Problems Book for Probabilistic Methods for the Theory of Structures with Complete Worked Through Solutions

Isaac Elishakoff

World Scientific

Problems Book for Probabilistic Methods for the Theory of Structures with Complete Worked Through Solutions

The first edition of the combined monograph and textbook *Probabilistic Methods in the Theory of Structures* was published by Wiley-Interscience in 1983. 1n 1999, Dover Publications, Inc. published its second edition under shorter title *Probabilistic Theory of Structures*. Now, World Scientific has expanded into a 3rd edition to include *Problems with Complete Worked-Through Solutions*. This compendium of solutions was written in response to requests by numerous university educators around the world, since it has been adopted as a textbook or an additional reading both undergraduate and graduate courses.

The author hopes that the availability of such solutions manual will further help to establish the courses dealing with probabilistic strength of materials, design, random buckling, and random vibration. The material itself was developed by author for various undergraduate and graduate courses, during years 1972–1989 at the Technion–Israel Institute of Technology, in Haifa, Israel, at the Delft University of Technology, The Netherlands, year 1979/80 at the University of Notre Dame, Indiana, USA and at the Florida Atlantic University, USA since 1994.

Already since mid-eighties, the author was informed that the book was adopted in numerous universities worldwide. Besides complete solutions to more than one hundred problems, additional material and remarks are included as Chapter 12, bringing some ideas down to the "number" level.

It is strongly hoped that this manual will promote much wider dissemination of probabilistic methods' courses at universities, and ultimately, in engineering practice worldwide.

World Scientific
www.worldscientific.com
10311 hc

ISBN 978-981-3201-10-1
9 789813 201101

Problems Book for Probabilistic Methods for the Theory of Structures with Complete Worked Through Solutions

Elishakoff

Problems Book for Probabilistic Methods for the Theory of Structures with Complete Worked Through Solutions

Isaac Elishakoff
Distinguished Research Professor
Florida Atlantic University, USA
&
Visiting Distinguished Professor
Technion–Israel Institute of Technology, Israel

World Scientific

NEW JERSEY · LONDON · SINGAPORE · BEIJING · SHANGHAI · HONG KONG · TAIPEI · CHENNAI · TOKYO

Published by

World Scientific Publishing Co. Pte. Ltd.
5 Toh Tuck Link, Singapore 596224
USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601
UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

PROBLEMS BOOK FOR PROBABILISTIC METHODS FOR THE THEORY OF STRUCTURES WITH COMPLETE WORKED THROUGH SOLUTIONS

ISBN 978-981-3201-10-1
ISBN 978-981-3201-11-8 (pbk)

For any available supplementary material, please visit
http://www.worldscientific.com/worldscibooks/10.1142/10311#t=suppl

Typeset by Stallion Press
Email: enquiries@stallionpress.com

Printed in Singapore

Problems Book for Probabilistic Methods for the Theory of Structures with Complete Worked Through Solutions

Preface

The first edition of the combined monograph and textbook *Probabilistic Methods in the Theory of Structures* was published by Wiley-Interscience in 1983. 1n 1999, Dover Publications, Inc. published its second edition under shorter title *Probabilistic Theory of Structures.* Now, World Scientific kindly publishes, simultaneously, the third, corrected and expanded edition, and its companion volume containing *Problems with Complete Worked-Through Solutions*. This compendium of solutions was written in response to requests by numerous university educators around the world, since it has been adopted as a textbook or an additional reading both undergraduate and graduate courses.

I hope that the availability of such solutions manual will further help to establish the courses dealing with probabilistic strength of materials, design, random buckling, and random vibration. The material itself was developed and used by me during teaching various undergraduate and graduate courses at the Technion, during years 1972-1989, at the Delft University of Technology during the academic year 1979/80, and at the University of Notre Dame during the academic year 1985/86, and at the Florida Atlantic University since 1989.

Already since mid-eighties I was informed that the book was adopted in numerous universities worldwide. Besides complete solutions to more than one hundred problems, additional material and remarks are included as Chapter 12, bringing some ideas down to the "number" level. Errors and misprints (as a random event) are obviously inevitable. I hope that they are few and that readers will be kind enough to call my attention to them so that they can be corrected.

It is a pleasure to acknowledge the help I received during the preparation of the manual. Amongst my colleagues at the Aeronautical Department at the Technion-I.T.T., I am indebted to Dr. Yehuda Stavsky, Gerard Swope Professor in Mechanics, and Professor Menahem Baruch, then Dean of the Department, for discussions on number of probabilistic topics. The manual was received much boost during my stay at the Department of Aerospace and Mechanical Engineering of the University of Notre Dame, as a Visiting Frank M. Freimann Chair Professor, during the academic year 1985/1986. My sincere thanks are due to Professor Albin A. Szewzyk, Chairman of the Department, as well as the staff, for providing an ever pleasant and encouraging atmosphere.

Among my numerous students, my sincere thanks are due to Dr. Gabriel Cederbaum, currently Associate Professor at the University of Ben Gurion, Beer-Sheva, Israel, providing a number of solutions and painstakingly going through most of them. Credit for editing the text goes to my friend of long standing Mr. Eliezer Goldberg, Eng. Last, but not least, I wish to thank Mr. Brandon Naar of the Florida Atlantic University, for typing the entire manuscript in Fall 2015 and Spring 2016 semsters. I am very appreciative to Ing. Damien Delbecq of IFMA- French Institute of Advanced Mechanics for creating the figures, during his stay at the Florida Atlantic University during academic year 2015/6. Special thanks are due to Ms. Rochelle Kronzek, Executive Editor of the World Scientific Publishing Company for her indefatigable insistence this solutions manual to be submitted for publication so as to help the lecturer and student alike in understanding the nitty-gritty of probabilistic methods in structural applications.

It is strongly hoped that this manual will promote much wider dissemination of probabilistic methods' courses at universities, and ultimately, in engineering practice worldwide.

Isaac Elishakoff
Boca Raton, Florida, May, 2016

Contents

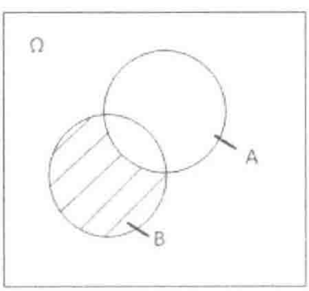
chapter 2

Probability Axioms

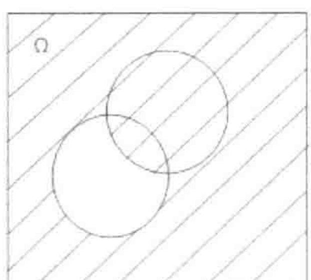
PROBLEM 2.1

Present a Venn diagram for $\bar{C}$, where $C = B \backslash A$.

SOLUTION 2.1



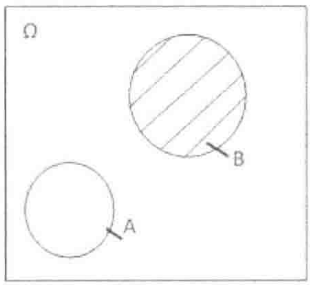
$C = B/A$ Hatched

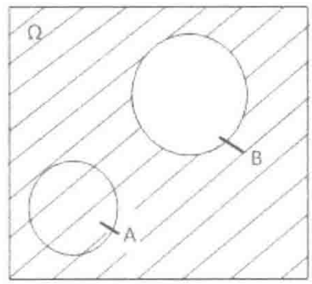


$\bar{C}$ hatched

A and B have common points

If A and B have no joint points

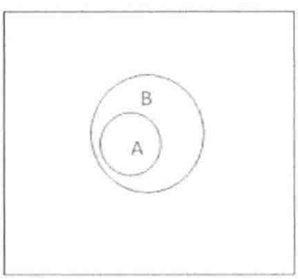




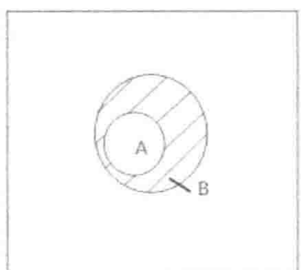
$C = B/A$ hatched

$\bar{C}$ hatched

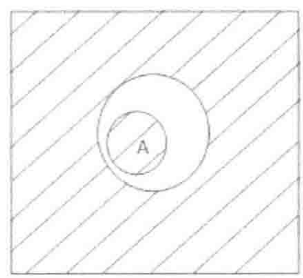
$A \subset B$



$A \subset B$



$C = B/A$ hatched



$\bar{C}$ hatched

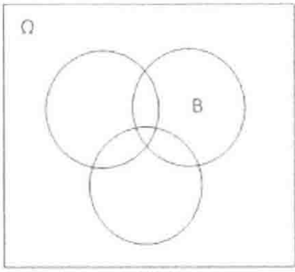
PROBLEM 2.2

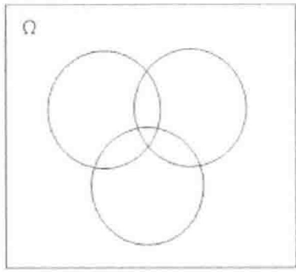
Verify by means of a Venn diagram that a union and an intersection of random events are distributive, that is,

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

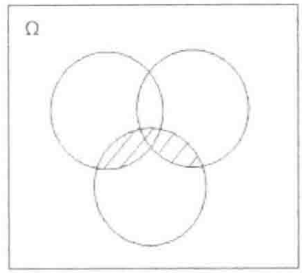
$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

SOLUTION 2.2

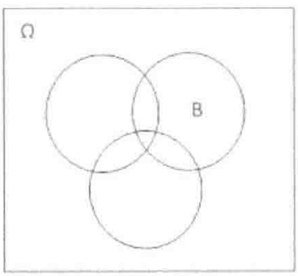




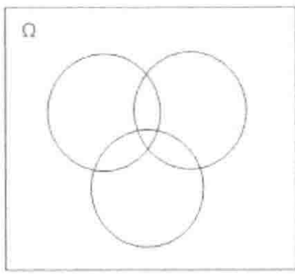
$A \cup B$



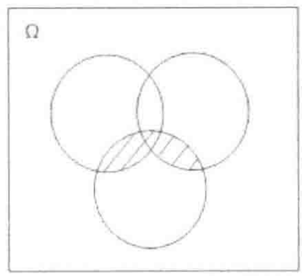
$(A \cup B) \cap C$



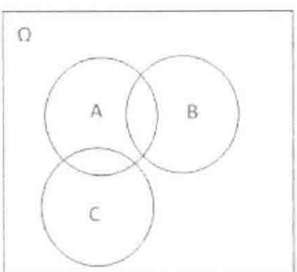
$A \cap C$

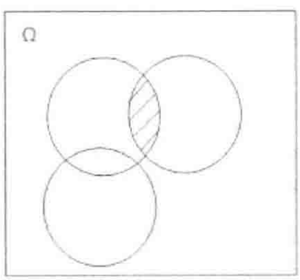


$B \cap C$

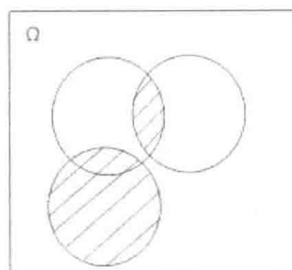


$(A \cap C) \cup (B \cap C)$

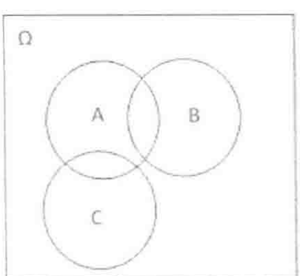




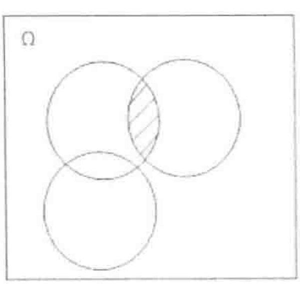
$A \cap B$



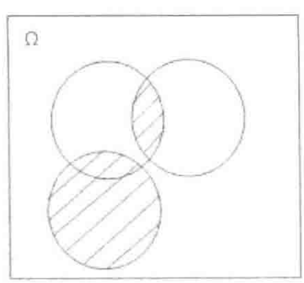
$(A \cap B) \cup C$



$A \cup C$



$B \cup C$



$(A \cup C) \cap (B \cup C)$

PROBLEM 2.3

A telephone relay satellite is known to have five malfunctioning channels out of 500 available. If a customer gets one of the malfunctioning channels on first dialing, what is the probability of his hitting on another malfunctioning channel on dialing again?

Note to Lecturer: Better to change "Another Malfunction Channel" to "Again Hitting on a Malfunctioning Channel."

SOLUTION 2.3

The random event (A_2) of hitting on a malfunctioning channel with second dialing is independent of what happened during previous dialings (A_1), therefore the probability of failure at the second dialing equals that at the first:

$$P(A_2|A_1) = P(A_2) = 0.01$$

The problem is equivalent to the following:

An opaque urn contains "a" white balls are "b" black balls. What is the probability of drawing a white ball in an honestly mixed lot in the course of Successive trials?

(1) The probability of drawing a white ball at the first trial is

$$P(A) = \frac{a}{a+b}$$

(2) The probability of drawing a white ball at the second trial depends on additional information: If the first ball is returned to the urn, then $P(B) = P(A) = \frac{a}{a+b}$. Such an experiment is designated "drawing" with replacement." In the original problem 2.3 the urn is analogous to the 500 channels; drawing a white ball is analogous to a hitting malfunctioning channel drawing a black ball into hitting a sound channel.

The channel hit on at the first dialing can be hit on again at the second. This is analogous to the possibility of drawing the same ball again in the urn experiment with replacement.

PROBLEM 2.4

Let m items be chosen at random from a lot containing $n > m$ items of which $p(m < p < n)$ are defective. Find the probability of all m items being nondefective. Consider also the particular case $m = 3$, $p = 4$, $n = 8$.

FIRST SOLUTION 2.4

The probability of choosing a nondefective item without returning (*WR*) is

$$P(A_1) = \frac{n - P}{n}$$

The probability of choosing another nondefective item, *WR* is:

$$P(A_2|A_1) = \frac{n - p - 1}{n - 1}$$

Therefore the probability of choosing two nondefective item, *WR* is:

$$P(A_1 A_2) = P(A_1)P(A_2)(A_1) = \frac{(n - p)(n - p - A)}{n(n - 1)}$$

The conditional probability of choosing the m-th nondefective item

$$P(A_m|A_1 A_2 \cdots A_{m-1}) = \frac{n - p - m + 1}{n - m + 1}$$

consequently, the probability of all items being nondefective equals

$$\begin{aligned} P(A_1 A_2 \cdots A_{m-1} A_m) &= P(A_m|A_1 A_2 \cdots A_{m-1})P(A_1 A_2 \cdots A_{m-1}) \\ &= \frac{(n - p)(n - p - 1) \cdots (n - p - m + 1)}{n(n - 1) \cdots (n - m + 1)} \end{aligned}$$

For $m = 3$, $p = 4$, $n = 8$

$$P = \frac{4 \times 3 \times 2}{8 \times 7 \times 6} = \frac{1}{14}$$

SECOND SOLUTION 2.4

The number of combinations of m items that can be chosen from a lot containing n items is

$$\binom{n}{m} = \frac{n!}{(n - m)!m!}$$

Our interest is confined to nondefective items, i.e. m items are to be from a lot of $(n - p)$ nondefective items. The number of such combinations is

$$\binom{n - p}{m} = \frac{(n - p)!}{(n - p - m)!m!}$$

The sought probability is therefore

$$P(A) = \frac{\binom{n-p}{m}}{\binom{n}{m}} = \frac{(n-p)!}{(n-p-m)!m!} \frac{(n-m)!m!}{n!} = \frac{(n-m)!(n-p)!}{n!(n-p-m)!}$$

$$= \frac{(n-m)!}{n!} \frac{(n-p)!}{(n-p-m)!} = \frac{1}{(n-m+1)(n-m+2)\cdots n}$$

$$\times \frac{(n-p-m+1)(n-p-m+2)\cdots(n-p)}{1}$$

which is identical in the result by the first method of solution.

SIMPLER SOLUTION FOR A PARTICULAR CASE 2.4

The probability of choosing the first nondefective item is

$$P(A_1) = \frac{8-4}{8} = \frac{4}{8} = \frac{1}{2}$$

The probability of choosing the second nondefective item, provided the first was nondefective as well

$$P(A_2|A_1) = \frac{4-1}{8-1} = \frac{3}{7}$$

The probability of choosing the third nondefective item provided the first two are a nondefective

$$P(A_3|A_1A_2) = \frac{4-2}{8-2} = \frac{2}{6} = \frac{1}{3}$$

Finally, the sought probability of all three being nondefective is

$$P(A_1A_2A_3) = \frac{1}{2} \times \frac{3}{7} \times \frac{1}{3} = \frac{1}{14}$$

PROBLEM 2.5

A single playing card is picked at random from a well-shuffled ordinary deck of 52. Consider the events A, king picked; B, ace picked; C, heart picked. Check whether (1) A and B, (2) A and C, (3) B and C, are dependent or independent.