

APPLIED PROBABILISTIC CALCULUS for FINANCIAL ENGINEERING

AN INTRODUCTION USING

R

BERTRAM K. C. CHAN

with website



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APPLIED PROBABILISTIC CALCULUS for FINANCIAL ENGINEERING

AN INTRODUCTION USING **R**

**Illustrates how R may be used successfully to solve problems
in quantitative finance**

Applied Probabilistic Calculus for Financial Engineering: An Introduction Using R provides R recipes for asset allocation and portfolio optimization problems. It begins by introducing all the necessary probabilistic and statistical foundations, before moving on to topics related to asset allocation and portfolio optimization with R codes illustrated for various examples.

This clear and concise book covers financial engineering, using R in data analysis, and univariate, bivariate, and multivariate data analysis. It examines probabilistic calculus for modeling financial engineering—walking the reader through building an effective financial model from the Geometric Brownian Motion (GBM) Model via probabilistic calculus, while also covering Ito Calculus. Classical mathematical models in financial engineering and modern portfolio theory are discussed—along with the Two Mutual Fund Theorem and The Sharpe Ratio. The book also looks at R as a calculator and using R in data analysis in financial engineering. Additionally, it covers asset allocation using R, financial risk modeling and portfolio optimization using R, global and local optimal values, locating functional maxima and minima, and portfolio optimization by performance analytics in CRAN.

- Covers optimization methodologies in probabilistic calculus for financial engineering
- Answers the question: What does a "Random Walk" Financial Theory look like?
- Covers the GBM Model and the Random Walk Model
- Examines modern theories of portfolio optimization, including The Markowitz Model of Modern Portfolio Theory (MPT), The Black-Litterman Model, and The Black-Scholes Option Pricing Model

Applied Probabilistic Calculus for Financial Engineering: An Introduction Using R is an ideal reference for professionals and students in economics, econometrics, and finance, as well as for financial investment quants and financial engineers.

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An Introduction Using R

Bertram K.C. Chan

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Applied Probabilistic Calculus for
Financial Engineering

Dedicated to the glory of God and to my better half

Marie Nashed Yacoub Chan

Preface

The Financial Challenges and Experience of a Typical Retiring Couple – Mr. and Mrs. Smith (not their real name)

About 10 years ago, after a lifetime of steady work for some 40 years, Mr. John A. Smith and Mrs. Mary B. Smith of California were preparing for a life of active retirement, including extensive traveling worldwide. To take care of their future financial needs, they had decided to obtain the services of a local professional financial engineering and investment management company – XYZ (fictitious) – of California that conducts its transactions through a large national financial engineering corporation: LPL (Linsco – 1968 and Private Ledger – 1973).

To that end, Mr. and Mrs. Smith invested a sum of approximately \$2,000,000 from their life savings, with the following twin goals:

- i) The preservation of their capital of \$2 M
- ii) Receiving a regular net monthly cash income of at least \$10,000 from XYZ

Thus, if the original capital of \$2 M were to be preserved (approximately unchanged), as well as to maintain a steady withdrawal of \$10,000 per month, the average annual return of the investment of the \$2 M will have to be on the order of $(10,000 \times 12) / 2,000,000 = 0.06$, or 6%.

The financial services management typically charges fees on the order of 1.5%. Thus, a rough estimate that the financial management should achieve would be on the order of $6\% + 1.5\%$, or 7.5%.

How does a service such as XYZ/LPL achieve such a goal?

Approximately 10 years after their retirement, on Tuesday, November 15, 2016, the financial markets closed at

Dow (DJIA)	18,923.06/+54.03/+0.29%
Nasdaq	5,275.62/+57.22/+1.10%
S&P 500	2,180.39/+16.19/+ 0.75%
Gold	\$1,229.00 per oz +0.37%

Over these 10 years, Mr. and Mrs. Smith had been receiving regularly a monthly payout from XYZ/LPL of \$10,947.03! And, on the same day, the net balance of their portfolio investment account is as follows:

Portfolio ending at : \$2,111,603.35, +\$6,152.47/ + 0.29%

In other words, the balance at the end of that day stood at approximately \$2.1 M! And the total payout received over these past 10 years comes to \$10,947.03 per month or $\$10,947.03 \times 12 = \$131,364.36$ per annum or $\$131,364.36 \times 10 = \$1,313,643.60$ over the past decade!

Exclusive of the financial management at 1.5%! How can such an investment management be achieved? Indeed, that is the central theme of this book:

The challenge in financial engineering

Whereas the nominal saving accounts of banks and credit unions in the United States have been paying at 0.1% to about 1.0%, how does a financial manager allocate the managed funds to generate, and sustain, an average return of about 7.5%? This is a typical simple example in Assets Allocation and Portfolio Optimization in Financial Engineering. It is the objective of this book to consider the underlying mathematical principles in meeting this challenge – in terms of Assets Allocation and Portfolio Optimization in Financial Engineering. This introductory text in financial engineering will include the use of the well-known and popular computer language R. Numerical worked examples are provided to illustrate the practical application of Applied Probabilistic Calculus in Financial Engineering leading to practical results in assets allocation and portfolio optimization in financial engineering using R.

About the Companion Website

This book is accompanied by a companion website:
www.wiley.com/go/chan/appliedprobabilisticcalculus

The website includes:

- Solutions to all the exercises in the body of the text, with some supportive comments

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