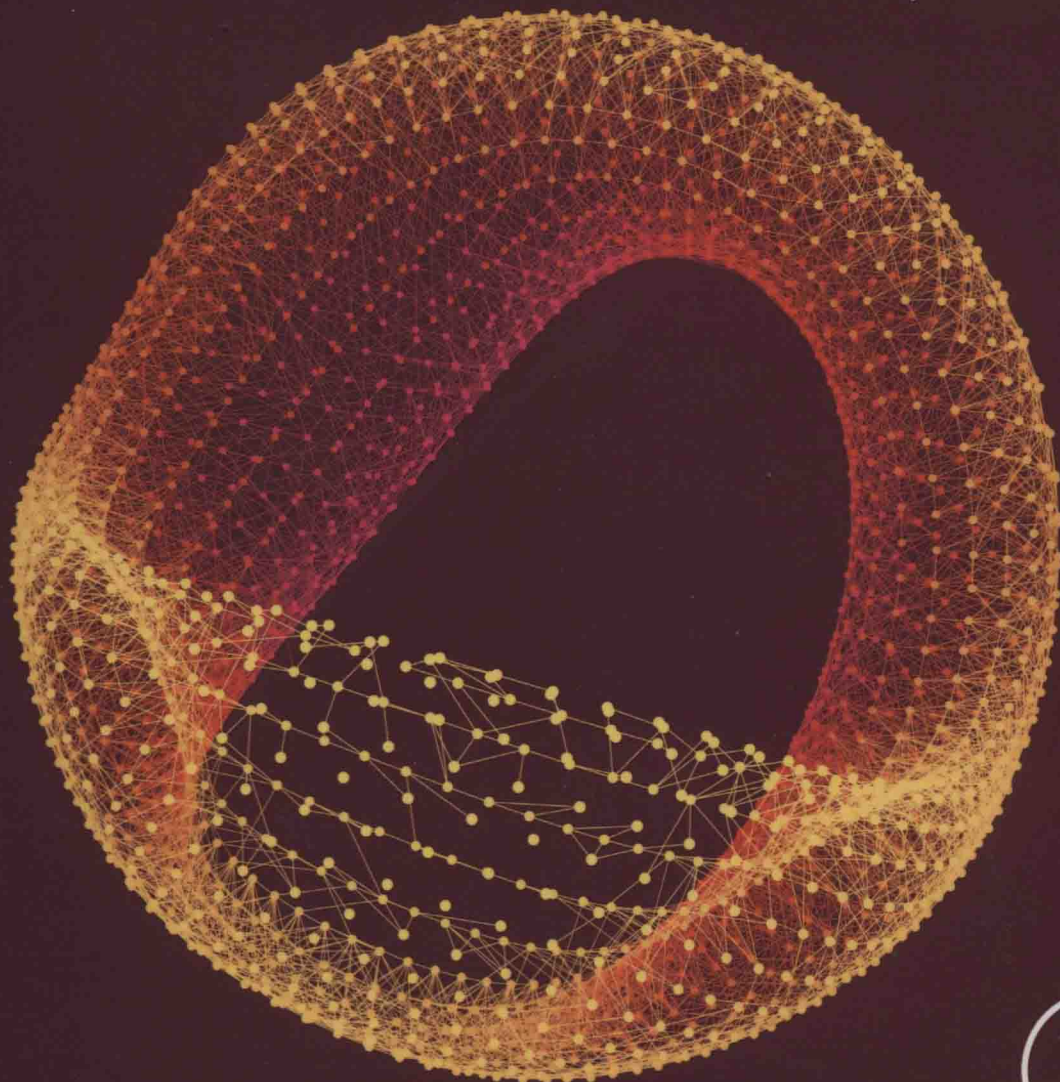




# Sliding Mode Control Using MATLAB

Jinkun Liu



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# Sliding Mode Control Using MATLAB



# Sliding mode control MATLAB simulation basic theory and design method

In the formulation of any control problem, there will typically be discrepancies between the actual plant and the mathematical model developed for controller design. This mismatch may be due to unmodelled dynamics, variation in system parameters or the approximation of complex plant behavior by a straightforward model. The engineer must ensure that the resulting controller has the ability to produce the required performance levels in practice despite such plant /model mismatches. This has led to intense interest in the development of robust control methods that seek to solve this problem. One particular approach to robust controller design is the sliding mode control methodology.

One of the most intriguing aspects of sliding mode is the discontinuous nature of the control action, whose primary function of each of the feedback channels is to switch between two distinctively different system structures (or components) such that a new type of system motion, called the sliding mode, exists in a manifold. This peculiar system characteristic is claimed to result in superb system performance, which includes insensitivity to parameter variations, and complete rejection of disturbances.

Sliding mode control is a particular type of variable structure control. In sliding mode control, the control system is designed to drive and then constrain the system state to lie within a neighborhood of the switching function. There are two main advantages to this approach. Firstly, the dynamic behavior of the system may be tailored by the particular choice of switching function. Secondly, the closed-loop response becomes totally insensitive to a particular class of uncertainty. The latter invariance property clearly makes the methodology an appropriate candidate for robust control. In addition, the ability to specify performance directly makes sliding mode control attractive from a design perspective.

The sliding mode design approach consists of two components. The first involves the design of a switching function so that the sliding motion satisfies design specifications. The second is concerned with the selection of a control law that will make the switching function attractive to the system state. Note that this control law is not necessarily discontinuous.

The chattering phenomenon is generally perceived as motion that oscillates about the sliding manifold. There are two possible mechanisms that produce such a motion. Firstly, in the absence of switching nonidealities such as delays, i.e., the switching device is switching ideally at an infinite frequency, the presence of parasitic dynamics in series with the plant causes a small amplitude high-frequency oscillation to appear in the neighborhood of the

sliding manifold. These parasitic dynamics represent the fast actuator and sensor dynamics. Secondly, the switching nonidealities alone can cause such high-frequency oscillations.

In this book, we aim to accomplish these objectives:

- Provide reasonable methods of the chattering phenomenon alleviating.
- Offer a catalogue of implementable robust sliding mode control design solutions for engineering applications.
- Provide advanced sliding mode controller design methods and their stability analysis.
- For each sliding mode control algorithm, we offer its simulation example and Matlab program.

This book provides the reader with a thorough grounding in sliding mode controller design. From this basis, more advanced theoretical results are developed. Typical sliding mode controller design is emphasized using Matlab simulation. In this book, concrete case studies, which present the results of sliding mode controller implementations, are used to illustrate the successful practical application of the theory.

The book is structured as follows.

Chapter 1, Basic sliding mode control principle and design, introduces the concept of sliding mode control and illustrates the attendant features of robustness and performance specification using a straightforward example and graphical exposition. Several typical sliding mode controllers for continuous system are introduced, and concrete stability analysis, simulation examples and Matlab programs are given.

In Chapter 2, Sliding mode control with high performance, firstly an adaptive sliding mode control is introduced for mechanical systems with tanh function; to avoid a control input value that is too big, a projection algorithm is used. Secondly, the problem of tracking control with prescribed performance guarantees is considered, the error evolution within prescribed performance bounds in both problems of regulation and tracking.

In Chapter 3, Sliding mode control based on a state observer, several kinds of state observer such as high gain observer, K observer, high gain differentiator, robust observer and separation theorem are introduced, and based on the different observer, sliding mode controller is designed.

In Chapter 4, Sliding mode control based on disturbance and a delayed observer, an exponential disturbance observer, delayed output observer for linear system and delayed output observer for nonlinear system are introduced, and closed system stability and convergence are analyzed.

In Chapter 5, Sliding mode control based on LMI, several kinds of sliding mode controller based on LMI technology are introduced, closed system stability and convergence are analyzed, and simulation examples are given.

In Chapter 6, Sliding mode control based on the RBF neural network, firstly, a simple adaptive sliding mode control based on RBF is introduced, then an adaptive sliding mode control based on RBF compensation is discussed. Sliding mode control based on RBF neural network with minimum parameter learning method is introduced, and finally, a sliding mode controller based on RBF with MPL is introduced.

In Chapter 7, Sliding mode control based on a fuzzy system, firstly, sliding mode control based on the fuzzy system approximation is introduced. Then, based on the fuzzy system with minimum parameter learning method, a sliding mode controller based on fuzzy system is designed.

In Chapter 8, Sliding mode control of a class of underactuated systems, considering several kinds of underactuated system, sliding mode controllers are designed, and the Lyapunov function and the Hurwitz method are used to analyze closed system stability.

In Chapter 9, Sliding mode control for underactuated system with decoupling algorithm, a general decoupling algorithm for underactuated system is introduced. With this decoupling algorithm, sliding mode control is designed, and the Lyapunov function and the Hurwitz method are used to analyze closed system stability.

All the control algorithms are described separately and classified by chapter name; all the programs can be run successfully in MATLAB and can be downloaded via <http://shi.buaa.edu.cn/liujinkun>.

If you have questions about algorithms and simulation programs, please contact the author at [lj@buaa.edu.cn](mailto:lj@buaa.edu.cn).





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# Basic sliding mode control principle and design

Sliding mode techniques are one approach to solving control problems and are an area of increasing interest.

This book provides the reader with classical sliding mode control design examples, based on [1] and [2].

Variable Structure Control (VSC) with Sliding Mode Control (SMC) was first proposed and elaborated in early 1950s in the Soviet Union by Emelyanov and several coresearchers such as Utkins and Itkis [3]. During the last decades significant interest on VSC and SMC have been generated in the control research community.

SMC has been applied including nonlinear system, multiinput multioutput (MIMO) systems, discrete-time models, large-scale and infinite-dimension systems, and stochastic systems. The most eminent feature of SMC is it is completely insensitive to parametric uncertainty and external disturbances during sliding mode [4].

Essentially, VSC utilizes a high-speed switching control law to drive the nonlinear plant's state trajectory onto a specified and user-chosen surface in the state space, which is called the sliding or switching surface, and to maintain the plant's state trajectory on this surface for all subsequent time. This surface is called the switching surface because if the state trajectory of the plant is "above" the surface, a control path has one gain and a different gain if the trajectory drops "below" the surface. During the process, the control system's structure varies from one to another, thus earning the name VSC. To emphasize the important role of the sliding mode, the control is also called SMC [5].

In SMC, the system is designed to drive and then constrain the system state to lie within a neighborhood of the switching function. Its two main advantages are (1) the dynamic behavior of the system may be tailored



by the particular choice of switching function, and (2) the closed-loop response becomes totally insensitive to a particular class of uncertainty. Also, the ability to specify performance directly makes SMC attractive from the design perspective.

Trajectory of a system can be stabilized by a sliding mode controller. After the initial reaching phase, the system states “slides” along the line  $s = 0$ . The particular  $s = 0$  surface is chosen because it has desirable reduced-order dynamics when constrained to it. In this case, the  $s = cx_1 + \dot{x}_1, c > 0$ . Surface corresponds to the first-order LTI system  $\dot{x}_1 = -cx_1$ , which has an exponentially stable origin.

There are two steps in the SMC design. The first step is designing a sliding surface so that the plant restricted to the sliding surface has a desired system response. This means the state variables of the plant dynamics are constrained to satisfy another set of equations which define the so-called switching surface. The second step is constructing a switched feedback gains necessary to drive the plant's state trajectory to the sliding surface. These constructions are built on the generalized Lyapunov stability theory.

Now we give a simple sliding mode controller design example as follows.

### 1.1 A SIMPLE SLIDING MODE CONTROLLER DESIGN

Consider a plant as

$$J\ddot{\theta}(t) = u(t) + d(t), \quad (1.1)$$

where  $J$  is the inertia moment,  $\ddot{\theta}(t)$  is the angle signal,  $u(t)$  is the control input,  $d(t)$  is the disturbance and  $|d(t)| \leq \eta$ .

Design the sliding mode function as

$$s(t) = ce(t) + \dot{e}(t) \quad (1.2)$$

where  $c$  must satisfy Hurwitz condition,  $c > 0$ .

The tracking error and its derivative value is

$$e(t) = \theta(t) - \theta_d(t), \quad \dot{e}(t) = \dot{\theta}(t) - \dot{\theta}_d(t)$$

where  $\theta_d(t)$  is the ideal position signal.

From Eq. (1.2), we can see that if  $s(t) = 0$ , then  $ce(t) + \dot{e}(t) = 0$ , and we can get  $e(t) = e(0)\exp(-ct)$ . That is, when  $t \rightarrow \infty$ , position tracking