Modeling and Simulation in Science, Engineering and Technology

Tomás Chacón Rebollo
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Mathematical and Numerical Foundations of Turbulence Models and Applications

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To my wife Mame and my children Carmen and Tomás
Tomás Chacón Rebollo

To my wife Sophie and my children Noé, Yann, Sarah, and Marie
Roger Lewandowski

Preface

This book is about turbulence in incompressible fluids.

We have asked people in the street what the word "turbulence" means for them. One woman replied: "Turbulence makes me think of the sea, because it makes one feel what is invisible, what cannot be predicted." More generally, people answered giving only one word, such as disorder, aircraft, clouds, weather forecast, power, and chemistry. Therefore, turbulence is something that anyone has experienced in one way or another. Mathematicians will answer that turbulence is about fluids, mixing, chaos, and connected scales. It may be a source of inspiration for painters or poets. One may attempt to control it for technological progress. It is however a source of concern because of its impact on environment and human life, the most critical environmental challenge being climate change.

Although understanding turbulence is of primary importance, there is no mathematical definition of it, and many physical mechanisms governing turbulent motions remain unknown. One could say that there is a chance for mankind to understand quantum physics someday, but not turbulence. Nevertheless, it is possible to simulate by means of computers some features of turbulent motions: weather forecasts are rather accurate over 5 days, the mean Gulf Stream path can be calculated, numerical flow simulations around an aircraft wing are in good agreement with experimental data, etc. All these numerical simulations are performed by means of "turbulence models."

Turbulence models aim to simulate statistical means of turbulent flows or some of their scales. It is however estimated that an accurate computation of all scales of such flows will be possible only by the end of the twenty-first century, if the improvement of the computational resources continues at the same rate.

We do not pretend to give a definition of what turbulence is. Our goal is to provide a comprehensive and innovative presentation of turbulence models, at the crossroads of modeling and mathematical and numerical analysis, including all these aspects in one single book, in complementarity with the other reference manuals in the field.

This book is the synthesis of almost 20 years of thoughts and works about turbulence models, through the meeting of a mathematician with a numerical analyst, leading to a long-term collaboration and friendship. This resulted in several joint research works, which gave us the opportunity to check that the complementarity of these specialities can be quite fruitful. Finally, it led us to the project of jointly writing a book from a comprehensive point of view on one of the most challenging scientific problems, as is the understanding of turbulence: we deliver here what we are able to understand from turbulence.

In mathematics, authors are always listed in alphabetical order, which is the case of this book.

Seville, Spain
Rennes, France
January 2014

Tomás Chacón Rebollo
Roger Lewandowski

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Roger Lewandowski

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Chapter 1
Introduction

Understanding turbulence is one of the oldest and most challenging scientific problems. Since the early works of Boussinesq and Reynolds in the late nineteenth century that formalized the basic characteristics of turbulent flows, the analysis of the extremely complex behavior of turbulence has raised the interest of many researchers. The issue of what is "turbulence" is still far from being solved, although some facts can be deduced from observations and experiments. Turbulent flows have a huge impact in human life, from weather forecasting to freshwater supply, energy generation, navigation, biological processes and so on.

Numerical simulation of turbulence is thus of primary importance to improve human life in many ways. Classical fluid mechanics establishes that the motion of a viscous fluid is governed by the Navier–Stokes equations, which in theory should be appropriate to perform numerical simulations of turbulent flows.

However, a turbulent flow is a highly irregular system, characterized by chaotic property changes involving a wide range of scales in nonlinear interaction with each other. These features yield a high computational complexity, which makes today direct numerical simulations of turbulent flows from the Navier–Stokes equations impossible. This is why turbulence models are introduced, in order to reduce this computational complexity.

Besides experiments and physics, mathematical modeling and analysis play a central role in the study of turbulent flows. Mathematics provide a permanent support to build turbulent models for weather forecasting and meteorology, oceanography, climatology, and environmental and industrial applications. Industrial flow softwares (Ansys-Fluent, Comsol, Femap,...) are deeply based upon mathematical and numerical analysis.

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