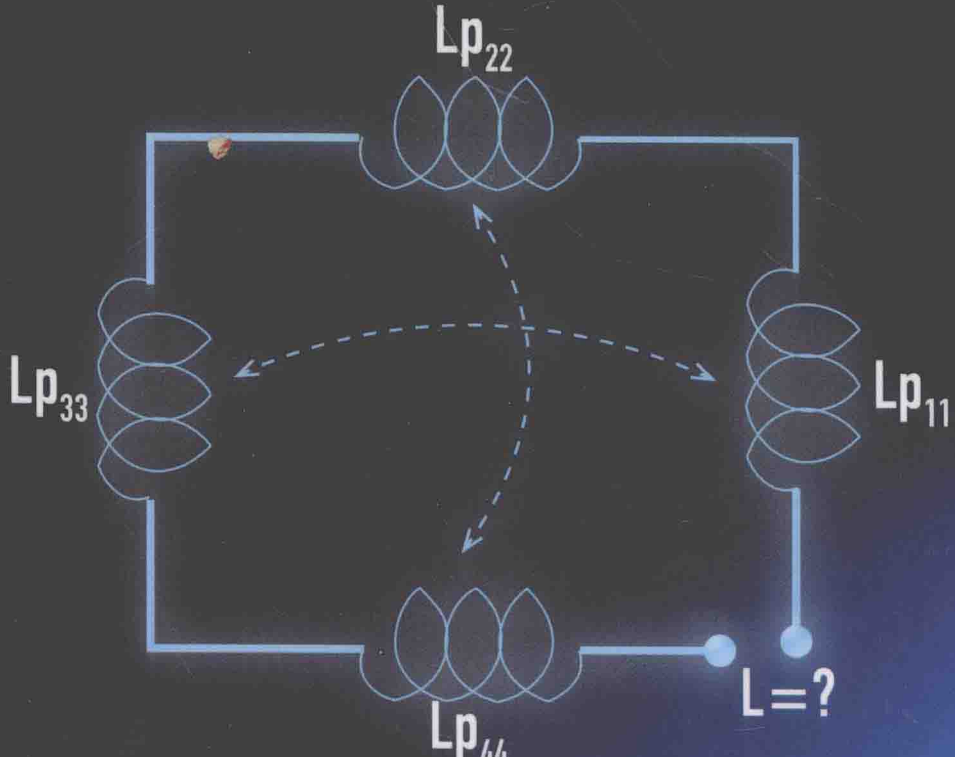
CIRCUIT ORIENTED ELECTROMAGNETIC MODELING USING THE PEEC TECHNIQUES

ALBERT RUEHLI • GIULIO ANTONINI • LIJUN JIANG



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The book is dedicated to our families without whose patience and support it would have been impossible to write.

Albert: to my wife, Kristina, and all members of my extended family. This includes not only the children but also the grandchildren who are our future.

Giulio: to my wife Francesca, "the one, the beloved one, the most beautiful" (Ancient Egyptian Poem) and to our loved son Andrea.

Lijun: to my dear wife, Tao, and to my blessed daughters.

PREFACE

GENERAL ASPECTS

Electromagnetic (EM) modeling has been of interest to the authors of this book for a large portion of their careers. Giulio Antonini has been involved with partial element equivalent circuit (PEEC) for over 15 years at the Università degli Studi dell'Aquila, Italy, where he is now a professor. Both Albert Ruehli and Lijun Jiang worked as Research Staff members at the IBM Research Laboratory in Yorktown Heights, New York, on electrical interconnect and package modeling and electromagnetic compatibility (EMC) issues. Lijun Jiang is now a professor at the University of Hong Kong, Hong Kong, and Albert Ruehli is now an adjunct professor at the University of Science and Technology, Rolla, Missouri. We all continue to work today on different aspects of the PEEC method.

We welcome the opportunity to share the product of our experience with our readers. Fortunately, electromagnetic modeling (EMM) is a field of increasing importance. Electronic systems have been and will continue to increase in complexity over the years leading to an ever increasing set of new problems in the EM and circuit modeling areas. The number of electronic systems and applications expands every day. This leads to an ever-increasing need for electrical modeling of such systems.

EMM has been a key area of interest to the authors for quite a while. About 40 years ago, the general field of EMM was very specialized and more theoretical. The number of tools in this area and consequent applications were much more limited. Research is driven by the desire to discover new ways and potential applications as well as the need for solutions of real life problems.

Waveguides that mostly were interesting mechanically complex structures were physically large due to the lower frequencies involved. Some of the main topics of interest were antennas and waveguides as well as transmission lines. EM textbooks usually demanded an already high level of education in the theory and they were sometimes removed from realistic problems.

Transmission lines were the most accessible devices from both a theoretical and a practical point of view. Very few tools were available for practical computations especially before computers were widely available. Computers were mostly used for specialized applications. Problems were solved with a combination of theoretical analysis and measurements as well as insight that was a result of years of experience.

In contrast, today electromagnetic solver tools are available for the solution of a multitude of problems. Hence, the theoretical and intuitively ascertained solutions have been replaced with numerical method-based results. However, this does not eliminate the need for a thorough understanding of the EM fundamentals and the methods used in EM tools. The advanced capabilities available in the tools require a deeper understanding of the formulations on which the tools are based. We are well aware that the interaction of tools and theory leads to advances.

Textbooks such as Ramo and Whinnery [1] have evolved over many years. Meanwhile, many new excellent introductory textbooks have been written that treat different special subjects such as EMC [2]. Our book is oriented toward a diverse group of students at the senior to graduate level as well as professionals working in this general area. In our text, we clearly want to emphasize the utility of the concepts for real-life applications, and we tried to include as many relevant references as possible.

FUNDAMENTALS OF EMM SOLUTION METHODS

We have to distinguish between two fundamentally different types of circuit models for electromagnetics. Some of them are based on a differential equation (DE) formulation of Maxwell's equations, while others are based on integral equation (IE) form.

The DE forms are commensurate with the system of equations that results from the formulation of a problem in terms of DEs. This results in circuit models that have neighbor-to-neighbor coupling only. The most well-known form is the finite difference time domain (FDTD) method, which is a direct numerical solution of Maxwell's equations. The advantages of DE methods is that very sparse systems of equation result. At the same time, these systems are larger than the ones obtained from IE-based methods.

On the other hand, the IE-based methods will result in systems that have element-to-element couplings. Hence, this results in smaller, denser systems of equations. The finite element (FE) method is a somewhat hybrid technique since it involves local integrations while the overall coupling is local as in the DE methods. This also results in a large and sparse system of equations. Among the formulations used today, there are two circuit-oriented ones: (a) the DE-based transmission line modeling (TLM) method; and (b) the PEEC method. In this text, we mainly consider the IE-based PEEC method.

The PEEC method has evolved over the years from its start in the early 1970s [3–5]. Interestingly, this is about the same time when the other circuit-oriented EM approach – the TLM method – was first published [6]. Some early circuit-oriented work for DE solutions of Maxwell's equations was done by Kron in the 1940s [7]. However, the solution of the large resultant systems was impossible to solve without a computer. Hence, the work was of little practical importance. Recently, matrix stamps for FDTD models have been presented [8].

Around the same time, numerical DE methods made important progress. The FDTD method was conceived in 1966 [9]. Also, the finite integration technique (FIT) technique was published in 1977 [10]. All these methods have made substantial progress since the early work was published.

MORE ABOUT THE PEEC METHOD

The PEEC method evolved in a time span of more than 40 years. From the start, the approach has been tailored for EMM of electronic packages or Electronic Interconnect and Packaging called signal integrity (SI). Power integrity (PI) and noise integrity (NI) as well as EMC problems. In the beginning, only high-performance computer system modeling needed accurate models for the electrical performance of the interconnects and power distribution in the package and chips. In main frame computes the speed of the circuits was much faster than that of conventional computer circuits such as the early personal computers.

Quasistatic solutions were adequate then even for the highest performance systems. Problems such as the transient voltage drops due to large switching currents were discovered very early. This prompted and extended the work on partial inductance calculations for problems of an ever-increasing size. In the 1990s, the modeling of higher performance chips and packages became an issue with the race for higher clock rates in computer chips. This led to the need for full-wave solutions. As a consequence, stability and passivity issues became important. Today, aspects such as skin-effect loss and dielectric loss models are required for realistic models.

Numerous problems can be solved besides package and interconnect and microwave problems. Approximate physics-based PEEC equivalent circuit models can be constructed, which are very helpful for a multitude of purposes. Further, PEEC is one of the methods used in some of the EMM tools. Fortunately, PEEC models can easily be augmented with a multitude of additional circuit models. This leads to other real advantages. Further, techniques have been found to improve the efficiency of these methods. As we show, PEEC is ideally suited for small simple models. Also, the wealth of circuit solution techniques that are available today can be employed. One example of this is the use of the modified nodal analysis (MNA) approach, which helps PEEC for low-frequency and a *dc* solution that other techniques may not provide.

TEACHING ASPECTS

We hope that this text can be used as an effective tool to introduce EM to new students. We think that a key advantage of the PEEC method is its suitability for an introductory course in EM.

The teaching of the PEEC method can be approached from several different points of view. It may be used as a way to introduce EMM, since most engineering students are more familiar with circuit theory rather than EM theory. This is also the case since circuit courses are taught at a lower level than EM courses. Alternatively, one may want to start with the introduction of the quasistatic PEEC models in a first EM course.

We prefer to use concepts that can be understood in lieu of the introduction of more advanced topics and mathematical notation. As a second course, general PEEC methods could be covered. This could be done, perhaps, in conjunction with introduction of concepts such as interconnect modeling and other chip and package design concepts.

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The authors of this book are aware that progress in any area, including science and technology, is driven by the requirement for innovation. As part of this contribution, we also learn that it is not pursued in isolation. Our own thinking is greatly impacted in this process in our various areas of research.

We acknowledge contributions by many people who helped to debug the book to various degrees. To remove the errors was a daunting challenge. In particular, Giulio Antonini wants to acknowledge Professor Antonio Orlandi for having initially introducing him to the PEEC method. Giulio Antonini is then deeply grateful to Albert E. Ruehli for his guidance in the PEEC modeling, his long lasting friendship and for the many evenings spent together, making integrals and solving equations, having a lot of fun.

Kristina Ruehli read and corrected each of the chapters multiple times and modified the text written by Swiss-German, Italian, and Chinese thinkers to conform with English grammatical forms of speech, syntax, and punctuation.

We decided to find an alternate approach for acknowledgment of contributions by other researchers, since it is clearly impossible to add all the appropriate references to the book chapters. We added a list of M.S. and Ph.D. works that have contributed to the partial element equivalent circuit (PEEC) method and thereby acknowledge the contributions by their advisors.

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ACRONYMS

ABC	absorbing boundary condition
BE	backward Euler method, BD1
BD2	backward differentiation method, Gear 2
CAD	computer-aided design
Ckt	circuit
EFIE	electric field integral equation
EM	electromagnetic
EMM	electromagnetic modeling
FDTD	finite difference time domain
FE	forward Euler method
FEM	finite element method
FFT	fast Fourier transform
FIR	digital filter nonrecursive
FIT	finite integration technique
IIR	digital filter with feedback
KCL	Kirchhoff's current law
KVL	Kirchhoff's voltage law
MFM	multifunction method
MFIE	magnetic field integral equation
MNA	modified nodal analysis
MOR	model order reduction
NI	noise integrity
PCB	printed circuit board
PEEC	partial element equivalent circuit
PI	power integrity
PDE	partial differential equation
PEC	perfect electric conductor
PMC	perfect magnetic conductor
PML	perfect matched layer
PPP	parallel plane PEEC model
PWTD	plane wave time domain

RCS	radar cross section
ROM	reduced order model
SI	signal integrity
SPICE	Simulation Program with Integrated Circuit Emphasis
TEM	transverse electromagnetic
Theta,Θ	theta integration method
TL	transmission line
TLM	transmission line modeling method
TR	trapezoidal method
VFI	volume filament
WRM	weighted residual method

CONTENTS

DEDICATION xv

PREFACE xvii

ACKNOWLEDGEMENTS xxi

ACRONYMS xxv

1 Introduction 1

References, 6

2 Circuit Analysis for PEEC Methods 9

2.1 Circuit Analysis Techniques, 9
2.2 Overall Electromagnetic and Circuit Solver Structure, 9
2.3 Circuit Laws, 11
2.3.1 Kirchoff's Current Law, 11
2.3.2 Kirchoff's Voltage Law, 11
2.3.3 Branch Impedances, 12
2.3.4 Incomplete Kirchhoff's Current Law, 12
2.4 Frequency and Time Domain Analyses, 13
2.5 Frequency Domain Analysis Formulation, 14
2.6 Time Domain Analysis Formulations, 17
2.6.1 Numerical Integration of Time Domain Differential Equations, 18
2.6.2 List of Integration Methods for PEEC Solver, 20
2.6.3 Initial Conditions for Time Solver with Delays, 22
2.7 General Modified Nodal Analysis (MNA), 22
2.7.1 Matrix Kirchhoff's Current Law and Stamps, 23
2.7.2 Matrix Kirchhoff's Voltage Law, 24
2.7.3 Matrix KCL Solution of MNA Equations for PEEC, 25
2.7.4 Matrix KCL for Conductor Example, 27

2.8 Including Frequency Dependent Models in Time Domain Solution, 28
2.9 Including Frequency Domain Models in Circuit Solution, 31
2.9.1 Equivalent Circuit for Rational Approximation of Transfer Functions, 31
2.9.2 Inclusion of Frequency Domain Models in a Time Domain Circuit Solver, 34
2.9.3 General Inclusion of Frequency Domain Admittance Models, 36
2.9.4 State-Space and Descriptor Representations, 37
2.10 Recursive Convolution Solution, 39
2.10.1 Conventional Convolution, 39
2.10.2 Recursive Convolution, 40
2.11 Circuit Models with Delays or Retardation, 41
2.11.1 Inclusion of Delays in the Circuit Domain, 42
Problems, 43
References, 44

3 Maxwell's Equations **47**

3.1 Maxwell's Equations for PEEC Solutions, 47
3.1.1 Maxwell's Equations in the Differential Form, 47
3.1.2 Maxwell's Equations in the Integral Form, 49
3.1.3 Maxwell's Equations and Kirchhoff's Circuit Laws, 50
3.1.4 Boundary Conditions, 51
3.2 Auxiliary Potentials, 52
3.2.1 Magnetic Vector Potential A and Electric Scalar Potential Φ_e, 52
3.2.2 Electric Vector Potential F and Magnetic Scalar Potential Φ_m, 53
3.2.3 Important Fundamental Relationships, 54
3.3 Wave Equations and Their Solutions, 54
3.3.1 Wave Equations for E and H, 54
3.3.2 Wave Equations for A, F, and Φ_e, 55
3.3.3 Solution of the Helmholtz Equation, 56
3.3.4 Electric Field Integral Equation, 57
3.4 Green's Function, 58
3.4.1 Notation Used for Wave Number and Fourier Transform, 58
3.4.2 Full Wave Free Space Green's Function, 59
3.5 Equivalence Principles, 60
3.5.1 Volume Equivalence Principle, 61
3.5.2 Huygens' Equivalence Principle, 62
3.6 Numerical Solution of Integral Equations, 63
Problems, 65
References, 66

4 Capacitance Computations **67**

4.1 Multiconductor Capacitance Concepts, 68
4.2 Capacitance Models, 69
4.2.1 Capacitance Models for Multiconductor Geometries, 69
4.2.2 Short Circuit Capacitances, 70
4.2.3 Coefficient of Potential Matrix Pp, 71

4.2.4 Capacitance of Conductor Systems, 72
4.2.5 Elimination of a Floating Conductor Node, 72
4.2.6 Floating or Reference Free Capacitances, 73
4.3 Solution Techniques for Capacitance Problems, 74
4.3.1 Differential Equation (DE) Methods for Capacitance Computations, 76
4.4 Meshing Related Accuracy Problems for PEEC Model, 79
4.4.1 Impact of Meshing on Capacitances and Stability and Passivity, 80
4.5 Representation of Capacitive Currents for PEEC Models, 82
4.5.1 Quasistatic Capacitance–based Model, 82
4.5.2 Current Source-Based Model for the Capacitances, 82
4.5.3 Potential-Based Model for the Capacitances, 84
Problems, 85
References, 86

5 Inductance Computations **89**

5.1 Loop Inductance Computations, 90
5.1.1 Loop Inductance Computation in Terms of Partial Inductances, 91
5.1.2 Circuit Model for Partial Inductance Loop, 93
5.2 Inductance Computation Using a Solution or a Circuit Solver, 95
5.3 Flux Loops for Partial Inductance, 95
5.4 Inductances of Incomplete Structures, 96
5.4.1 Open-Loop Inductances, 96
5.4.2 Open-Loop Macromodels, 97
5.4.3 Examples for Open-Loop Inductances, 98
5.5 Computation of Partial Inductances, 99
5.5.1 Approximate Formulas for Partial Inductances, 100
5.5.2 Inductance Computations for Large Aspect Ratio Conductors, 101
5.6 General Inductance Computations Using Partial Inductances and Open Loop Inductance, 107
5.6.1 Closing the Loop for Open-Loop Problems, 108
5.7 Difference Cell Pair Inductance Models, 109
5.7.1 Inductances for Transmission Line-Type Geometries, 109
5.7.2 Approximate Inductive Coupling Calculation Between Difference Cell Pairs, 111
5.7.3 Inductance of Finite and Semi-Infinite Length TL, 113
5.7.4 Plane Pair PEEC Models Based on Difference Currents, 114
5.7.5 Parallel Plane PEEC Modeling, 114
5.7.6 PEEC Inductance Plane Model with Orthogonal Meshing, 115
5.7.7 Mesh Reduction Without Couplings of Nonparallel Inductances, 117
5.8 Partial Inductances with Frequency Domain Retardation, 119
5.8.1 Thin Wire Example for Retarded Partial Inductances, 122
5.8.2 General Case for Separated Conductor Partial Inductances with Retardation, 123
Problems, 125
References, 131