

$$\therefore R_{L0} \text{ linear } (L_0 \rightarrow L_6) \tag{1}$$

$$\overset{\curvearrowleft}{\Sigma M_{L0}}: \mathrm{R}_{L0} \cdot 240 - 1 \cdot \mathrm{x} = 0 \rightarrow R_{L6} = 1 \cdot \frac{x}{240}$$

$$\therefore R_{L6} \text{ linear } (L_0 \rightarrow L_6) \tag{2}$$