

Holm Altenbach  
Tomasz Sadowski *Editors*

# Failure and Damage Analysis of Advanced Materials



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for Mechanical Sciences



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## PREFACE

*Failure as a limit state of the material behavior is well known from engineering practice. Different types of failure can be identified: transition from the elastic to plastic state, loss of stiffness, loss of fracture resistance at different scale levels, ultimate strength, and fatigue. In addition, failure can be accompanied by various types of damage. The course was discussed basic concepts and new developments in failure and damage analysis with focus on advanced materials such as composites, laminates, sandwiches and foams, and also new metallic materials. Starting from some mathematical foundations (limit surfaces, symmetry considerations, invariants) new experimental results and their analysis will be presented. Finally, new concepts for failure prediction and analysis were introduced and discussed.*

*The classical strength criteria developed intensively in the 19<sup>th</sup> and 20<sup>th</sup> century are mostly based on the comparison of the stress state (usually three-dimensional) with some scalar-valued properties estimated in tests. Such a phenomenological approach can be easily extended to other types of limit states of a material (for example, plastic behavior, and damage or fracture toughness). But even in the case of classical, but anisotropic structural materials, predictions are not always satisfactory and the effort required for their experimental confirmation can increase dramatically. Furthermore, in the case of advanced materials additional effects such as load dependent material response should be taken into account. These effects can induce mechanisms leading to different behavior in tension and compression.*

*Considering advanced metallic and non-metallic materials new methods of failure and damage prediction were discussed. Based on experimental results the traditional methods will be revised. In some cases it is enough to extend the classical approaches (for example, for metallic sheet material). In other situations (foams, composites) this is not satisfying since the different mechanisms cannot be adequately presented.*

*The lecture notes contains 5 parts. Part 1 (Classical and Non-Classical Failure Criteria) was prepared by Holm Altenbach & Vladimir Kolupaev. The following items are discussed: examples of failure behavior, theory of invariants and symmetry, classical isotropic models, compressibility and incompressibility, non-classical , and anisotropic*

models. Part 2 (*Constitutive Description of Isotropic and Anisotropic Plasticity for Metals*) is written by Frédéric Barlat & Myoung-Gyu Lee and contains: modeling of advanced metallic materials, plasticity in metallic materials, isotropic and anisotropic yield criteria, state variable evolution and hardening, influence of constitutive description on failure prediction. Liviu Marsavina presented in his Part 3 (*Failure and Damage in Cellular Materials*): behavior of cellular materials in compression and tensile, fracture toughness of cellular materials under static and dynamic loading, effect of density, forming direction, loading speed and size effect, predicting properties of cellular materials using micromechanical models, comparison between polymer and metallic foams behavior. Neil McCartney (Part 4: *Analytical Methods of Predicting Performance of Composite Materials*) presents: predicting properties of undamaged lamina, predicting properties of undamaged laminates, principles controlling fracture processes in composites, prediction of ply cracking in general symmetric laminates, prediction of ply cracking in laminates subject to loading that includes bend deformation, some other important issues. Ramesh Talreja (Part 5: *Analysis of Failure in Composite Structures*) discusses the following problems: clarification of strength, fracture and damage in heterogeneous solids, role of constraint in lamina failure, homogenization and representative volume element concepts, continuum damage and internal variables, damage modes, thermodynamics framework for composite response with damage, damage evolution, synergistic damage mechanics. During the course were presented 6 lectures by Tomasz Sadowski on damage and failure criteria for micromechanical modeling of multiphase polycrystalline composites and joints of different materials, multiscale approach in material modeling, deformation damage theory defects initiation and propagation, experimental verification of damage and failure criteria in complex materials, modeling of hybrid joints of structural parts degradation with application of cohesive zone model. The lectures were not published by health reasons. People interested in these lectures can contact directly Tomasz Sadowski ([sadowski.t@gmail.com](mailto:sadowski.t@gmail.com)).

Last but not least we have to thank Mrs. Dr.-Ing. Anna Girchenko. She unified all manuscripts, which were finally submitted as  $\text{\LaTeX}$  files.

Holm Altenbach and Tomasz Sadowski

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# Classical and Non-Classical Failure Criteria

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**Abstract** In material science or structural mechanics, failure is generally the loss of load carrying capacity of a material unit or structural element. This definition introduces the fact that failure can be examined in different scales (microscopic, mesoscopic, macroscopic). In addition, one has to distinguish among brittle, ductile, and intermediate material behavior. In structural mechanics, if the structural response is beyond the initiation of nonlinear material behavior, failure is related to the determination of the integrity of the structure.

In principle, failure criteria correspond to phenomenological material behavior modeling. They describe the occurrence of failure at different loading conditions. Although there are no physical principles on which failure criteria can be based on, there are still a lot of suggestions available in the literature. Similarly due to the lack of generally accepted failure criteria, the formulation is up to now under research.

The criteria based on the introduction of some empirical assumptions for critical values defined by the stress or strain state are denoted as the engineering one. In addition, characteristics of the stored strain energy or power can also be used. Based on some of these hypotheses and their consequences failure criteria will be discussed here.

## 1 Examples of Failure Behavior

As mentioned earlier, regarding failure behavior, one has to distinguish among absolute brittle, ideal ductile, and intermediate material behavior. The first one is related to fracture, while the second one to yield. The intermediate behavior includes the combined occurrence of the brittle and ductile failure and is related to the majority of materials. In addition to above failures, the variety of other types of failure will be briefly discussed.

## 1.1 Failure

Failure is related to the material and to the structure. In the first case the observation scale plays an important role hence various failure definitions exist and we have various evidences. The microscopic material failure is related to crack initiation, growth and propagation. As usually this approach can be applied to the fracturing of specimens and simple structures affected by well defined global loadings.

The most popular failure models are micro-mechanical models, which combine continuum mechanics and classical fracture mechanics (Besson et al., 2003). These models are based on the assumption that during inelastic deformation one should observe:

- microvoid nucleation and growth until local plastic neck or fracture of the intervoid matrix occurs, and
- coalescence of neighboring voids.

Finally, the macroscopic fracture results when macrocracks occurs. It is known that the first model of this type was proposed by Gurson (1977) and extended by Tvergaard and Needleman (Tvergaard, 1981, 1982; Needleman and Tvergaard, 1984; Tvergaard and Needleman, 1984; Needleman and Tvergaard, 1987). Another approach is based on continuum damage mechanics (CDM) and thermodynamics and was proposed by Rousselier (1981, 2001a,b).

Both models can be characterized as a modification of the von Mises yield potential (von Mises, 1913). The modification is based on the inclusion the damage behavior. The damage is represented by void volume fraction of cavities (porosity  $f$ ). In this sense this concept is a combination of the phenomenological classical approach with some micromechanical elements.

Macroscopic material failure is defined in terms of critical load, strain or energy storage. Li (2001) presented the following classification of macroscopic failure:

- stress or strain failure,
- energy type failure,
- damage failure, and
- empirical described failure.

With respect to this classification different failure criteria can be formulated.

Regarding material behavior models as usual five observation scales are considered Li (2001):

- the structural element scale,
- the macroscopic scale where engineering stresses and strains are defined,
- the mesoscale which is represented by a typical void, small crack or inclusion,

- the microscale (scale of crystallites or grains), and
- the atomic scale.

In modern theories the material behavior at one level is considered as a collective of its behavior at a sublevel which corresponds to the Curie-Neumann principle (Neumann, 1885; Paufler, 1986; Voigt, 1910). An efficient deformation and failure model should be consistent at every level. Below the attention will be paid only on phenomenological criteria on the macroscopic or structural level because they reflect a lot of effects of the material behavior in a relatively simple way in engineering applications.

Different types of "failure" can be identified in the engineering practice:

- transition from the elastic to plastic state,
- loss of stiffness,
- loss of fracture resistance at different scale levels,
- ultimate strength,
- fatigue, etc.

In this sense failure means that the material approaches a certain limit state.

It is not so easy to find a suitable definition of failure since the its formulation depends, for example, on the application field. WIKIPEDIA offers the following explanation<sup>1</sup>:

**Definition 1.1** (Failure - General statement). Failure is the state or condition of not meeting a desirable or intended objective, and may be viewed as the opposite of success.

The same source gives another explanation for engineering applications.

**Definition 1.2** (Failure - Engineering statement). A engineering failure analysis is focussed on the questions how a component or product fails in service or if failure occurs in manufacturing or during production processing.

Last but not least let us introduce a specific statement.

**Definition 1.3** (Failure in the Sense of the Course). Failure is a limit state of the material behavior and/or loss of carrying capacity of structural element or the whole structure.

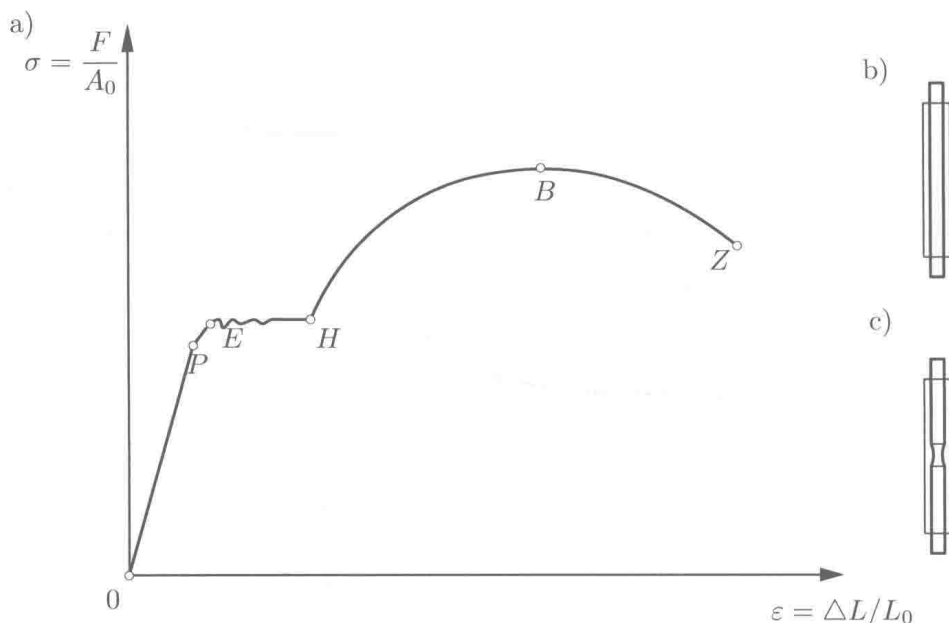
The last statement corresponds to the engineering practice. It means that the structure or elements of the structure are unable to fulfil all prescribed functions for some time. The limit state is defined with respect to the application case.

Such a statement can be related to the stress-strain diagram (Fig. 1). For example, if a structure can be exploited only in the elastic range the

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<sup>1</sup><http://en.wikipedia.org/wiki/Failure> (August 18<sup>th</sup>, 2014)





**Figure 1.** Stress-strain diagram for a ductile material: a) Engineering stresses  $\sigma$  vs. strains  $\varepsilon$  ( $P$  - proportional limit,  $E$  - elastic limit,  $H$  - beginning of hardening,  $B$  - ultimate strength,  $Z$  - rupture strength), b) Proportional elongation, c) Necking.

point  $P$  in the stress-strain diagram is the limit state. Other limit states are the transition from the elastic to the plastic range (point  $E$ ), the beginning of necking (point  $B$ ), the fracture (point  $Z$ ), etc. Note that all these limit cases are related to the diagram which is experimentally estimated in an one-dimensional tension experiment. But this is an exceptional loading case in mechanical or civil engineering.

As usual we have multi-axial loading cases resulting various values of the stress tensor. The limit state should be independent from the values of the stress tensor components. That means we need invariant limit estimates instead of the limit values for each tensorial component which vary with the change of the coordinate system. In addition we have to notice, that for different materials we obtain different experimental stress-strain curves (Fig. 2). In Fig. 2 the following symbols are used:  $\sigma_m$  is the ultimate stress (strength) and  $\sigma_y$  is the yield stress.  $x$  denotes fracture at the fracture stress  $\sigma_b$ .

In the classical theory the material behavior at tension and compression is assumed to be the same (different signs, but the absolute values of the