Mathematics for Industry 7

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Analysis and Control of Complex Dynamical Systems

Robust Bifurcation, Dynamic Attractors, and Network Complexity

Springer

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Aims & Scope

The meaning of "Mathematics for Industry" (sometimes abbreviated as MI or MfI) is different from that of "Mathematics in Industry" (or of "Industrial Mathematics"). The latter is restrictive: it tends to be identified with the actual mathematics that specifically arises in the daily management and operation of manufacturing. The former, however, denotes a new research field in mathematics that may serve as a foundation for creating future technologies. This concept was born from the integration and reorganization of pure and applied mathematics in the present day into a fluid and versatile form capable of stimulating awareness of the importance of mathematics in industry, as well as responding to the needs of industrial technologies. The history of this integration and reorganization indicates that this basic idea will someday find increasing utility. Mathematics can be a key technology in modern society.

The series aims to promote this trend by (1) providing comprehensive content on applications of mathematics, especially to industry technologies via various types of scientific research, (2) introducing basic, useful, necessary and crucial knowledge for several applications through concrete subjects, and (3) introducing new research results and developments for applications of mathematics in the real world. These points may provide the basis for opening a new mathematics-oriented technological world and even new research fields of mathematics.

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Preface

Recent advanced communication and information technologies have attracted our attention to developments of highly dependable, strongly resilient, and energy-efficient systems with applications to intelligent transportation systems (ITS), smart grids, high-level medical diagnosis/treatment systems, and so on. Such systems in general involve complex behavior induced by interactions among subsystems, and complex network structure as well as a large number of components. We need to analyze such complex behavior for capturing the intrinsic properties of the systems and to design control systems for realizing the desirable behavior. Both dynamical systems theory and control systems theory will play indispensable and central roles in addressing such issues.

Dynamical systems theory originated from Newton's motion equations in the seventeenth century, and has been founded by Poincaré's great contributions late in the nineteenth century. After that, various mathematical methods such as ergodic theory, stability theory of periodic solutions including equilibrium points, and bifurcation theory for nonlinear dynamical systems have been developed, and since the late 1970s, they have been extended to different research topics on more complex phenomena/control such as bifurcations to chaos, chaos control, and chaos synchronization.

On the other hand, James Watt's steam engine at the industrial revolution in the eighteenth century has opened the gate to feedback control, and Maxwell's stability analysis late in the nineteenth century, which theoretically analyzed the instability phenomena of steam engines, was the occasion of developing control systems theory. Continuing upon classical control theory dealing with control system design mainly in the frequency domain since the 1920s, modern control theory has been advancing since the 1960s, which enables us to analyze controllability/observability and to design optimal control by means of state equations in the time domain. Moreover, a deep understanding on robustness of the system behavior for dynamic uncertainty including unmodeled dynamics in addition to parametric uncertainty has been gained, and then robust control theory has been developed since the 1980s.

The above two research fields, however, have been developing almost independently so far, although there have been several successes to be related in both fields such as Pontryagin's maximum principle and R.E. Kalman's pioneering contribution on chaos and control theory. The main focus in dynamical systems theory is nonlinear autonomous dynamics with a kind of unstable phenomena like bifurcations and chaos, while the focus in control systems theory is feedback stabilization of linear non-autonomous dynamics at an equilibrium point. This motivates us to develop a new paradigm on analysis and control of complex/large-scale dynamical systems throughout collaborative research between dynamical systems theory and control systems theory.

This book, which is the first trial toward developments of such a new paradigm, presents fundamental and theoretical breakthroughs on analysis and control of complex/large-scale dynamical systems toward their applications to various engineering fields. In particular, this book focuses on the following three topics:

1. Analysis and control of bifurcation under model uncertainty.
2. Analysis and control of complex behavior including quasi-periodic/chaotic orbits.
3. Modeling of network complexity emerging from dynamical interaction among subsystems.

According to the above three topics, this book is organized as follows: In Part I, robust bifurcation analysis, which deals with bifurcation analysis for dynamical systems subject to uncertainty due to unmodeled dynamics, is presented and various kinds of bifurcation control methods based on the degree of stability are proposed. Part II begins with the analysis of chaotic behavior of triangle-folding maps, and presents novel attempts for controlling various kinds of complex behavior, namely feedback stabilization of quasi-periodic orbits and spatial patterns, chaos control, ultra-discretization based control, and control of unstabilizable switched systems. Finally, Part III includes research topics on network model reduction and network structure identification toward control of large-scale network systems.

This book can be beneficial to mathematicians, physicists, biophysicist as well as researchers on nonlinear science and control engineering for a better fundamental understanding of analysis and control synthesis of such complex systems.

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Contents

Part I
Robust Bifurcation and Control

Chapter 1
Dynamic Robust Bifurcation Analysis

Masaki Inoue, Jun-ichi Imura, Kenji Kashima and Kazuyuki Aihara

1.1 Introduction

In dynamical systems theory, bifurcation phenomena have been studied extensively [1–3]. Bifurcation is a phenomenon whereby a slight parametric perturbation in a dynamical system produces qualitative changes in structure of the solutions. It can be interpreted as bifurcation that because of a slight parameter change a stable equilibrium of differential equations is suddenly destabilized, and a stable periodic orbit can arise near the equilibrium. In order to analyze such phenomena, bifurcation theory has been studied and widely used for analysis and synthesis of complex behavior in many research fields; systems biology and synthetic biology [4–14], power system

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analysis [15–18], epidemic model analysis [19–22], and so on. For example, bifurcation theory has contributed to recent breakthroughs in systems biology and synthetic biology. Bifurcation analysis methods have been adopted to study the functions or characteristics of artificial bio-molecular systems, such as bio-molecular oscillators [4, 10, 11] and bio-molecular switches [5, 9, 14]. In addition, the robustness of such functions is identified with the volume of a parameter region in which the system has oscillatory property or bistable equilibria.

Conventional bifurcation theory is not always applicable to the analysis and synthesis of dynamical systems with uncertainties. Bifurcation analysis methods assume that mathematical models such as differential equations are completely known. Hence, they are not applicable to dynamical systems with uncertainties, in particular, large dynamic uncertainties . However, practical systems in the real world inevitably involve not only static but dynamic uncertainties [23, 24]. In order to apply the theory to such real-world systems, bifurcation analysis methods for uncertain dynamical systems are required.

In this chapter, we study local bifurcation of an equilibrium for systems with dynamic uncertainties. Note that a bifurcation point, i.e., a parameter value on which bifurcation occurs, depends on each model in general. If a system contains uncertainties and is described by a model set, we cannot find the specific bifurcation point. Therefore, we evaluate the potential bifurcation region: the parameter region that consists of all possible bifurcation points for a given model set. In other words, the region consists of all parameter points on which bifurcation can potentially occur. Evaluating the potential bifurcation region is referred to as the dynamic robust bifurcation analysis problem in this chapter. To this end, we first propose a condition for existence of equilibria independently of uncertainties and evaluate their location. Then, we derive a condition for robust hyperbolicity of potential equilibrium points, which implies that the dimension of unstable manifolds is independent of uncertainties. We consider parameter-dependent nonlinear systems with dynamic uncertainties, and using the robust hyperbolicity condition we identify the region that contains all potential bifurcation points. Finally, illustrative examples for robustness analysis of normal forms for various types of bifurcation are presented.

Notation: The symbols $\overline{\sigma}\{\cdot\}$ and $\rho\{\cdot\}$ represent the maximum singular value and the spectrum radius of a matrix, respectively. RH_∞ is the space that consists of all proper and complex rational stable transfer function matrices. The H_∞ norm and L_∞ norm of a linear system S are defined by

$$\|S\|_{H_\infty} := \sup_{\mathrm{Re}[s]>0} \overline{\sigma}\{\bar{S}(s)\}, \quad \|S\|_{L_\infty} := \sup_{\mathrm{Re}[s]=0} \overline{\sigma}\{\bar{S}(s)\},$$

where $\bar{S}(s)$ is a transfer function matrix representation of S. The poles (system poles) of a linear system $\dot{x} = Ax$ are defined by the roots of the characteristic polynomial $\phi(s) := \det(sI - A)$. In addition, a stable pole, an unstable pole, and a neutral pole are defined as poles lie in the open left half-plane, open right half-plane, and the imaginary axis of the complex plane, respectively.