Market Design

A Linear Programming Approach to Auctions and Matching

MARTIN BICHLER

"For nearly two decades, Martin Bichler has been a world-wide thought leader in the ways to design those markets that resist traditional, over-simplified analyses. In this book, he develops and articulates a unique approach which, informed by economics and mathematical programming, promises to bring both of those disciplines back to their exciting early roots."

Paul R. Milgrom, Department of Economics, Stanford University

"The textbook by Martin Bichler takes a mathematical programming approach to market design. By linking pricing issues to duality theory, he provides a framework for analyzing the outcomes of complex mechanisms. This book is suitable for students in economics, operations research, and applied mathematics."

Karla Hoffman, Department of Systems Engineering and Operations Research, George Mason University

"*Market Design* provides a fresh introduction and unique perspective to this rapidly growing research area at the intersection of economics and operations research. Martin Bichler, who has made seminal contributions in market design himself, introduces a linear programming approach to solve – frequently computationally complex – allocation problems among decentralized and privately informed economic agents. This highly readable introduction is equally well suited for advanced undergraduate or introductory graduate courses in economics, operations research, and computer science."

Dirk Bergemann, Department of Economics, Yale University

"Computational integer programming techniques play the central role in running sourcing auctions, spectrum auctions, kidney exchanges, and many other markets. Martin Bichler's book covers the foundations of this topic in a broad range of markets in a highly understandable way. This is must-know information for any market designer."

Tuomas Sandholm, Department of Computer Science, Carnegie Mellon University

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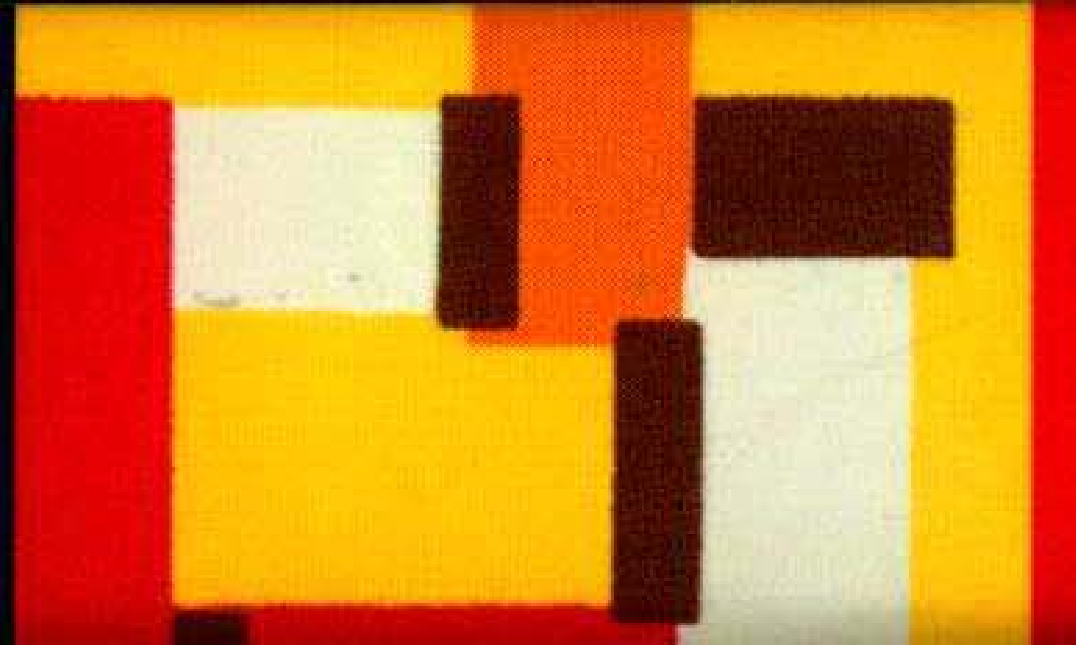

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The digital economy has led to many new services where supply is matched with demand for various types of goods and services. More and more people and organizations are now in a position to design market rules that are being implemented in software.

The design of markets is challenging as it is necessary to consider the strategic behavior of market participants, psychological factors, and computational problems in order to implement the objectives of a designer. Market models in economics have not lost their importance, but recent years have led to many new insights and principles for the design of markets which are beyond traditional economic theory. This book introduces the fundamentals of market design, an engineering field concerned with the design of real-world markets.

Martin Bichler is Professor of Informatics at the Technical University of Munich (TUM), and a faculty member at the TUM School of Management. He is known for his academic work on market design, and he has acted as a consultant for private and public organizations including regulators, telecoms, and procurement organizations. Projects in which he is involved include the design of auctions for industrial procurement, logistics, advertising, fishery access rights, and spectrum sales. His research addresses algorithmic, game-theoretical, and behavioral questions and has appeared in leading journals in computer science, economics, operations research, and management science. He is currently Editor of *Business and Information Systems Engineering* and serves on the editorial boards of several academic journals.

To my wife Claudia and my daughters Mona and Sonja

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1 Introduction

> Market design is a kind of economic engineering, utilizing laboratory research, game theory, algorithms, simulations, and more. Its challenges inspire us to rethink longstanding fundamentals of economic theory.
>
> Paul Milgrom, 2008

The digital economy has led to many new services where supply is matched with demand for various types of goods and services. While only a few years ago academics were mainly concerned with models describing markets, more and more people and organizations are now in a position to design market rules that are being implemented in software. The design of markets is challenging as it is necessary to consider the strategic behavior of market participants, psychological and cognitive factors, and computational problems in order to implement the objectives of a designer. Market models in economics have not lost their importance, but recent years have led to many new insights and principles for the design of markets which are beyond traditional economic theory. In this book, we study *market design*, an engineering field at the intersection of computer science, economics, information systems, and operations research concerned with the design of real-world markets.

Consider a transportation market with multiple lanes on a transportation network, one shipper and multiple carriers. The shipper has a set of truckloads to be shipped from different origins to various destinations. The carriers are available to meet the transportation demand, and they are invited by the shipper to submit sealed bids. Carriers typically have preferences for bundles of lanes on a route such that there are no empty lanes without a shipment, and they also submit respective bids on packages of lanes. This simple logistics procurement example leads to challenges for the shipper.

First, the shipper needs to determine an optimal allocation of bundle bids such that his costs are minimized. Cost minimization is a computationally hard optimization problem in this setting. Second, carriers want to maximize their payoff and it is not necessarily in their interest to reveal their costs truthfully. They might charge a high markup on some lanes where they expect no competition or they might not bid on other lanes where they expect high competition. However, strategic manipulation of this sort can lead to suboptimal allocations of lanes to carriers, high procurement costs for the shipper, and high bid preparation costs for the carriers, who would benefit from information about their competitors. Ideally, a shipper would have an economic mechanism where carriers have incentives to reveal their costs truthfully, and he or she can then determine the cost-minimal allocation optimally. In sales auctions the objectives are typically the maximization of auctioneer revenue or overall welfare. The latter is also referred to as (allocative) efficiency.

Overall, there are fundamental strategic and computational challenges in the design of such multi-object auctions, which are at the core of market design. The 2012 Nobel Memorial Prize in Economic Sciences for Alvin Roth and Lloyd Shapley honored research in this field. Market design uses economic theory, mathematical optimization, systems design, experiments, and empirical analysis to design market rules and institutions. Fundamentally, it asks how the design of the rules and regulations of a market affects the functioning and outcomes of that market. The study includes auction markets but also markets without money such as matching markets, which are used in the assignment of students to courses, in school choice programs, and in kidney exchanges.

This textbook focuses on the design and analysis of efficient multi-object market mechanisms.

1.1 Market Design and Mechanism Design

Market design has theoretical foundations in *mechanism design theory*, an analytical framework for thinking about what a given economic institution can achieve when the information necessary to make decisions is dispersed and privately held. The 2007 Nobel Memorial Prize in Economic Sciences to Leonid Hurwicz, Eric S. Maskin, and Roger B. Myerson was awarded in this field of economic theory. Mechanism design uses an axiomatic method, deriving results from a number of basic assumptions about utility functions or overall design goals of a mechanism. For example, mechanism design has been used to characterize the utility functions of market participants and mechanisms for which the truthful revelation of preferences is an equilibrium, i.e., a situation from which no participant wants to deviate (aka incentive-compatible mechanisms). The mechanism design literature shows that environments which allow for welfare maximization such that participants cannot make a loss and have strong incentives to bid truthfully are limited and require strong assumptions. The celebrated Vickrey–Clarke–Groves mechanism provides dominant strategies for agents to reveal valuations truthfully if they have independent and private values and they can maximize their payoff. If these assumptions are relaxed, so that bidders can have private budget constraints or valuations that are not independent, such positive results no longer exist (unless preferences can be characterized by a single parameter only). Overall, environments where truthful bidding satisfies a strong equilibrium solution concept are quite restricted. Often neither the bidders' preferences in real markets satisfy the assumptions necessary for truthful mechanisms nor is it possible to implement the required mechanisms for practical reasons. Actually, the Vickrey–Clarke–Groves mechanism is rarely used in the field. Still, it is important to understand the assumptions and mechanisms which would make truthful bidding an equilibrium bidding strategy.

Market design starts from the requirements in the field, eliciting the preferences and financial constraints of participants (which might be different from pure payoff maximization), information about competitors available to participants, objectives, and constraints on the allocation problem and on the award process. In many cases these requirements differ from those assumed in mechanism design theory, demanding the development of new models that capture the specifics of a certain type of market. Examples are models of markets for procurement and logistics, for spectrum sales, or for display advertising. Often the specifics of these markets might not allow for mechanisms which are incentive-compatible according to the established equilibrium solution concepts. So, market design takes the market environment as given and derives designs that satisfy some design goals (such as stability or budget balance of the outcome), while relaxing others. In other words, it analyzes tradeoffs and aims for "satisficing" solutions (Simon, 1996) to real types of market design problems rather than finding sets of assumptions that allow for "optimal" designs according to some design desiderata.

Market design complements mechanism design in the level of detail that is specified and analyzed. For example, in mechanism design valuations are typically described as an abstract function of the objects to be allocated, while the parametric forms of the valuation or cost functions, of the utility functions, and of the corresponding bid languages play a significant role in market design. Moreover, market designs almost always need to consider allocation constraints such as restrictions on the number of winners or the quantity awarded to individual bidders or groups of bidders. Therefore market design typically starts out as a mathematical programming task concerned with the design of an appropriate objective function, of appropriate constraints for the overall allocation, and of activity rules and with the definition of a bid language that lets bidders express their preferences in a succinct way. Milgrom (2017) provides an excellent and up-to-date view on market design and how it relates to economic theory.

1.2 Market Design and Mathematical Optimization

One thesis of this book is that *mathematical optimization*, in particular linear and integer linear programming, plays a central role in the design and analysis of multi-object markets and also provides a central foundation for theoretical models of multi-object markets. Many techniques and models introduced in this book use the theory of linear and integer linear programming in one way or another. Compared with algorithms designed for specific problems, there is an amazing versatility in linear and integer linear programming, which makes these techniques very important for market design. Algorithms for the solution of integer linear programs have seen substantial progress in the past two decades, allowing for totally new types of markets in different domains. Before that, in 1975 Leonid Kantorovich and Tjalling Koopmans had already received the Nobel Memorial Prize in Economic Sciences for their work on linear programming and optimal resource allocation. However, the advances in recent years have been staggering and have made many new markets possible that were not considered tractable just a few years ago. The subtitle of this book is reminiscent of the seminal book *Mechanism Design: A Linear Programming Approach* by Vohra (2011), who emphasized the role of linear programming in mechanism design theory.

However, market design is more than mathematical programming, as it takes into consideration bidding strategies and human behavior. Ignoring bidding strategies and different types of manipulation in market-based resource allocation is like optimizing a problem with the wrong parameters. As indicated earlier, strong game-theoretical solution concepts such as dominant-strategy or ex post incentive compatibility are desirable but they might not always be attainable, as a number of impossibility results in mechanism design show. These properties rest on assumptions about bidders' utility functions, which are often not given in the field. Still, in many markets it is possible to devise market rules for which manipulation becomes hard, given the uncertainties about other bidders and their valuations.

Overall, market design aims at design principles and rules for market institutions which are *robust against strategic manipulation and allow bidders to express their preferences*, so that the designer can aim for good or even optimal allocations. This is similar to network security, where designers aim for secure network protocols that are hard to tamper with, knowing that there is no absolute security. Note that "optimality" often refers to social welfare maximization, but the market designer might have different objectives or policy goals. In this sense, market design extends mathematical programming to situations with multiple decision makers. It is a *design science* in the sense of Herb Simon who received both the Nobel Memorial Prize in Economic Sciences (1978) and the Turing Award in Computer Science (1975). Simon's book *The Sciences of the Artificial* (Simon, 1996) motivated the development of systematic and formalized design methodologies relevant to many design disciplines, for example architecture, engineering, urban planning, medicine, computer science, and management science. Market design is therefore a very suitable name for the study of principles and methods in designing markets in the spirit of design science.

1.3 Outline of the Book

This textbook is intended to provide a one-semester course on market design. I am primarily targeting students with a background in computer science, information systems, mathematics, and management science. Hence, I will first introduce in Part I necessary concepts from game theory and mechanism design, as these students typically have not had the respective introductory courses. Parts II and III cover material which is more recent and is often not covered in microeconomics.

One prerequisite for this book is a familiarity with *linear and integer linear programming* and an introductory course in calculus and probability theory. There are many introductory textbooks on these subjects which would provide an excellent start for the topics discussed throughout this book. The appendices summarize important results from mathematical optimization and should serve as a convenient reference for the reader and a brief introduction for those who have not studied the respective courses.

As outlined earlier, this book focuses on the allocation of multiple objects with distributed decision makers. While single-object auctions are fairly well understood, multi-object auctions provide many more design options and they are more challenging to design and analyze. The goal of this book is to introduce important models and principles from game theory, mechanism design, and single-object auction theory for which there are several excellent textbooks. After basic concepts have been introduced, the book focuses on multi-object markets and their properties.

This leads to the three parts of the book: *Part I (Microeconomic Fundamentals)* discusses basic concepts from game theory, mechanism design, and single-object auction theory. The goal of this part is not to discuss each of these topics in breadth but to introduce the terminology and theory required for Parts II and III. The above topics are typically not taught in computer science and management science, and this makes it hard for students from these fields to understand the literature in market design. *Part II (Multi-Object Auction Design)* introduces the more recent theory and specific designs for multi-object markets. *Part III (Approximation and Matching Markets)* analyzes approximation mechanisms which have been designed for markets where the designer cannot hope to solve the allocation problem optimally. Then the design of matching markets where monetary transfers are not possible is discussed. Approximation and randomization play an important role in the recent matching literature. *Part IV (Appendices: Mathematical Optimization)* summarizes the main results from linear and integer linear optimization, on which I draw in Parts II and III.

There is always a tradeoff between the breadth and the depth of a textbook, and it is a matter of choice how much space is devoted to each single topic. If all the various literature streams in computer science, economics, and operations research were covered then a book on market design could easily cover thousands of pages, and it would clearly be beyond a one-semester course. This book provides a fair amount of detail in Part II and also in Part III, but Parts I and IV are limited to a necessary minimum for readers who have not taken the respective introductory courses. Selected references to the original literature are included in the text for those interested in learning more about a specific topic and in reading the original literature.

1.3.1 Part I Microeconomic Fundamentals

Chapter 2 introduces basic game-theoretical notions and solution concepts relevant for market design problems. Solution concepts for normal-form, extensive-form, and Bayesian games are introduced. There are many excellent textbooks on game theory. For example, Shoham and Leyton-Brown (2011) provides a comprehensive introduction for computer scientists, while I cover only a subset of the topics relevant for later chapters.

In Chapter 3 I discuss mechanism design theory, also known as inverse game theory. While the rules of a game are given in game theory, mechanism design tries to design rules such that certain goals and solution concepts are achieved. Market design problems can be considered as games where participants should have incentives to reveal their preferences for objects truthfully. If participants were truthful then the market designer would only need to solve an optimal allocation problem. The first section shows that aggregating general and unrestricted preferences is hard and that simple truthful mechanisms for general preferences are impossible. Then preferences are restricted to