TEXTBOOKS IN MATHEMATICS

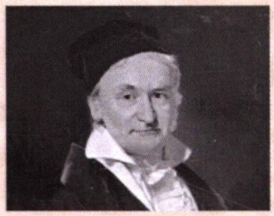
A CONCRETE INTRODUCTION TO REAL ANALYSIS

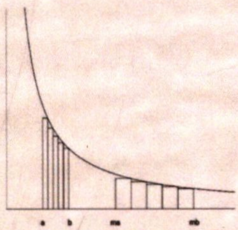
SECOND EDITION

$$F(b) - F(a) = \int_a^b f(x)\, dx$$

$$\sqrt{\pi} = \int_{-\infty}^{\infty} e^{-x^2}\, dx$$

$$e^{i\pi} = -1$$





Robert Carlson

CRC Press
Taylor & Francis Group
A CHAPMAN & HALL BOOK

TEXTBOOKS in MATHEMATICS

A Concrete Introduction to Real Analysis

Second Edition

Robert Carlson

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business
A CHAPMAN & HALL BOOK

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

International Standard Book Number-13: 978-1-498-77813-8 (Hardback)

Visit the Taylor & Francis Web site at
http://www.taylorandfrancis.com

and the CRC Press Web site at
http://www.crcpress.com

FSC www.fsc.org MIX Paper from responsible sources FSC® C013056

Printed and bound in Great Britain by
TJ International Ltd, Padstow, Cornwall

A Concrete Introduction to Real Analysis

Second Edition

TEXTBOOKS in MATHEMATICS

Series Editors: Al Boggess and Ken Rosen

PUBLISHED TITLES

ABSTRACT ALGEBRA: A GENTLE INTRODUCTION
Gary L. Mullen and James A. Sellers

ABSTRACT ALGEBRA: AN INTERACTIVE APPROACH, SECOND EDITION
William Paulsen

ABSTRACT ALGEBRA: AN INQUIRY-BASED APPROACH
Jonathan K. Hodge, Steven Schlicker, and Ted Sundstrom

ADVANCED LINEAR ALGEBRA
Hugo Woerdeman

ADVANCED LINEAR ALGEBRA
Nicholas Loehr

ADVANCED LINEAR ALGEBRA, SECOND EDITION
Bruce Cooperstein

APPLIED ABSTRACT ALGEBRA WITH MAPLE™ AND MATLAB®, THIRD EDITION
Richard Klima, Neil Sigmon, and Ernest Stitzinger

APPLIED DIFFERENTIAL EQUATIONS: THE PRIMARY COURSE
Vladimir Dobrushkin

APPLIED DIFFERENTIAL EQUATIONS WITH BOUNDARY VALUE PROBLEMS
Vladimir Dobrushkin

APPLIED FUNCTIONAL ANALYSIS, THIRD EDITION
J. Tinsley Oden and Leszek Demkowicz

A BRIDGE TO HIGHER MATHEMATICS
Valentin Deaconu and Donald C. Pfaff

COMPUTATIONAL MATHEMATICS: MODELS, METHODS, AND ANALYSIS WITH MATLAB® AND MPI, SECOND EDITION
Robert E. White

A COURSE IN DIFFERENTIAL EQUATIONS WITH BOUNDARY VALUE PROBLEMS, SECOND EDITION
Stephen A. Wirkus, Randall J. Swift, and Ryan Szypowski

A COURSE IN ORDINARY DIFFERENTIAL EQUATIONS, SECOND EDITION
Stephen A. Wirkus and Randall J. Swift

DIFFERENTIAL EQUATIONS: THEORY, TECHNIQUE, AND PRACTICE, SECOND EDITION
Steven G. Krantz

DIFFERENTIAL EQUATIONS: THEORY, TECHNIQUE, AND PRACTICE WITH BOUNDARY VALUE PROBLEMS
Steven G. Krantz

DIFFERENTIAL EQUATIONS WITH APPLICATIONS AND HISTORICAL NOTES, THIRD EDITION
George F. Simmons

DIFFERENTIAL EQUATIONS WITH MATLAB®: EXPLORATION, APPLICATIONS, AND THEORY
Mark A. McKibben and Micah D. Webster

DISCOVERING GROUP THEORY: A TRANSITION TO ADVANCED MATHEMATICS
Tony Barnard and Hugh Neill

DISCRETE MATHEMATICS, SECOND EDITION
Kevin Ferland

ELEMENTARY DIFFERENTIAL EQUATIONS
Kenneth Kuttler

ELEMENTARY NUMBER THEORY
James S. Kraft and Lawrence C. Washington

THE ELEMENTS OF ADVANCED MATHEMATICS: FOURTH EDITION
Steven G. Krantz

ESSENTIALS OF MATHEMATICAL THINKING
Steven G. Krantz

EXPLORING CALCULUS: LABS AND PROJECTS WITH MATHEMATICA®
Crista Arangala and Karen A. Yokley

EXPLORING GEOMETRY, SECOND EDITION
Michael Hvidsten

EXPLORING LINEAR ALGEBRA: LABS AND PROJECTS WITH MATHEMATICA®
Crista Arangala

EXPLORING THE INFINITE: AN INTRODUCTION TO PROOF AND ANALYSIS
Jennifer Brooks

GRAPHS & DIGRAPHS, SIXTH EDITION
Gary Chartrand, Linda Lesniak, and Ping Zhang

INTRODUCTION TO ABSTRACT ALGEBRA, SECOND EDITION
Jonathan D. H. Smith

INTRODUCTION TO ANALYSIS
Corey M. Dunn

INTRODUCTION TO MATHEMATICAL PROOFS: A TRANSITION TO ADVANCED MATHEMATICS, SECOND EDITION
Charles E. Roberts, Jr.

PUBLISHED TITLES CONTINUED

INTRODUCTION TO NUMBER THEORY, SECOND EDITION
Marty Erickson, Anthony Vazzana, and David Garth

INVITATION TO LINEAR ALGEBRA
David C. Mello

LINEAR ALGEBRA, GEOMETRY AND TRANSFORMATION
Bruce Solomon

MATHEMATICAL MODELLING WITH CASE STUDIES: USING MAPLE™ AND MATLAB®, THIRD EDITION
B. Barnes and G. R. Fulford

MATHEMATICS IN GAMES, SPORTS, AND GAMBLING–THE GAMES PEOPLE PLAY, SECOND EDITION
Ronald J. Gould

THE MATHEMATICS OF GAMES: AN INTRODUCTION TO PROBABILITY
David G. Taylor

A MATLAB® COMPANION TO COMPLEX VARIABLES
A. David Wunsch

MEASURE AND INTEGRAL: AN INTRODUCTION TO REAL ANALYSIS, SECOND EDITION
Richard L. Wheeden

MEASURE THEORY AND FINE PROPERTIES OF FUNCTIONS, REVISED EDITION
Lawrence C. Evans and Ronald F. Gariepy

NUMERICAL ANALYSIS FOR ENGINEERS: METHODS AND APPLICATIONS, SECOND EDITION
Bilal Ayyub and Richard H. McCuen

ORDINARY DIFFERENTIAL EQUATIONS: AN INTRODUCTION TO THE FUNDAMENTALS
Kenneth B. Howell

PRINCIPLES OF FOURIER ANALYSIS, SECOND EDITION
Kenneth B. Howell

REAL ANALYSIS AND FOUNDATIONS, FOURTH EDITION
Steven G. Krantz

RISK ANALYSIS IN ENGINEERING AND ECONOMICS, SECOND EDITION
Bilal M. Ayyub

SPORTS MATH: AN INTRODUCTORY COURSE IN THE MATHEMATICS OF SPORTS SCIENCE AND SPORTS ANALYTICS
Roland B. Minton

A TOUR THROUGH GRAPH THEORY
Karin R. Saoub

PUBLISHED TITLES CONTINUED

TRANSITION TO ANALYSIS WITH PROOF
Steven G. Krantz

TRANSFORMATIONAL PLANE GEOMETRY
Ronald N. Umble and Zhigang Han

Preface

This book is an introduction to real analysis, which might be briefly defined as the part of mathematics dealing with the theory of calculus and its more or less immediate extensions. Some of these extensions include infinite series, differential equations, and numerical analysis. This brief description is accurate, but somewhat misleading, since analysis is a huge subject which has been developing for more than three hundred years, and has deep connections with many subjects beyond mathematics, including physics, chemistry, biology, engineering, computer science, and even business and some of the social sciences.

The development of analytic (or coordinate) geometry and then calculus in the seventeenth century launched a revolution in science and world view. Within one or two lifetimes scientists developed successful mathematical descriptions of motion, gravitation, and the reaction of objects to various forces. The orbits of planets and comets could be predicted, tides explained, artillery shell trajectories optimized. Subsequent developments built on this foundation include the quantitative descriptions of fluid motion and heat flow. The ability to give many new and interesting quantitatively accurate predictions seems to have altered the way people conceived the world. What could be predicted might well be controlled.

During this initial period of somewhat over one hundred years, the foundations of calculus were understood on a largely intuitive basis. This seemed adequate for handling the physical problems of the day, and the very successes of the theory provided a substantial justification for the procedures. The situation changed considerably in the beginning of the nineteenth century. Two landmark events were the systematic use of infinite series of sines and cosines by Fourier in his analysis of heat flow, and the use of complex numbers and complex valued functions of a complex variable. Despite their ability to make powerful and accurate predictions of physical phenomenon, these tools were difficult to understand intuitively. Particularly in the area of Fourier series, some nonsensical results resulted from reasonable operations. The resolution of these problems took decades of effort, and involved a careful reexamination of the foundations of calculus. The ancient Greek treatment of geometry, with its explicit axioms, careful definitions, and emphasis on proof as a reliable foundation for reasoning, was used successfully as a model for the development of analysis.

A modern course in analysis usually presents the material in an efficient but austere manner. The student is plunged into a new mathematical environment, replete with definitions, axioms, powerful abstractions, and an overriding emphasis on formal proof. Those students able to find their way in these new surroundings are rewarded with greatly increased sophistication, particularly in their ability to reason effectively about mathematics and its applications to such fields as physics, engineering and scientific computation. Unfortunately, the standard approach often produces large numbers of casualties, students with a solid aptitude for mathematics who are discouraged by the difficulties, or who emerge with only a vague impression of a theoretical treatment whose importance is accepted as a matter of faith.

This text is intended to remedy some of the drawbacks in the treatment of analysis, while providing the necessary transition from a view of mathematics focused on calculations to a view of mathematics where proofs have the central position. Our goal is to provide students with a basic understanding of analysis as they might need it to solve typical problems of science or engineering, or to explain calculus to a high school class. The treatment is designed to be rewarding for the many students who will never take another class in analysis, while also providing a solid foundation for those students who will continue in the "standard" analysis sequence.

Contents

List of Figures